

sequence and series this lecture is intended to explore some more problems on these topics what is the sum of or three digit numbers that leave a remainder of two when divided by three to start with let us have the following observation let a , $a+1$, $a+2$, $a+3$ consecutive positive integers further if a is divisible by three then $a+1$ will leave remainder 1 when divided with 3 whereas $a+2$ will leave a remainder 2 when divided by three this is a trivial but useful observation let me repeat if you have a , $a+1$, $a+2$ etc to be consecutive positive integers with a divisible by 3 meaning it leaves remainder 0 when divided by 3 then $a+1$ will leave remainder 1, $a+2$ will leave remainder 2 then the next number $a+3$ will be again divisible by 3 and so on on the other hand if a is not divisible by 3 but leaves a remainder 1 when divided by 3 then $a+1$ will leave remainder 2 when divided by 3 whereas $a+2$ will be exactly divisible by 3 and

so on keeping this observation in mind let us proceed with solution of given problem you are asked to find some of all three digit numbers that leave a remainder of two when divided by three the first three digit number namely hundred leaves remainder 1 when divided by three therefore the next three digit number leaves remainder two when divided by three that is our observation is that the first three digit number which leaves remainder 2 when divided by 3 is 101 then the next number namely 104 or 107 will be divisible by 3 one not three will leave remainder one when divided by three whereas one not four value remainder 2 when divided by 3 thus the numbers more specifically 3 digit numbers which leaves remainder 2 when divided by 3 are one not one one not four one not seven etcetera let us try to find the last three digit number which leaves remainder 2 when divided by 3 note that the last 3 digit number or the highest 3 digit number is 999 which is divisible by 3.

so the next preceding number namely 998 will be leaving a remainder 2 when divided by 3 therefore the last number in this succession of numbers is 998 consequently the question reduces to find the sum of all terms of the sequence one not one one not four one not seven etc up to 998 can you observe that this sequence is in arithmetic progression with common difference three therefore we are asked to find some of finite number of terms of an ap with first term one not one and common difference 3 recall the formula that is available for sum of first n terms of an ap there are two formulas available one being sum of first n terms of an ap equal to n by 2 multiplied with the sum of first term and last term there is one more formula however note that any of these two formula requires n the number of terms that you are summing therefore the first task is to find how many terms are there in the ap starting from 101 and ending with 998 in other words what is n involved here to tackle this let 998 be the n th then 998 will be equal to $a + n - 1$ times d we know the first term a is one not one and the common difference is 3 simplifying this equation we get $n - 1$ equal to $998 - 101$ divided by 3 which is 299 therefore n will be equal to 300 thus in the given sequence 101, 104 etc 998 which is an arithmetic progression there are in fact 300 terms

hence we are asked to find sum of first 300 terms of an arithmetic progression therefore required sum is equal to n by two into first term plus last term is equal to 300 by 2 multiplied with $101 + 998$ with a little computation one can obtain the answer as one six four eight five zero one lakh sixty four thousand and eight fifty this solves the given problem let us proceed with a similar problem obtain the sum of or positive integers up to thousand which are divisible by ϕ and not divisible by two this is again a problem dealing with sum of first n terms of an ap as one can see let us formally solve this note that positive integers up to thousand that are divisible by ϕ are 5, 10, 15 etc

thousand note that 1000 is divisible by 5.

the preceding number divisible by 5 will be 995.

in this list note that 10 20 and

so on are divisible by 2.

hence we should not consider 10 20 and

so on while listing positive integers that are divisible by 5 but not divisible by 2.

5 15 etc we are asked to positive integer up to 1000

so the last positive integer under consideration which are divisible by phi but not by 2 will be 995 thus the problem boils down to find the sum of terms of the sequence 5 15

so on up to 995 one can easily observe that this sequence is an arithmetic progression with first term phi and common difference 10.

as in the previous problem let us recall that sum of first n terms of an ap has two formulae however both needs the number of terms that are required to be summed therefore as next step we shall find how many terms are there in this ap starting from 5 up to 995 to this end let 995 be the nth term using the formula for nth term we get 995 is equal to a plus n minus 1 times d which is 5 plus n minus 1 times 10 isolating n we get n equal to 995 minus 5 by 10 plus 1.

that implies n equal to 100 thus 995 in the given list is in fact the 100th using this required sum that is sum of first 100 terms of an ap with first term 5 and common difference 10 will be n by 2 into first term plus last term we shall rely on this formula since last term is known to us this is 100 by 2 into first term is 5 and last term is 995 with some simple calculation one can get the answer to be 50 000 here is your next problem if a is given to be 2 power 65 and b is given to be 2 power 64 plus 2 power 63 plus etc plus 2 power 0 then is a greater than b in this problem you are asked to compare a and b towards the solution first let us observe that b which is given to be 2 power 64 plus 2 power 63 plus etc plus 2 power 0 is in fact sum of first few terms of a gp with first term 2 power 0 and common ratio 2.

b is sum of terms of a gp with first term a equal to 2 power 0 which is 1 and common ratio 2 see that 2 power 0 is the last term last but one will be 2 power 1 proceeding to it will be 2 square and

so on up to 2 power 64 if you read from the other side therefore you can observe that the first term is 1 and the common ratio is 2.

now let us use the formula for sum of terms of a gp recall that sum of first n terms of a gp with first term a and common ratio r is s n equal to a into r power n minus 1 by r minus 1 assuming r not equal to 1 therefore similar to previous two problems first task will be to find out how many terms are there in this series to find out let us assume that 2 power 64 is nth term when we read from the other side let 2 power 64 be the nth term using the formula for nth term of a gp a into r power n minus 1 is equal to 2 power 64.

note that a is 1 and r is 2.

recall the law of integers here the base is same two and numbers are same therefore comparing the exponents one get n minus 1 equal to 64 isolating n equal to 65 this concludes that there are in fact 65 terms in the sum 2 power 0 plus etc up to 2 power 64.

using this let us find b b is equal to a into r power n minus 1 by r minus 1 being sum of n terms of a gp which is equal to a is 1 times r is 22

so 2 power 65 minus 1 by 2 minus 1 which is 2 power 65 minus 1 therefore we obtain b to be 2 power 65 minus 1 recall that 2 power 65 is the value of a therefore b is a minus 1 which says that a equal to b plus 1 consequently a is greater than b note that a b are positive thus answer to the question is yes a is greater than b here is the next problem for you a piece of equipment cost a

certain factory rupees 6 lakh if this equipment depreciates in value 15 percentage the first year 13.

5 percentage the next year 12 percentage the third year and

so on what will be its value at the end of 10 years all percentages applying to the original cost you have a piece of equipment with certain costs it's depreciates in value every year we are asked to find the value at the end of 10 years since all the depreciation is given in percentage for the sake of simplicity let us assume that the cost is 100 in that case the percentage of depreciation at the end of one two three years are given in the list 15 13. 5 12 etcetera this list of percentage of depreciation can be observed to be in arithmetic progression with first term a equal to 15 and common difference d equal to minus 1.

5 it is difference the second term minus first term or the third term minus second term and

so on keeping this observation let us find what is the percentage depreciation at the end of 10 years therefore percentage of depreciation in tenth year this is simply asking the tenth term of this ap consequently the percentage of depreciation in 10th year can be obtained from the formula $a + (n-1)d$ substituting the value of a and d we obtain the percentage depreciation in 10th year to be 1.

5 therefore the successive depreciation in the first 10 years would be 15 13.

5 12 etc up to 1.

5 using this total value depreciated in 10 years assuming the cost to be 100 is 15 plus 13.

5 plus etc plus 1.

5 can you see that this sum is in fact sum of first 10 terms of an arithmetic progression therefore its value will be $10 \times \frac{10+1}{2}$ being the number of terms in this sum multiplied with 15 plus 1.

5 which is 82.

5 therefore assuming the value to be 100 total value depreciated in 10 years is 82.

5 as a consequence value of the equipment at the end of 10 years would be the cost 100 minus total depreciated 82.

5 thus the value of the equipment after 10 years would be 17.

5 this is the case if the cost is 100 rupees now let us scale it with the actual cost total cost being 6 lakh its value at the end of 10 years would be 6 lakh into 17.

5 by 100 this is because 17.

5 is the depreciation if the cost is 100 therefore 17.

5 by 100 is the depreciation if the cost is 1 rupee multiply it with 60 000 the actual cost this can be simplified to one lakh and five thousand this completes the solution let us proceed with few more problems if $\log_2 \log_2$ power x minus 1 and logarithm of 2 power x plus 3 are in ap find the value of x this is an interesting problem based on arithmetic progression and the concept of logarithm to set the stage for it first let me remind you some basics on logarithm recall that logarithm is inverse of the process of exponentiation more precisely logarithm of a positive real number x is the exponent to which another positive real number should be raised to obtain x in simple logarithm of a positive real number x to base b where b is a positive real number other than 1 is y if b power y is x i repeat logarithm of a positive real number x to the base b where b is a fixed positive real number not equal to 1 is said to be y if b exponentiated with y gives x for instance we know 2 power 3 is 8 in logarithmic language we say this log of 8 to the base 2 is 3 as another example we know phi square is 25 in the language of logarithm logarithm of the real number 25 to the

base 5 is 2.

we know 25^1 is 25 therefore logarithm of 25 to the base 25 is 1.

please compare logarithm of 25 to the base phi is 2 as 5 when squared gives 25 at the same time logarithm of 25 to another base namely 25 is 1.

let me give you one more instant just to give some specific example we say logarithm of 9 to the base 3 is 2 that is because 3 when squared gives 9.

i urge you to practice more with exponentiation and its inverse process logarithm let me just remark that though i have defined logarithm of a positive real number x with respect to a positive real number b not equal to 1.

or more specifically with base b not equal to 1 it is customary to take b as the number e in that case we call the logarithm as natural logarithm logarithm of a number x to base e will be called natural logarithm of x it is worth to recall that in calculus it is preferred to work with natural logarithm than logarithm to an arbitrary base b and natural logarithm is denoted by the simple \ln next let me recall two basic properties of logarithm logarithm of product of two positive real numbers x and y to a fixed base b is sum of logarithm of the individual logarithm of a product gets transformed into some of the logarithm in fact this simplifies the calculations and this property is one of the motivations for defining logarithm the complicated process of multiplication can be changed to a relatively simple process of addition by taking logarithm and as next property of logarithm let me recall that logarithm of a power x^p to some base b not equal to 1 is p times logarithm of x with respect to base b to put in simple language logarithm changes product to some of the logarithms and logarithm changes power to the product keeping this in mind let us come back to the problem given that $\log_2 \log_2^x - 1$ and $\log_2^x + 3$ are in ap since these 3 numbers are in ap the number appearing in the middle namely $\log_2^x - 1$ would be arithmetic mean of the numbers occurring in the first and third place that is therefore twice logarithm of $2^{\log_2^x - 1}$ is $\log_2^x + 3$ now let us use the property of logarithm logarithm of 2 plus logarithm of $2^{\log_2^x + 3}$ can be written as logarithm of the product 2 into $2^{\log_2^x + 3}$.

similarly in the left hand side 2 times logarithm of $2^{\log_2^x - 1}$ can be written as logarithm of $2^{\log_2^x - 1}$ the whole square thus the given information translates to logarithm of $2^{\log_2^x - 1}$ the whole square is equal to logarithm of 2 into $2^{\log_2^x + 3}$ now taking exponents and bearing that logarithm and exponentiation are inverse process we get $2^{\log_2^x - 1}$ the whole square is equal to $2^{\log_2^x + 3}$ let us expand this to yield $2^{\log_2^x - 1} \times 2^{\log_2^x - 1} = 2^{\log_2^x + 3}$ with simple manipulation this transformed to $2^{\log_2^x - 2} = 2^{\log_2^x + 3}$ times $2^{\log_2^x - 5} = 2^0$.

if you let $2^{\log_2^x}$ to be y it can be seen that the previous equation is a quadratic equation $y^2 - 4y - \phi = 0$ solving we get y equal to 5 or y equal to minus 1 substituting y equal to $2^{\log_2^x}$ this reduces to $2^{\log_2^x} = 5$ or $2^{\log_2^x} = -1$ for a real x note that $2^{\log_2^x}$ cannot be minus 1 therefore $2^{\log_2^x}$ will be equal to 5 recalling the definition of logarithm this is same as saying x equal to logarithm of phi to the base 2 let us conclude with this problem thank you you