

welcome students in previous class we have discussed about slope of a line and so many things now we continued this

so in slope of line we discuss what is slope of line how can you find slope of a line and if slope of a line is zero it means line is parallel to x axis slope of a line is not defined it means parallel to y axis if slope of a line is equal then what will happen if two lines have equal slopes then what will happen

so today we discussed about the slope of perpendicular and parallel lines here this line  $l_1$  and  $l_2$  are two parallel lines and this line makes angle  $\theta_1$  and  $\theta_2$  with the positive direction of x axis since line  $l_1$  is parallel to  $l_2$  this implies  $\theta_1$  is equal to  $\theta_2$  why because these two angles have corresponding angles if  $\theta_1$  equal to  $\theta_2$  it means  $\tan \theta_1$  equal to  $\tan \theta_2$  this implies  $\tan \theta_1$  is what slope of line  $l_1$  and  $\tan \theta_2$  is slope of line  $l_2$

so if two lines are parallel then their slopes are equal

so if it means  $l_1$  parallel to  $l_2$  implies  $m_1$  equal to  $m_2$  what does it mean if line is parallel then its slope is equal and if slope is equal then lines are parallel what will happen when two lines are perpendicular say this is line  $l_1$  and this is line  $l_2$  this is line  $l_1$  this is line  $l_2$  here  $l_1$  perpendicular to  $l_2$   $l_1$  perpendicular to  $l_2$  it means this angle is ninety degree and we draw a line this dotted line which is parallel to what which is parallel to x axis

so say this line  $l_1$  max angle  $\theta_1$  with x axis and this line  $l_2$  max  $\theta_2$  with x axis  $\theta_n$  is  $\theta_1$  is equal to what  $\theta_1$  is equal to we can say ninety degree plus  $\theta_2$   $\theta_1$  is equal to ninety degree plus  $\theta_2$  this implies  $\tan \theta_1$  is equal to  $\tan (90^\circ + \theta_2)$

so this implies  $m_1$  is equal to  $-\cot \theta_2$  is equal to  $-\frac{1}{\tan \theta_2}$   $\tan \theta_2$  is equal to  $-\frac{1}{m_2}$  this implies  $m_1$  is equal to  $-\frac{1}{m_2}$  this implies  $m_1 m_2$  is equal to  $-1$  this will be the condition for two perpendicular lines when two perpendicular line  $l_1$   $l_2$  are perpendicular then the product of their slopes is equal to  $-1$  or you can say when product of slopes is equal to  $-1$  then that two lines are perpendicular

so in this way we can use the concept of slope of a line decide whether line is parallel or perpendicular lines now we see another example

so that the line joining given two points  $(-3, -5)$  and  $(1, 3)$  is parallel to the line joining  $(-1, 7)$  and  $(3, 0)$  and second perpendicular to the line joining  $(5, 4)$  and  $(2, 0)$

so what we have to do here first of all find the slope of this line which passing through these two point  $(-3, -5)$  and  $(1, 3)$

so we just name these two point say this is  $p$   $(-3, -5)$  and  $q$   $(1, 3)$

so slope of  $p$   $q$  is equal to what slope of  $p$   $q$  is  $\frac{y_2 - y_1}{x_2 - x_1}$  means  $\frac{3 - (-5)}{1 - (-3)}$  plus plus 3 and minus 5 minus 2 means 1 plus 3 and minus 5 minus 2 is 4 by minus seven now we have to show that this line  $p$   $q$  is parallel to this line which passing through this two point  $(-1, 7)$  and  $(3, 0)$  again name these two points

so ah say  $a$  is  $(-1, 7)$  and  $b$   $(3, 0)$

so slope of  $a$   $b$  is equal to  $\frac{0 - 7}{3 - (-1)}$  means three plus one by zero minus seven

so four by minus seven

so we see that here the slope of  $p$   $q$  is four by minus seven and slope of  $a$   $b$  is also four by minus seven

so slope of  $p$   $q$  equal to slope of  $a$   $b$  equal to  $\frac{4}{-7}$

so  $p$   $q$  parallel to  $a$   $b$  now we have to show that the line passing through these

two point four five and zero minus two is perpendicular to p q again name it c four five and d zero minus two

so slope of c d is equal to again  $y_2 - y_1$  means  $-2 - 5$  by  $0 - 4$

so  $-7$  by  $-4$  it means  $7/4$  now slope of p q that we have already find slope of p q is equal to  $4$  by  $-7$  say this is  $m_1$  and this is  $m_2$  now find  $m_1 \times m_2$

so  $7/4$  into  $4$  by  $-7$  is equal to  $-1$

so product of the slope of c d and p q is equal to  $-1$  this implies c d perpendicular to p q as we have already discussed it

so in this way we can see the application of slope of a line to find whether line is parallel or line is perpendicular now how can you find angle between two lines l one and l two are two lines which makes angle  $\theta_1$  and  $\theta_2$  with the x axis

so this is x because this if this line is this line l one makes angle  $\theta_1$  then this angle is also  $\theta_1$  and this line makes angle  $\theta_2$  then this angle is also  $\theta_2$  because this two lines are parallel lines

so just suppose this is x axis we have to find the acute angle between these two lines because when two line intersect each other then it will max acute angle and obtuse angle both if that two lines are not perpendicular to each other

so we just find what is the acute angle  $\theta$  between these two line l one and l two

so let angle with x axis by the line lines l one and l two are  $\theta_1$  and  $\theta_2$  respectively

so slope of l one that is  $m_1$  is equal to  $\tan \theta_1$  and slope of l two that is  $m_2$  is equal to  $\tan \theta_2$  in the figure we see  $\theta$  is equal to  $\theta_2 - \theta_1$

so  $\tan \theta$  is equal to  $\tan \theta_2 - \theta_1$  by trigonometry  $\tan \theta_2 - \tan \theta_1$  by  $1 + \tan \theta_2 \tan \theta_1$

so  $\tan \theta$  is equal to  $m_2 - m_1$  by  $1 + m_2 m_1$  since the sign of this is plus

so the acute angle if sin also is minus this is obtuse angle

so in this way we can find

so finally we can say the angle between any two line when slopes is known is  $\tan^{-1} \left( \frac{m_2 - m_1}{1 + m_1 m_2} \right)$

so when you open this mod you will get plus minus sign

so this is plus minus plus minus  $m_2 - m_1$  by  $1 + m_1 m_2$  plus is four plus means acute angle between l one and l two and minus means obtuse angle between l one and l two in this way we can find the angle between any two lines again we have problem find the angle between the straight line whose slope are  $-7/3$  and  $5/2$  means when slope is known

so  $\tan \theta$  given  $m_1$  equal to  $-7/3$  and  $m_2$  equal to  $5/2$  by two

so let  $\theta$  be the angle between the lines

so  $\tan \theta$  is equal to  $m_2 - m_1$  by  $1 + m_1 m_2$  mod mod  $5/2 - 7/3$  by  $1 + (-7/3)(5/2)$  plus it says  $15/6 - 35/6$

by  $6$  and this is  $6$  and  $-35$

so  $29/6$

so  $-29/6$  by six

so  $-1$  mod of  $-1$  line will make

so situation is like this these two lines are like this

so if you take plus one it means these two line makes forty five degree or if

you take minus one then these two lines makes 135 degree

so by concept of the slope of line we can also find exact angle between these two lines another problem if the angle between two line is  $\pi/4$  and the slope of one the line is  $1/2$  find the slope of other line

so given here  $\theta$  is equal to  $\pi/4$  and say  $m_1$  equal to  $1/2$  then  $m_2$  equal to what question is like this

so we know the formula  $\tan \theta$  is equal to  $\pi/4$

so we take positive sign

so  $m_1 m_2 - 1 = 1 + m_1 m_2$

so  $\tan \pi/4$  is equal to  $m_2 - 1/2$  by  $1 + m_2/2$  and  $\tan \pi/4$  equal to one

so  $2m_2 - 1$  by  $2 + m_2$  this implies  $2m_2 - 1 = 2 + m_2$  so  $m_2 = 3$

so  $m_2$  is equal to three

so slope of second line is three

now equation of straight line before you start with state line equation of straight line we must have the idea about slope of a line

so we have already discussed slope of a line passing through two points and what is slope of a line now equation of line means equation of line is an equation in  $x, y$  which is satisfied by every point of the line

so this is a simple definition of equation of a straight line now the very basic equation of a line parallel to  $x$  axis

so here we have coordinate system this is  $x$  axis is  $y$  axis and this is a line  $l$  which is parallel to  $x$  axis then what will the equation of this line  $l$

so equation of line parallel to  $x$  axis means the locus of this point  $P(x, y)$  which satisfy the condition that the distance between this line with  $x$  axis is always constant that is  $b$

so distance if you take distance from here you will get  $b$  distance you will get  $b$

so the distance between these two line is always constant and that constant is say  $b$  here

so equation of line parallel to  $x$  axis is  $y = b$  because its  $y$  coordinate is fixed it never change whatever the value of  $x$  we just take here value of  $y$  is  $b$  it may be  $1, 2, 3, 4, \dots$  or minus  $2, -3, -4, \dots$   $x$  sector means we can take example

so  $y = 1, y = -2, y = 13, y = 5$  all these are equation of line parallel to  $x$  axis similarly we must have a line which is parallel to  $y$  axis

so here situation again  $x$  axis  $y$  axis and this line  $l$  is parallel to  $y$  axis

so again this is locus of point  $P(x, y)$  which moves in such way the distance between these this line with  $y$  axis is always constant that is this is also a this is also  $a$

so this distance is always fixed it means the value of this  $x$  is fixed here

so equation of  $l$  parallel to  $y$  axis is  $x = a$  this is a condition

so this condition will give the equation of line  $l$  means for example say  $x = -1, x = 7, x = -1/2$  all these are the example of equation of line parallel to  $y$  axis again we have example excel problem find the equation of the line passing through  $(2, 3)$  and is parallel to  $x$  axis and parallel to  $y$  axis

so here is situation we have to find the equation of these two lines these two lines say this is  $l_1$  and say this is  $l_2$

so now first equation of line  $l_1$  one

so when we see this line equation of line  $l_1$  one this is  $x$  and this is line  $l_1$  one

so this point is what the distance between  $x$  axis and this line  $l_1$  one is what three since this line  $l_1$  one is passing through say any point two three and parallel to  $x$  axis it means this the value of  $y$  coordinate is always fixed that is equal to three

so here  $y$  equal to three this value never change for this line  $l_1$  one

so equation of  $l_1$  one is  $y$  equal to three similarly if you take another line  $l_2$  say this is  $l_2$  and the distance between this  $x$  axis and  $y$  axis is again fixed and this distance is the coordinate of  $y$  axis is two three

so this is two

so it means the value of the intercept made by this line  $l_2$  on  $x$  axis is  $x$  equal to  $x$  equal to two

so equation of line  $l_2$  is  $x$  equal to two whenever line is parallel to  $x$  axis or parallel to  $y$  axis we have to just find the distance between that line and axis will give the equation of line parallel to  $x$  axis or parallel to  $y$  axis this is very important equation of straight line in various standard forms or now we shall discuss very important part of this chapter equation of a straight line or a straight line chapter

so equation of a straight line in various standard form

so we have different forms of equation of a straight line

so first form is point slope form point slope form means when the information about this line that this line passing through a particular point  $p(x_1, y_1)$  and its slope is also given its slope is also given

so let line  $l$  passing through  $p(x_1, y_1)$  with slope  $m$  let us take an arbitrary point  $q(x, y)$  on the line then and draw a right angle triangle that is  $pqr$

so in this  $pqr$  we see this  $pr = x - x_1$  and this  $qr = y - y_1$  and slope of line means we know that the tangent of this angle  $\theta$

so  $m$  is what  $m$  is equal to  $\tan \theta$  means  $\tan \theta$  means what  $qr$  by  $pr$  means  $y - y_1$  by  $x - x_1$  this implies  $y - y_1$  is equal to  $m(x - x_1)$  this will be the equation of line in point slope form means when two information is given or two information is known about a line that is line is passing through some given points and the slope is also known

so we can find the equation of line by using this equation  $y - y_1 = m(x - x_1)$  now another important form that is slope intercept form in this form as clear from the heading slope intercept means slope of line is again known

so let line  $l$  having slope  $m$  and intercept means this line makes some  $y$  intercept or at what point this line intersect  $y$  axis

so intercept means  $y$  intercept and  $y$  intercept  $y$  intercept  $c$

so  $y$  intercept  $c$  means what it means this line passing through a point  $q$  which coordinate is zero  $c$  again you just look back in previous form point slope form

so again we have slope is known that is  $m$  and one point  $q(0, c)$  is known

so we just use that concept

so this implies line  $l$  passing through  $q(0, c)$

so equation of line equation of line  $y - c = m(x - 0)$

so this implies  $y - c = mx$  or we can say  $y$  is equal to  $mx + c$  this is very important form  $y$  is equal to  $mx + c$

so whenever we have to find the slope of any line we have to use this type of equation we have to reduce any equation in this form we will get the coefficient of  $x$  will give the slope of a line

so very important we have two or three observation on this equation that is for

$y$  equal to  $m x$  plus  $c$  say when  $m$  equal to zero means slope is  $m$  is not equal to  $0$  and  $c$  equal to  $0$  it means in that situation this line will reduce to  $y$  equal to  $m x$  and  $y$  equal to  $m x$  this is a line passing through origin because  $c$  is equal to zero here

so this line does not make any  $y$  intercept

so this line passing through this implies line passing through origin

so whenever you find any line in this form  $y$  equal to  $m x$  you can easily say this line passing through origin and its slope is what its slope is  $m$  it will give the angle formed by line with  $x$  axis now when both  $y$   $m$  and  $c$  are  $0$  when both  $m$  and  $c$   $j$  it means  $y$  equal to  $0$

so  $y$  equal to  $0$  is nothing but the equation of  $x$  axis this is means line coincide with  $x$  axis line coincides with  $x$  axis now we have third possibilities that is when  $m$  equal to zero and  $c$  is not equal to zero

so when  $m$  equal to  $0$  this implies  $y$  is equal to  $c$  and  $y$  equal to  $c$  is nothing but the line parallel to  $x$  axis this is line parallel to  $x$  axis

so that's why this form  $y$  equal to  $m x$  plus  $c$  is very important form

so on the basis of this three observation we can say the different situation of line now we have another form that is two point form two point form means when line passing through two given points say  $p x$  one  $y$  one and  $q x$  two  $y$  two

so since this line  $l$  is passing through two points  $p x$  one  $y$  one and  $q x$  two  $y$  two so first of all find slope because we know that whenever you have to find equation of line first of all you target what is the slope of line

so slope of line  $m$  obviously  $m$  is what slope of line

so slope of line  $m$  is equal to  $y$  two minus  $y$  one by  $x$  two minus  $x$  one now when we have slope of a line  $y$  two minus  $y$  one  $x$  two minus  $x$  one say we just take an  $r$   $b$  tree point say this arbitrary point is  $a x$   $y$  because we have to find equation with respect to any arbitrary points

so  $a x$  one  $y$  one now you have a choice either you should take  $p$  or  $q$  because we have already slope is known

so now let line passing through passing through  $p x$  one  $y$  one and slope  $m$  then equation of line by point slope form but this is  $y$  minus  $y$  one equal to  $m$   $x$  minus  $x$  one  $m$  is what  $y$  two minus  $y$  one  $x$  two minus  $x$  one  $x$  minus  $x$  one this implies  $y$  minus  $y$  one by  $x$  minus  $x$  one is equal to  $y$  two minus  $y$  one by  $x$  two minus  $x$  one this is the equation of line passing through two point or you can say two point form another form that is intercept form this is very important form again in this form this line  $l$  which makes  $x$  and  $y$  intercept both only then we can find the equation of line in this form this line  $l$  makes  $x$  intercept  $a$  and  $y$  intercept  $b$

so  $a$  intercept means this line passing through  $a$   $0$  and this line passing through  $0$   $b$  it means this line passing through 2 points

so let line  $l$   $x$  intercept and  $y$  intercept  $a$  and  $b$  respectively that is line passing through  $a$   $0$  and  $0$   $b$  it means we just discuss how can you find the equation of line passing through two points again we have two point  $a$  zero and  $b$  zero  $b$

so first of all find the slope of line

so slope of  $a$   $b$  that is  $m$  is equal to  $y$  two minus  $y$  one

so  $b$  minus zero  $b$  minus  $0$  by  $x$  two minus  $x$  one

so  $0$  minus  $a$  means minus  $b$  by  $a$  minus  $b$   $y$   $a$

so equation of line through  $a$   $0$   $a$   $0$  and slope equal to minus  $b$  by  $a$  is  $y$  minus zero minus  $b$  by  $a$   $x$  minus  $a$

so this is what this is  $y$  by  $b$  equal to minus  $x$  by  $a$  and minus minus  $a$  by equal to one

so this is this implies  $x$  by  $a$  plus  $y$  by  $b$  equal to one

so this is the equation of line in intercept form

so in this way we can find equation of line when intercept means x intercept and y intercept is given now this is very important for perpendicular normal form

so some different types of information about line is given we have to find the equation of this line l and this way is a perpendicular or normal you can say normal which makes angle alpha with the x axis means in formation about the normal to the line is given length of normal is given and its angle formed with x axis is given and then we have to find the equation of line l

so this is very peculiar type of information is given here let us see now draw perpendicular from this a to x axis say this is a m

so this m is perpendicular to x axis now this o makes angle alpha and this o a length of this y is p

so in right angle triangle o a m we have two information that is length of hypotenuse is given and one acute angle is given

so it is more than enough to find the coordinate of this point a

so we by using these two we just find o m equal to p cos alpha and a m is equal to p sin alpha it means the coordinate of this point is p cos alpha and p sine alpha now this line o a max angle alpha with the x axis

so slope of a slope of o a is equal to  $\tan \alpha$  since o a is perpendicular to l already given this implies slope of l that is m is equal to minus 1 by  $\tan \alpha$  because you know that the product of perpendicular slope of perpendicular line is minus one

so slope of this line l is minus one by  $\tan \alpha$  that is minus  $\cot \alpha$  now you see we have two information about this line coordinate of one point is known and slope of line l is known

so equation of l passing through passing through a p cos alpha p sine alpha with slope m equal to minus  $\cot \alpha$  is

so we have these two information about the line l it means y minus we use just this concept y minus y one equal to m x minus means point slope form by using point slope form this implies y minus y one means p sine alpha and m x minus p cos alpha

so y minus p sine alpha and m means what that means  $\cot \alpha$

so we can write it as cos alpha by minus sin alpha cos alpha by because m is equal to minus  $\cot \alpha$

so we can write m as a minus cos alpha by sine alpha now cross multiply it

so y sin alpha minus p sin square alpha minus x cos alpha and plus p cos square alpha this implies x cos alpha plus y sin alpha is equal to p sin square alpha plus p cos square alpha

so p sine square alpha plus cos square alpha equal to p

so finally we will get x cos alpha plus y sin alpha equal to p this is the equation of line in normal form or you can say perpendicular form we just give another form that is general form general for means any equation of the form a x plus b y plus c equal to zero here a b c all are real numbers but one most important condition that is a and b are not simultaneously 0 this is very important condition either a may be 0 or b may be 0 but note both a and b equal to 0 at same time

so this is this makes meaningless

so this is most important condition that is a and b not both equal to zero and a b and c belongs to R

so if a x plus b y plus c equal to 0 satisfy these two condition then only we can say this represent a straight line now we just see one or two more most important things that the first say when a equal to 0 then what we will see when a equal to 0 this implies b y plus c equal to zero it means y is equal to minus c by b

so when  $a$  equal to zero means coefficient of  $x$  is  $0$  then this will give the line parallel to  $x$  axis

so simply you say when the coefficient of  $x$  is  $0$  then we will have line parallel to  $x$  axis similarly when  $b$  equal to  $0$  then  $ax + c$  equal to  $0$  this implies  $x$  is equal to  $-\frac{c}{a}$

so this will gives when  $b$  equal to  $0$  implies will get equation of line parallel to  $y$  axis now third when means  $a$  is not equal to zero and you just put and  $b$  is not equal to zero  $b$  is equal to zero and  $a$  is not equal to zero and third is when both not equal to zero when  $a$  is not equal to zero and  $b$  is not equal to zero when both is not equal to zero then we have  $ax + by + c$  equal to zero or we can say  $by$  is equal to  $-ax - c$  are  $y$  is equal to  $-\frac{ax + c}{b}$

so when both is not equal to zero then this equation will gives the slope of a line and at what point this intersect  $y$  axis

so two very important information can be drawn when both  $a$  is not eq and  $b$  is not equal to zero and when  $a$  equal to zero will get line parallel to  $x$  axis when  $b$  is not equal to  $0$  will get line parallel to  $y$  axis

so in this way we see the importance of this general form reduce this equation  $ax + by + c$  equal to zero in various form that we will discuss some problems and more things in next session okay thank you