

welcome students to the third lecture on infinite series in the last lecture i was talking about binomial expansions of the form one minus x whole to the power minus n n is an integer or one minus x to the power p by q which is a rational number in particular we were looking at one minus x whole to the power minus half and we have seen the coefficients of its expansion for different powers of x let us take a simpler problem what is the series expansion for one minus x to the power half

so this is a rational number let us see how to go about it we have one minus x whole to the power half into 1 minus x whole to the power half is equal to 1 minus x therefore if we write one minus x whole to the power half is equal to a zero plus a one x plus a two x square etcetera etcetera then by multiplying it with itself we should get one minus x

so let us try that therefore a 0 plus a 1 x plus a 2 x square plus multiplied by a 0 plus a 1 x plus a two x square is equal to one minus x therefore coeff of coefficient of x to the power zero is equal to a zero square is equal to one again taking the positive root a zero is equal to one coefficient of x is equal to a zero a one plus a one a zero is equal to two a zero a one which is is equal to minus one therefore 2 a 1 is equal to minus 1 therefore a 1 is equal to minus half coefficient of x square is equal to a 0 a 2 plus a 1 square plus a 2 a 0 is equal to 0 y 0 because there is no x square in 1 minus x therefore a zero is equal to one we have already got therefore two a two plus a one square is equal to zero or two a two plus a one is equal to minus half

so u square is equal to one by four is equal to zero therefore a two is equal to minus one by eight or we can write 1 minus x to the power half is equal to square root of 1 minus x as 1 minus half x minus 1 by 8 x square plus higher powers of x we are going only up to the second degree of x but if x is small we often ignore higher powers ok example what is consider root over seventeen we can write in a similar way we can find root over one plus x say root over one plus x is equal to a zero plus a one x plus a two x square etcetera therefore in a similar way we can write that a zero plus a one x plus a two x square into a zero plus a one x plus a two x square plus is equal to one plus x now by multiplying them and equating the powers of x we find a 0 square is equal to 1 or a zero is equal to one by taking positive value a zero a one plus a 1 a 0 is equal to 1 or 2 a 0 a 1 is equal to 1 therefore a one is equal to half a zero a two plus a one square plus a zero a two is equal to 0 or 2 a 2 plus a 1 square is equal to 0 therefore 2 a 2 is equal to minus a one square is equal to minus one by four therefore a two is equal to minus one by eight therefore we get square root of one plus x is equal to one plus half x minus 1 by 8 x square plus other terms which we are ignoring for the time being let us apply this find root over 17 suppose we need to find out the square root of 17 we can write it as but if we write it like that we are making a mistake what is the mistake because this x in 1 plus x whole to the power some p in the expansion modulus of x has to be less than one but if we write it like one plus sixteen whole to the power half then we are making a mistake

so we write it in a different way we write it as 16 power half into 1 plus 1 by 16 whole to the power half right

so that gives us a term 1 by 16 whose mod value is less than 1 therefore to obtain root 17 we write it as 16 to the power half into 1 plus 1 upon 16 whole to the power half is equal to 4 into 1 plus 1 by 16 whole to the power half and now let us expand it using binomial we have found out that in the expansion of one plus x whole to the power half we have found out this is is equal to one plus half x minus one by eight x square plus other terms which we have ignored putting x is equal to 1 by 16 we get this is is equal to 1 plus 1 by 32 minus 1 by 8 into 16 square therefore root 17 is equal to 4 into 1 plus 1 by 32 minus 1

by 8 into 16 square

so  $4^2 = 16$   $4^4 = 16^2 = 256$   $4^8 = 256^2 = 65536$   $4^{16} = 65536^2 = 4294967296$  therefore square root of 17 is equal to 4.

125 minus 0.

0019 is equal to four point one two three one now if we consider root 17 i suggest all of you use your calculator to compute root 17 and you see it comes so close to four point one two three one

so this is a verification that this expansion works in a correct manner therefore the final expansion is  $1 + \binom{p}{q} x^q + \binom{p}{2q} x^{2q} + \dots$  we will write in a very similar way as we did with respect to negative integral index this is is equal to  $1 + \binom{p}{q} x^q + \binom{p}{2q} x^{2q} + \dots$  upon factorial 2 to the power  $x^2 + \binom{p}{3q} x^{3q} + \dots$  irrespective of when we have a binomial expansion of the form  $1 + x$  whole to the power sum index irrespective of whether it is positive integral negative integral or a rational we can write it in the same way and only thing that we have to remember that for positive integral we can write it as  $n$  choose  $r$  or  $n C r$  that we cannot do when we have a negative integral index or a rational index like  $p$  by  $q$  but we can rewrite it in the following form and we can get the series expansion for any binomial expression with power as an integer or a rational number again this is not a proof right what we have done

so far is not a proof we have just verified certain results and the assumption as i said in the very first class is y stars approximation theorem which suggest that every continuous function in a closed interval can be approximated as can be approximated as closely as possible by a polynomial function therefore what we have done given a continuous function we have tried to compute the coefficients of first few powers of  $x$  and thereby we try to find the expansion of one plus  $x$  whole to the power  $k$  where  $k$  can be negative integral or rational let us see how to obtain the coefficients how to obtain the coefficients we make such an assumption that given a function  $f$  of  $x$  it is possible to have such an such a polynomial expansion about a point  $a$

so let us write  $f(x)$  is equal to  $a_0 + a_1(x-a) + a_2(x-a)^2 + a_3(x-a)^3 + \dots$  the advantage of polynomial is that if it is of degree  $n$  it can be differentiated  $n$  plus one times and if we take an infinite polynomial then we can differentiate it in finite number of times

so with this assumption we shall try to find out the polynomial expansion for  $f(x)$   $f(x)$  is equal to we have assumed as  $a_0 + a_1(x-a) + a_2(x-a)^2 + a_3(x-a)^3 + \dots$  therefore  $f(a)$  is equal to  $a_0$  because all other terms will become 0 therefore  $a_0$  is equal to  $f(a)$  thus the constant term will come the functional value at a point  $a$  about which we are expanding the polynomial what is the first derivative of  $a$

so i am writing it as  $f'(x)$  which means i am differentiating  $f$  with respect to  $x$  once this is is equal to  $a_1 + 2a_2(x-a) + 3a_3(x-a)^2 + 4a_4(x-a)^3 + \dots$  like that therefore what is the second derivative second derivative of  $x$  is two  $a_2 + 6a_3(x-a) + 12a_4(x-a)^2 + \dots$  etcetera therefore if two at  $a$  is equal to twice  $a_2$  or  $a_2$  is equal to  $f''(a)/2$  in a similar way the third derivative  $f'''(x)$  is equal to  $6a_3 + 24a_4(x-a) + \dots$  plus terms with higher powers of  $x-a$  therefore if three at  $a$  is equal to three factorial times  $a_3$

three therefore a three is equal to  $f^3$  a upon three factorial in a similar way we can see that if i differentiate it once more i will get the term  $f^4$  at  $x$  is equal to  $4$  into  $3$  into  $2$  into  $1$  plus powers of  $x$  minus  $a$  therefore a four is equal to fourth derivative of  $x$  at  $a$  divided by factorial four therefore we can find that  $f$  of  $x$  can be written as  $f$  at  $a$  plus  $f'$  at  $a$  into  $x$  minus  $a$  plus  $f''$  second derivative of  $f$  at  $a$  into  $x$  minus  $a$  whole square upon factorial two plus third derivative of  $f$  at  $a$  into  $x$  minus  $a$  whole cube upon factorial three plus fourth derivative of  $f$  at  $a$  into  $x$  minus  $a$  whole to the power four upon factorial four it is not mandatory that you have to go to infinity we can always make an approximation by expanding it to a fixed power say  $k$  is equal to  $4$  and then the remaining term will be the error term in the approximation but if the difference between  $x$  and  $a$  is very small then as the power increases then the error term will go to zero

so this expansion is called Taylor series expansion of  $f(x)$  when  $f$  is continuous and differentiable in finite number of times at the point  $a$  you shall stand study more about Taylor series in your higher classes of mathematics but in this class we shall see how it helps us in solving certain problems let us consider  $1 - x$  whole to the power minus  $2$  therefore  $f(x)$  is equal to  $1 - x$  to the power minus  $2$   $f'$  first derivative of  $x$  is equal to  $-2$  into  $1 - x$  to the power minus  $3$   $f''$  second derivative of  $x$  is equal to  $-3$  into  $2$  into  $1 - x$  to the power minus  $4$  is equal to factorial  $3$  into  $1 - x$  to the power minus  $4$   $f'''$  third derivative of  $x$  is equal to in a similar way factorial  $4$  into  $1 - x$  whole to the power minus  $5$  therefore if at zero is equal to one  $f'$  at zero is equal to  $-2$  if second derivative at zero is equal to factorial  $3$  and if fourth derivative at zero is equal to factorial  $4$  therefore we get  $f(x)$  is equal to  $f(a)$  plus  $f'(a)$  into  $x$  minus  $a$  plus  $f''(a)$  into  $x$  minus  $a$  whole square upon factorial two plus  $f'''(a)$  into  $x$  minus  $a$  whole cube upon factorial three plus  $f^{(4)}(a)$  into  $x$  minus  $a$  whole to the power four upon factorial four like that and putting the values  $f(x)$  is equal to  $1 - x$  whole to the power minus two is equal to  $f(0)$  plus  $f'(0)$  into  $x$  plus  $f''(0)$  into  $x$  square upon factorial two plus  $f'''(0)$  into  $x$  cube upon factorial three plus  $f^{(4)}(0)$  into  $x$  four upon factorial four is equal to  $f(0)$  plus  $2$  times  $x$  plus factorial three into  $x$  minus zero whole square upon factorial two plus factorial four into  $x$  minus zero whole cube upon factorial three etcetera is equal to  $1$  plus  $2x$  plus  $3x^2$  plus  $4x^3$  plus and these are the terms that we have already seen right we have already seen that  $1 - x$  whole to the power minus two is equal to  $1$  plus  $2x$  plus  $3x^2$  plus  $4x^3$  like that

so Taylor series expansion works for  $1 - x$  whole to the power minus  $2$  i want you to verify the same with other polynomial expansions that we have done already in this class and i shall go beyond polynomials if i ask you what are the other functions that you can remember very easily which is differentiable smoothly for different values of  $x$  the first one that comes to my mind is the trigonometric functions in particular let us look at  $\sin x$   $\cos x$   $\tan x$  right let us see whether we can expand this using Taylor series expansion and find a way to compute  $\sin$  of  $x$  or  $\cos$  of  $x$  or  $\tan$  of  $x$  etcetera because if you remember in classes we see the values of  $\sin x$   $\cos x$  etcetera are only for a fixed set of values right we have seen for zero degree for  $\pi/6$  for  $\pi/4$  for  $\pi/3$  for  $\pi/2$  and  $\pi$  and typically we work with their multiples or

perhaps with some more trigonometric manipulations we can get for 15 degrees 18 degrees etcetera right what if i ask you what is the sign of one degree or what is the sign of five degrees it is not easy to compute those values unless we use a taylor series expansion that is why this formula is very very important for analysis

so for illustration consider sine x  $f(x)$  is equal to sine x therefore  $f(0)$  is equal to zero first derivative of x is equal to cos x therefore first derivative at zero is equal to cos zero which is is equal to one second derivative of x is equal to minus sin x therefore second derivative at zero is equal to zero third derivative of x is equal to minus cos x therefore third derivative at zero is equal to minus one i will go for a few more the fourth derivative of x is equal to sine x therefore fourth derivative at zero is equal to zero fifth derivative of x is equal to cos x therefore fifth derivative at zero is equal to one

so let us stop here and we can we know that from taylor's theorem from taylor's series  $f(x)$  is equal to  $f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5 + \dots$  therefore from taylor series  $f(x)$  is equal to  $f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5 + \dots$  like that we will go i am not going any further let us replace the values  $f(0)$  is equal to zero  $f'(0)$  is equal to one if second derivative at zero is equal to zero if third derivative at zero is equal to minus one a fourth derivative at zero is equal to zero and a fifth derivative at zero is equal to plus one this is what we have got putting these values we get sine of x is equal to  $f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5 + \dots$  is equal to  $0 + 1x + \frac{0}{2!}x^2 + \frac{-1}{3!}x^3 + \frac{0}{4!}x^4 + \frac{1}{5!}x^5 + \dots$  is equal to  $x - \frac{x^3}{6} + \frac{x^5}{120} - \dots$  if we continue further we shall see that this is is equal to  $x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \dots$  like that that is we shall get a polynomial where only all powers of x are there and the coefficient for x to the power k is one upon factorial k and the signs of them are going to be plus minus plus minus in an alternative way similarly we can work out for cos x and i like you to verify that cos x is equal to  $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$  like that that is when we look at cos x we find only even powers of x and again like sign we get alternatively positive and negative signs next we look at tan x again we expand about 0  $f(0)$  is equal to 0  $f'(0)$  is equal to 1 at x is equal to 0  $f''(0)$  of tan x is equal to  $2 \tan x$  is equal to 0  $f'''(0)$  of tan x is equal to  $2 + 6 \tan^2 x$  is equal to 2  $f^{(4)}(0)$  of tan x is equal to  $6 + 12 \tan^2 x + 12 \tan^4 x$  is equal to 6  $f^{(5)}(0)$  of tan x is equal to  $12 + 24 \tan^2 x + 24 \tan^4 x$  is equal to 12  $f^{(6)}(0)$  of tan x is equal to  $24 + 48 \tan^2 x + 48 \tan^4 x + 24 \tan^6 x$  is equal to 24  $f^{(7)}(0)$  of tan x is equal to  $48 + 96 \tan^2 x + 96 \tan^4 x + 48 \tan^6 x$  is equal to 48  $f^{(8)}(0)$  of tan x is equal to  $96 + 192 \tan^2 x + 192 \tan^4 x + 96 \tan^6 x$  is equal to 96  $f^{(9)}(0)$  of tan x is equal to  $192 + 384 \tan^2 x + 384 \tan^4 x + 192 \tan^6 x$  is equal to 192  $f^{(10)}(0)$  of tan x is equal to  $384 + 768 \tan^2 x + 768 \tan^4 x + 384 \tan^6 x$  is equal to 384  $f^{(11)}(0)$  of tan x is equal to  $768 + 1536 \tan^2 x + 1536 \tan^4 x + 768 \tan^6 x$  is equal to 768  $f^{(12)}(0)$  of tan x is equal to  $1536 + 3072 \tan^2 x + 3072 \tan^4 x + 1536 \tan^6 x$  is equal to 1536  $f^{(13)}(0)$  of tan x is equal to  $3072 + 6144 \tan^2 x + 6144 \tan^4 x + 3072 \tan^6 x$  is equal to 3072  $f^{(14)}(0)$  of tan x is equal to  $6144 + 12288 \tan^2 x + 12288 \tan^4 x + 6144 \tan^6 x$  is equal to 6144  $f^{(15)}(0)$  of tan x is equal to  $12288 + 24576 \tan^2 x + 24576 \tan^4 x + 12288 \tan^6 x$  is equal to 12288  $f^{(16)}(0)$  of tan x is equal to  $24576 + 49152 \tan^2 x + 49152 \tan^4 x + 24576 \tan^6 x$  is equal to 24576  $f^{(17)}(0)$  of tan x is equal to  $49152 + 98304 \tan^2 x + 98304 \tan^4 x + 49152 \tan^6 x$  is equal to 49152  $f^{(18)}(0)$  of tan x is equal to  $98304 + 196608 \tan^2 x + 196608 \tan^4 x + 98304 \tan^6 x$  is equal to 98304  $f^{(19)}(0)$  of tan x is equal to  $196608 + 393216 \tan^2 x + 393216 \tan^4 x + 196608 \tan^6 x$  is equal to 196608  $f^{(20)}(0)$  of tan x is equal to  $393216 + 786432 \tan^2 x + 786432 \tan^4 x + 393216 \tan^6 x$  is equal to 393216  $f^{(21)}(0)$  of tan x is equal to  $786432 + 1572864 \tan^2 x + 1572864 \tan^4 x + 786432 \tan^6 x$  is equal to 786432  $f^{(22)}(0)$  of tan x is equal to  $1572864 + 3145728 \tan^2 x + 3145728 \tan^4 x + 1572864 \tan^6 x$  is equal to 1572864  $f^{(23)}(0)$  of tan x is equal to  $3145728 + 6291456 \tan^2 x + 6291456 \tan^4 x + 3145728 \tan^6 x$  is equal to 3145728  $f^{(24)}(0)$  of tan x is equal to  $6291456 + 12582912 \tan^2 x + 12582912 \tan^4 x + 6291456 \tan^6 x$  is equal to 6291456  $f^{(25)}(0)$  of tan x is equal to  $12582912 + 25165824 \tan^2 x + 25165824 \tan^4 x + 12582912 \tan^6 x$  is equal to 12582912  $f^{(26)}(0)$  of tan x is equal to  $25165824 + 50331648 \tan^2 x + 50331648 \tan^4 x + 25165824 \tan^6 x$  is equal to 25165824  $f^{(27)}(0)$  of tan x is equal to  $50331648 + 100663296 \tan^2 x + 100663296 \tan^4 x + 50331648 \tan^6 x$  is equal to 50331648  $f^{(28)}(0)$  of tan x is equal to  $100663296 + 201326592 \tan^2 x + 201326592 \tan^4 x + 100663296 \tan^6 x$  is equal to 100663296  $f^{(29)}(0)$  of tan x is equal to  $201326592 + 402653184 \tan^2 x + 402653184 \tan^4 x + 201326592 \tan^6 x$  is equal to 201326592  $f^{(30)}(0)$  of tan x is equal to  $402653184 + 805306368 \tan^2 x + 805306368 \tan^4 x + 402653184 \tan^6 x$  is equal to 402653184  $f^{(31)}(0)$  of tan x is equal to  $805306368 + 1610612736 \tan^2 x + 1610612736 \tan^4 x + 805306368 \tan^6 x$  is equal to 805306368  $f^{(32)}(0)$  of tan x is equal to  $1610612736 + 3221225472 \tan^2 x + 3221225472 \tan^4 x + 1610612736 \tan^6 x$  is equal to 1610612736  $f^{(33)}(0)$  of tan x is equal to  $3221225472 + 6442450944 \tan^2 x + 6442450944 \tan^4 x + 3221225472 \tan^6 x$  is equal to 3221225472  $f^{(34)}(0)$  of tan x is equal to  $6442450944 + 12884901888 \tan^2 x + 12884901888 \tan^4 x + 6442450944 \tan^6 x$  is equal to 6442450944  $f^{(35)}(0)$  of tan x is equal to  $12884901888 + 25769803776 \tan^2 x + 25769803776 \tan^4 x + 12884901888 \tan^6 x$  is equal to 12884901888  $f^{(36)}(0)$  of tan x is equal to  $25769803776 + 51539607552 \tan^2 x + 51539607552 \tan^4 x + 25769803776 \tan^6 x$  is equal to 25769803776  $f^{(37)}(0)$  of tan x is equal to  $51539607552 + 103079215104 \tan^2 x + 103079215104 \tan^4 x + 51539607552 \tan^6 x$  is equal to 51539607552  $f^{(38)}(0)$  of tan x is equal to  $103079215104 + 206158430208 \tan^2 x + 206158430208 \tan^4 x + 103079215104 \tan^6 x$  is equal to 103079215104  $f^{(39)}(0)$  of tan x is equal to  $206158430208 + 412316860416 \tan^2 x + 412316860416 \tan^4 x + 206158430208 \tan^6 x$  is equal to 206158430208  $f^{(40)}(0)$  of tan x is equal to  $412316860416 + 824633720832 \tan^2 x + 824633720832 \tan^4 x + 412316860416 \tan^6 x$  is equal to 412316860416  $f^{(41)}(0)$  of tan x is equal to  $824633720832 + 1649267441664 \tan^2 x + 1649267441664 \tan^4 x + 824633720832 \tan^6 x$  is equal to 824633720832  $f^{(42)}(0)$  of tan x is equal to  $1649267441664 + 3298534883328 \tan^2 x + 3298534883328 \tan^4 x + 1649267441664 \tan^6 x$  is equal to 1649267441664  $f^{(43)}(0)$  of tan x is equal to  $3298534883328 + 6597069766656 \tan^2 x + 6597069766656 \tan^4 x + 3298534883328 \tan^6 x$  is equal to 3298534883328  $f^{(44)}(0)$  of tan x is equal to  $6597069766656 + 13194139533312 \tan^2 x + 13194139533312 \tan^4 x + 6597069766656 \tan^6 x$  is equal to 6597069766656  $f^{(45)}(0)$  of tan x is equal to  $13194139533312 + 26388279066624 \tan^2 x + 26388279066624 \tan^4 x + 13194139533312 \tan^6 x$  is equal to 13194139533312  $f^{(46)}(0)$  of tan x is equal to  $26388279066624 + 52776558133248 \tan^2 x + 52776558133248 \tan^4 x + 26388279066624 \tan^6 x$  is equal to 26388279066624  $f^{(47)}(0)$  of tan x is equal to  $52776558133248 + 105553116266496 \tan^2 x + 105553116266496 \tan^4 x + 52776558133248 \tan^6 x$  is equal to 52776558133248  $f^{(48)}(0)$  of tan x is equal to  $105553116266496 + 211106232532992 \tan^2 x + 211106232532992 \tan^4 x + 105553116266496 \tan^6 x$  is equal to 105553116266496  $f^{(49)}(0)$  of tan x is equal to  $211106232532992 + 422212465065984 \tan^2 x + 422212465065984 \tan^4 x + 211106232532992 \tan^6 x$  is equal to 211106232532992  $f^{(50)}(0)$  of tan x is equal to  $422212465065984 + 844424930131968 \tan^2 x + 844424930131968 \tan^4 x + 422212465065984 \tan^6 x$  is equal to 422212465065984  $f^{(51)}(0)$  of tan x is equal to  $844424930131968 + 1688849860263936 \tan^2 x + 1688849860263936 \tan^4 x + 844424930131968 \tan^6 x$  is equal to 844424930131968  $f^{(52)}(0)$  of tan x is equal to  $1688849860263936 + 3377699720527872 \tan^2 x + 3377699720527872 \tan^4 x + 1688849860263936 \tan^6 x$  is equal to 1688849860263936  $f^{(53)}(0)$  of tan x is equal to  $3377699720527872 + 6755399441055744 \tan^2 x + 6755399441055744 \tan^4 x + 3377699720527872 \tan^6 x$  is equal to 3377699720527872  $f^{(54)}(0)$  of tan x is equal to  $6755399441055744 + 13510798882111488 \tan^2 x + 13510798882111488 \tan^4 x + 6755399441055744 \tan^6 x$  is equal to 6755399441055744  $f^{(55)}(0)$  of tan x is equal to  $13510798882111488 + 27021597764222976 \tan^2 x + 27021597764222976 \tan^4 x + 13510798882111488 \tan^6 x$  is equal to 13510798882111488  $f^{(56)}(0)$  of tan x is equal to  $27021597764222976 + 54043195528445952 \tan^2 x + 54043195528445952 \tan^4 x + 27021597764222976 \tan^6 x$  is equal to 27021597764222976  $f^{(57)}(0)$  of tan x is equal to  $54043195528445952 + 108086391056891904 \tan^2 x + 108086391056891904 \tan^4 x + 54043195528445952 \tan^6 x$  is equal to 54043195528445952  $f^{(58)}(0)$  of tan x is equal to  $108086391056891904 + 216172782113783808 \tan^2 x + 216172782113783808 \tan^4 x + 108086391056891904 \tan^6 x$  is equal to 108086391056891904  $f^{(59)}(0)$  of tan x is equal to  $216172782113783808 + 432345564227567616 \tan^2 x + 432345564227567616 \tan^4 x + 216172782113783808 \tan^6 x$  is equal to 216172782113783808  $f^{(60)}(0)$  of tan x is equal to  $432345564227567616 + 864691128455135232 \tan^2 x + 864691128455135232 \tan^4 x + 432345564227567616 \tan^6 x$  is equal to 432345564227567616  $f^{(61)}(0)$  of tan x is equal to  $864691128455135232 + 1729382256910270464 \tan^2 x + 1729382256910270464 \tan^4 x + 864691128455135232 \tan^6 x$  is equal to 864691128455135232  $f^{(62)}(0)$  of tan x is equal to  $1729382256910270464 + 3458764513820540928 \tan^2 x + 3458764513820540928 \tan^4 x + 1729382256910270464 \tan^6 x$  is equal to 1729382256910270464  $f^{(63)}(0)$  of tan x is equal to  $3458764513820540928 + 6917529027641081856 \tan^2 x + 6917529027641081856 \tan^4 x + 3458764513820540928 \tan^6 x$  is equal to 3458764513820540928  $f^{(64)}(0)$  of tan x is equal to  $6917529027641081856 + 13835058055282163712 \tan^2 x + 13835058055282163712 \tan^4 x + 6917529027641081856 \tan^6 x$  is equal to 6917529027641081856  $f^{(65)}(0)$  of tan x is equal to  $13835058055282163712 + 27670116110564327424 \tan^2 x + 27670116110564327424 \tan^4 x + 13835058055282163712 \tan^6 x$  is equal to 13835058055282163712  $f^{(66)}(0)$  of tan x is equal to  $27670116110564327424 + 55340232221128654848 \tan^2 x + 55340232221128654848 \tan^4 x + 27670116110564327424 \tan^6 x$  is equal to 27670116110564327424  $f^{(67)}(0)$  of tan x is equal to  $55340232221128654848 + 110680464442257309696 \tan^2 x + 110680464442257309696 \tan^4 x + 55340232221128654848 \tan^6 x$  is equal to 55340232221128654848  $f^{(68)}(0)$  of tan x is equal to  $110680464442257309696 + 221360928884514619392 \tan^2 x + 221360928884514619392 \tan^4 x + 110680464442257309696 \tan^6 x$  is equal to 110680464442257309696  $f^{(69)}(0)$  of tan x is equal to  $221360928884514619392 + 442721857769029238784 \tan^2 x + 442721857769029238784 \tan^4 x + 221360928884514619392 \tan^6 x$  is equal to 221360928884514619392  $f^{(70)}(0)$  of tan x is equal to  $442721857769029238784 + 885443715538058477568 \tan^2 x + 885443715538058477568 \tan^4 x + 442721857769029238784 \tan^6 x$  is equal to 442721857769029238784  $f^{(71)}(0)$  of tan x is equal to  $885443715538058477568 + 1770887431076116955136 \tan^2 x + 1770887431076116955136 \tan^4 x + 885443715538058477568 \tan^6 x$  is equal to 885443715538058477568  $f^{(72)}(0)$  of tan x is equal to  $1770887431076116955136 + 3541774862152233910272 \tan^2 x + 3541774862152233910272 \tan^4 x + 1770887431076116955136 \tan^6 x$  is equal to 1770887431076116955136  $f^{(73)}(0)$  of tan x is equal to  $3541774862152233910272 + 7083549724304467820544 \tan^2 x + 7083549724304467820544 \tan^4 x + 3541774862152233910272 \tan^6 x$  is equal to 3541774862152233910272  $f^{(74)}(0)$  of tan x is equal to  $7083549724304467820544 + 14167099448608935641088 \tan^2 x + 14167099448608935641088 \tan^4 x + 7083549724304467820544 \tan^6 x$  is equal to 7083549724304467820544  $f^{(75)}(0)$  of tan x is equal to  $14167099448608935641088 + 28334198897217871282176 \tan^2 x + 28334198897217871282176 \tan^4 x + 14167099448608935641088 \tan^6 x$  is equal to 14167099448608935641088  $f^{(76)}(0)$  of tan x is equal to  $28334198897217871282176 + 56668397794435742564352 \tan^2 x + 56668397794435742564352 \tan^4 x + 28334198897217871282176 \tan^6 x$  is equal to 28334198897217871282176  $f^{(77)}(0)$  of tan x is equal to  $56668397794435742564352 + 113336795588871485128704 \tan^2 x + 113336795588871485128704 \tan^4 x + 56668397794435742564352 \tan^6 x$  is equal to 56668397794435742564352  $f^{(78)}(0)$  of tan x is equal to  $113336795588871485128704 + 226673591177742970257408 \tan^2 x + 226673591177742970257408 \tan^4 x + 113336795588871485128704 \tan^6 x$  is equal to 113336795588871485128704  $f^{(79)}(0)$  of tan x is equal to  $226673591177742970257408 + 453347182355485940514816 \tan^2 x + 453347182355485940514816 \tan^4 x + 226673591177742970257408 \tan^6 x$  is equal to 226673591177742970257408  $f^{(80)}(0)$  of tan x is equal to  $453347182355485940514816 + 906694364710971881029632 \tan^2 x + 906694364710971881029632 \tan^4 x + 453347182355485940514816 \tan^6 x$  is equal to 453347182355485940514816  $f^{(81)}(0)$  of tan x is equal to  $906694364710971881029632 + 1813388729421943762059264 \tan^2 x + 1813388729421943762059264 \tan^4 x + 906694364710971881029632 \tan^6 x$  is equal to 906694364710971881029632  $f^{(82)}(0)$  of tan x is equal to  $1813388729421943762059264 + 3626777458843887524118528 \tan^2 x + 3626777458843887524118528 \tan^4 x + 1813388729421943762059264 \tan^6 x$  is equal to 1813388729421943762059264  $f^{(83)}(0)$  of tan x is equal to  $3626777458843887524118528 + 7253554917687775048237056 \tan^2 x + 7253554917687775048237056 \tan^4 x + 3626777458843887524118528 \tan^6 x$  is equal to 3626777458843887524118528  $f^{(84)}(0)$  of tan x is equal to  $7253554917687775048237056 + 14507109835375550096474112 \tan^2 x + 14507109835375550096474112 \tan^4 x + 7253554917687775048237056 \tan^6 x$  is equal to 7253554917687775048237056  $f^{(85)}(0)$  of tan x is equal to  $14507109835375550096474112 + 29014219670751100192948224 \tan^2 x + 29014219670751100192948224 \tan^4 x + 14507109835375550096474112 \tan^6 x$  is equal to 14507109835375550096474112  $f^{(86)}(0)$  of tan x is equal to  $29014219670751100192948224 + 58028439341502200385896448 \tan^2 x + 58028439341502200385896448 \tan^4 x + 29014219670751100192948224 \tan^6 x$  is equal to 29014219670751100192948224  $f^{(87)}(0)$  of tan x is equal to  $58028439341502200385896448 + 116056878683004400771792896 \tan^2 x + 116056878683004400771792896 \tan^4 x + 58028439341502200385896448 \tan^6 x$  is equal to 58028439341502200385896448  $f^{(88)}(0)$  of tan x is equal to  $116056878683004400771792896 + 232113757366008801543585792 \tan^2 x + 232113757366008801543585792 \tan^4 x + 116056878683004400771792896 \tan^6 x$  is equal to 116056878683004400771792896  $f^{(89)}(0)$  of tan x is equal to  $232113757366008801543585792 + 464227514732017603087171584 \tan^2 x + 464227514732017603087171584 \tan^4 x + 232113757366008801543585792 \tan^6 x$  is equal to 232113757366008801543585792  $f^{(90)}(0)$  of tan x is equal to  $464227514732017603087171584 + 928455029464035206174343168 \tan^2 x + 928455029464035206174343168 \tan^4 x + 464227514732017603087171584 \tan^6 x$  is equal to 464227514732017603087171584  $f^{(91)}(0)$  of tan x is equal to  $928455029464035206174343168 + 1856910058928070412348686336 \tan^2 x + 1856910058928070412348686336 \tan^4 x + 928455029464035206174343168 \tan^6 x$  is equal to 928455029464035206174343168  $f^{(92)}(0)$  of tan x is equal to  $1856910058928070412348686336 + 3713820117856140824697372672 \tan^2 x + 3713820117856140824697372672 \tan^4 x + 1856910058928070412348686336 \tan^6 x$  is equal to 1856910058928070412348686336  $f^{(93)}(0)$  of tan x is equal to  $3713820117856140824697372672 + 7427640235712281649394745344 \tan^2 x + 7427640235712281649394745344 \tan^4 x + 3713820117856140824697372672 \tan^6 x$  is equal to 3713820117856140824697372672  $f^{(94)}(0)$  of tan x is equal to  $7427640235712281649394745344 + 14855280471424563298789490688 \tan^2 x + 14855280471424563298789490688 \tan^4 x + 7427640235$

can get if five  $x$  is equal to 16 into 1 plus tan square  $x$  plus terms with tan  $x$  therefore  $f$  five at zero is equal to sixteen therefore by expanding up to this point we can get that tan  $x$  is equal to zero plus one upon one factorial  $x$  plus 0 time  $x$  square upon factorial 2 plus 2 times  $x$  cube upon factorial 3 plus 0 times  $x$  4 upon factorial 4 plus 16 times  $x$  to the power 5 upon factorial 5 plus higher powers of  $x$  on simplification we get tan of  $x$  is equal to 0 plus  $x$  plus 0 times  $x$  square plus 2 into  $x$  cube upon factorial 3 plus 0 times  $x$  to the power 4 plus sixteen times  $x$  to the power five upon factorial five now this is is equal to  $x$  plus  $x$  cube upon three plus sixteen upon factorial five is equal to one twenty

so this is four upon thirty is equal to two upon fifteen is equal to two upon fifteen times  $x$  to the power five plus higher powers of thus we get an approximation of tan  $x$  in the form of a polynomial up to fifth power of  $x$  okay students i stop here today in the next class i look at some more problems in particular i will look at how to obtain the taylor series expansion for tan inverse  $x$  for logarithmic functions in particular log of 1 plus  $x$  and more importantly the expansion of  $e$  to the power  $x$  which is very important in analysis okay you