

welcome back to sequence and series we shall continue to tackle some more problems on this topic here is your first problem in this lecture you drop a ball from a meters above a flat surface each time the ball hits the surface after falling a distance h it rebounds a distance rh where r is positive but less than one let me repeat the given information you drop a ball from a meters above a flat surface each time the ball hits the surface after falling a distance h it rebounds a distance rh find the total distance the ball travels up and down find the total number of seconds the ball is travelling assuming the height is equal to 4 meters let us try to solve this problem as follows the crux of the matter is whenever the ball travels a distance h and hits the surface it will rebound from the surface to a height rh recall that first the ball is being dropped above a flat surface with a height a meters

so this height is a from this point you are dropping the ball

so it will travel a distance down once it is on the floor then it will rebound if it travels a height h to hit the ground it rebounds us to a height rh

so since it travels first through a distance a meters it will rebound to a height ra and then it will come down travelling ra distance and it will rebound since to come down it travel ra distance it will rebound the distance r^2a up it will come down the same distance and then rebounds r^3a and

so on first it is dropped from a height of a distance

so it travels down a distance a it hits the floor it rebounds us how much it rebounds us it depends upon how much it travel to come down since it is a it will rebound to a height ra once it rebounds same distance it covers down and then rebounds us r times the distance it travelled down way which is r^2a and

so on

so the total distance will be let me denote it as S will be the first a height travel down plus rebounds up ra and then down ra plus rebounds up r^2a then same distance r^2a and

so on which will be $a + 2ra + 2r^2a + \dots$ etcetera now if you see the second term onwards or second summand onwards in this infinite sum it's a gp $2ra + 2r^2a + 2r^3a + \dots$ etc is a gp with the first term $2ra$ and common ratio r are gp with first term $2ra$ and common ratio r given r is positive and less than one

so this infinite sum is in fact convergent and sum to infinity of a gp with the first term a and common ratio r is given by the formula $\frac{a}{1-r}$ which we developed in previous lectures therefore the required distance is a plus the remaining is a gp with the first term $2ra$ common ratio r therefore sum to infinity of that gp is $\frac{2ra}{1-r}$

so once a is given we can find the value for the total distance second part of the question demands you to find total number of second the ball is travelling here we shall recall the law of motion $s = ut + \frac{1}{2}at^2$ freely falling body

so acceleration is acceleration due to gravity initial velocity is zero therefore $s = \frac{1}{2}gt^2$ giving the approximate value of g as nine point eight meter per second square this is $4t^2$

so once we know what is s the total distance covered by the ball we can isolate t as follows $t = \sqrt{\frac{s}{4}}$

9 in fact to get a value for s we need a and r second part of the question tells you a equal to 4 meters r is not given

so we will get t in terms of r this solves the problem here is at another for you can you make an infinite series of non-zero terms that converges to any number you want the question is interesting for the following reason first note that in contrast to a finite series or a finite sum an infinite sum may not have

always a finite value in rigorous language an infinite series of real numbers may not be convergent as we remarked in our previous lectures even if we know by some way or the other that an infinite series is convergent finding some of an infinite series may not be that easy in other words a formula like sum of an infinite terms of a gp may not be available for an arbitrary infinite series

so in our attempt to make an infinite series of non zero terms that converge to some number say l we will try to confine ourselves to geometric progressions for the obvious reason that for geometric progression we know condition for convergence and we know the sum of infinite terms of a geometric series in case of convergence keeping this in mind let l be a given number you are asked to find some series which is convergent infinite series and the sum of that series should be equal to l let us search the series in the domain of geometric series let a, ar, ar^2, \dots be a geometric progression therefore $a + ar + ar^2 + \dots$ would be a geometric series and it is convergent for $|r| < 1$ in case of convergent that sum summation ar^{n-1} from $n=1$ to infinity is a by $1 - r$ what we are required to find is an infinite series whose sum is l given number

so let us assume that the series is geometric series

so we need a by $1 - r$ to be equal to l where for the time being restriction of r is that $|r| < 1$

so to get a geometric progression and the corresponding geometric series with the sum as l only condition is that a by $1 - r$ should be equal to l with r some number between minus 1 and 1

so there is only one condition but for determining such a geometric progression you need first term a and common ratio are two unknowns are there first fix are some arbitrary value between minus one and one and use this formula to get a thus selecting r such that $|r| < 1$ say for instance $r = \frac{1}{2}$ and $a = 1$ into $1 - r$ we get a geometric series namely $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ and the observation that we had

so far says that this series will be convergent because we selected r less than one and greater than minus one and the sum of the series would be l

so its always possible to find an infinite series whose sum is given number l for our convenience we worked on geometric series it's interesting to note that l can be positive can be zero or can be negative if it is zero our series reduces to the trivial series $0 + 0 + 0 + \dots$

so this like a inverse problem but instead of giving you a geometric series and asking the sum the question is given some number can we construct a geometric series whose sum is the given number let us proceed you have a pattern of squares first four of the squares in that pattern is given outermost square has area four meter square each of the other square is obtained by joining the midpoints of sides of squares before it find the sum of areas of all squares

so you are given with a pattern of squares outermost square has area 4 meter square how do you get the next square that is by joining the midpoints of each side of the previous square this pattern is continued

so this will be your fifth square in the pattern

so this will be the sixth square in the pattern you are asked to find sum of areas of all the rectangles only thing given is area of outermost square place 4 meter square let us solve this note that if a square has each side length a the next square obtained in this pattern by joining the midpoint will have side length as follows remember the outer square had side length a and this is midpoint

so this distance is $\frac{a}{2}$ this distance is $\frac{a}{2}$ you have a right angled triangle here let me denote it as a, b, c that gives you length of bc for which

you can use pythagoras theorem bc will be square root of a b square plus ac square which gives you a by 2 square plus a by 2 square that is this is the main observation in this problem if you have a square with the side length a the next square in our pattern will have side length a by root 2.

next will be having side length a by root 2 by root 2 and

so on whenever we have square of side length x the immediate next to it in our pattern will have side length x by root 2.

therefore corresponding areas would be a square a by root 2 square a by root 2 by root 2 square and

so on that is a square a square by 2 a square by 4 and

so on we are asked to find sum of areas of all the rectangle that is we have to sum these numbers therefore sum of areas equal to a square plus a square by 2 plus a square by 4 plus etcetera it can be easily observed that this infinite sum corresponds to a geometric progression with first term a square and common ratio 1 by 2.

so the sum would be first term by 1 minus common ratio since the common ratio is less than 1 in fact it is convergent and that is how we could write a finite value for that infinite sum amounts to two a square it is given that the outermost square has area four meter square

so a square is four meter square a square equal to four numerical value therefore sum of areas is equal to two into four which is eight with the unit eight meter square basically it is a problem on gp let us continue second term of a gp is 1000 and common ratio is one by n were n element of n lets p and b product of n terms of this gp if p six is greater than p phi and p six is greater than p seven what is sum of all possible values of n a careful observation should reveal you that it is concerned with gp and product of terms in gp further one term i do not mean the first term in fact here it is second term is positive and the common ratio is 1 by n and n is natural number therefore common ratio is also positive if one term is positive and common ratio is also positive then all terms of that geometric progression should be positive that's one observation that may help us given p n denote the product of n terms therefore p six is equal to product of six terms first six terms which is equal to p phi the product of first five terms into the sixth term let me denote the sixth term as t six t n denote nth term

so p six the product of six terms is product of phi term first five terms into t6 for brevity we may add that p n denote product of first n terms given p 6 is greater than p 5 and that implies t 6 is greater than one recall that all terms are positive and p six is equal to p phi into t six p6 is larger than p5 therefore p6 by p5 will be larger than 1 that is t6 will be larger than 1 similarly p seven is p six the product of first six terms into the seventh term it's given that p seven is less than p six therefore t seven is less than one thus we observe that the sixth term of given gp is greater than one and seventh term is less than one the only information remains is that second term of this gp is 1000

so let us connect this t6 and t7 with second term remember a gp is of the form a a r a r square a r cube and

so on therefore let me denote these things by first term this is second term third term fourth term and

so on let us connect every terms with the second term which is given to us note that the third term is r times the second term fourth term is r square times the second terms fifth term will be r cube times the second term and

so on therefore t six equal to the sixth term equal to second term times how much it will be r power 4 see this third term is r times second term fourth term

is r^2 times second term and
so on

so sixth term will be r^4 times second term and second term is given to be thousand similarly t_7 would be the seventh term that is just a notation which is equal to second term into r^5 which is thousand into r^5 thus $t_6 > 1$ and that is $r^4 > 1$ by thousand recall that the common ratio is $1/n$ therefore this implies $1/n^4 > 1$ by thousand taking reciprocal this gives $n^4 < 1$ thousand using the second piece of information namely $t_7 < 1$ we get $1000 r^5 < 1$ that implies $1/n^5 < 1$ thousand thus we are searching for value of n all possible values of n with $n^4 < 1000$ and $n^5 > 1000$ its not hard to see that $n^4 < 1000$ if n is less than six you can think of fourth powers of one two three four and five which are all less than thousands and it is not hard to see that fifth power is greater than thousand only if n is greater than or equal to four thus we are searching for those values of n which are less than 6 which is same as saying $n^4 < 1000$ and which are greater than or equal to four which are same as saying fifth power is strictly greater than thousand thus the possible values of n are four and five now the answer to the question is immediate what are all the sum of possible values of n the required sum is nine four and five are the possible values therefore the required sum is nine here you have your next problem some of first 12 terms of a gp is equal to sum of first 14 terms of same gp first 12 terms sum and first 14 term sum are the same given that sum of first 17 terms is 92 what is the third term in the gp the question concerns basically on sum of n terms of a gp let us solve it recall that S_n is used as a standard notation for sum of n terms of a a p or gp

so what's given to us is $S_{12} = S_{14}$ sum of first dual terms equal to sum of first fourteen terms but $S_{14} = S_{12} + t_{13} + t_{14}$ that gives $S_{14} = S_{12} + t_{13} + t_{14}$ that is $t_{13} + t_{14} = 0$ since it is a gp 14th term will be constant multiple of 13th term where that constant is called common ratio that is $t_{13} + r t_{13} = 0$ which amounts to $t_{13}(1+r) = 0$ that is $t_{13} = 0$ or $r = -1$ if $t_{13} = 0$ the 13th term is 0 in a gp all other terms will be 0 i mean 14 15 every term will be 0 because it is obtained by multiplying 13th term with r r^2 and

so on

so once 13th term is 0 all other successive terms will be 0.

in that case the sum of first 17 terms would be same as sum of first 13 terms which is same as sum of first 12 terms and

so on since $t_{13} = 0$ gives all terms to be 0 this case is ruled out and we are left with $r = -1$ thus we observe that the gp that we are concerned about has common ratio minus 1 let the first term be a then geometric progression would be $a, a r, a r^2$ and

so on which would be $a, -a, a, -a$ and

so on thus our gp reduces to this simple form $a, -a, a, -a$ and

so on alternatively positive and negative therefore sum of n terms of this gp would be same as adding a and minus a several times

so it will be 0 if n is even each a will cancel with minus a if n is even in case n is out each a will cancel with minus a but we will be left with the last a giving sum to a

so our observation is that sum of n terms of this particular gp would be 0 if n

is even and a if n is owed given sum of first 17 terms is 92 sum of first 17 terms is equal to 92 first n terms when n is owed some would be a since 17 is odd we have a equal to 92 the first term of our geometric progression is 92 and common ratio we already got as minus 1 therefore third term would be 92 let us proceed with the next problem this next problem reads as follows the sum of first 25 terms of an arithmetic progression is 525 and the sum of next 25 terms is 725 what is the common difference of this ap this problem concerns sum of n terms of an ap let us try to solve this problem let $t_1, t_2, \dots, t_{25}, t_{26}, \dots, t_{50}$ etc be the terms of given arithmetic progression next let us translate the information given in the question in symbol given that sum of first 25 terms denoted s_{25} is 525 recall that we have a ready made formula for sum of first n terms of an arithmetic progression further it is given that sum of next 25 terms which we denote as k_{25} to be 725 that is k_{25} which is equal to $t_{26} + t_{27} + \dots$ up to t_{50} is given to be 725 let me remind you that we don't have a ready-made formula to find some of terms of an ap starting from 26 term and ending with 52 however let us proceed as follows recall that 26th term t_{26} will be given by first term plus 25 into common difference similarly 27th term in notation t_{27} is given by first term plus 26 d with the intention of connecting k_{25} namely the sum of next 25 terms of the ap with s_{25} namely sum of first 25 terms of the same ap let us do the following the 27th term i can write as $t_1 + d + 25d$ is being simply decomposed into $d + 25d$ from this one can observe that 27th term is second term plus 25d in a similar fashion 28th term which is given by the formula first term plus 27 d can be rewritten as first term plus 2 d plus 25 d now observe that $t_1 + 2d$ is 30 therefore 28 term t_{28} can be written as third term t_3 plus 25d proceeding like this the 50th term t_{50} can be written as 25th term t_{25} plus 25d using this let us write expression for k_{25} which is equal to sum of $t_{26} + t_{27} + \dots + t_{50}$ can be written as $t_1 + 25d + t_2 + 25d + \dots$

so on $t_{25} + 25d$ let us regroup this as $t_1 + t_2 + \dots + t_{25} + 25d + 25d + \dots + 25d$ there are 25 such terms from this one can gather that k_{25} is equal to sum of first 25 terms that is $s_{25} + 25 \times 25d$ from this d can be isolated as follows $d = \frac{k_{25} - s_{25}}{625}$ substituting the value of given k_{25} and s_{25} we obtain $d = \frac{725 - 525}{625}$ by 625 with simple calculation d reduces to $\frac{200}{625}$ which can be written as $\frac{8}{25}$ thus the common difference of given arithmetic progression is $\frac{8}{25}$ we shall conclude this lecture with this problem we shall continue to explore more problems in coming lectures thank you