welcome you all to this eighth lecture on sequence and series in this lecture we shall explore more problems on sequence and series this is intended to enhance your understanding of concepts that we discussed

so far on this topic let us quickly recall that ap is a sequence in which difference between any two successive terms remains the same and this constant is referred to as common difference and arithmetic progression with first term as a and common difference as d can be represented as a a plus d a plus 2 d

so in this ap the nth term is given by the formula a plus n minus 1 into d we shall denote the n term by a n or sometime by t n and sum of first n terms of an ap denoted by sn has the formula sn equal to n by 2 into first term plus last term that's the nth term alternatively sn the sum of first n terms receives the formula s n equal to n by 2 into 2 a plus n minus 1 into d were as usual a is the first term and d is the common difference of the concerned ap here is your next problem which reads as follows if s 1 is the sum of first n terms of an ap were n is odd and s two is the sum of terms of this series in order places find the ratio s1 by s2 this problem concerns the sum of n terms of an arithmetic progression as you can see without further delay let us solve this problem lets a one a two and

so on b the arithmetic progression let us denote its common difference by the symbol d

so note that the arithmetic progression under consideration has first term a1 and common difference d now let us try to translate the given information using this simple what is given to you is the sum of first n terms of this ap further it is given that n is owed thus a 1 plus a 2 plus etcetera plus a n is given to be s1 we have a ready-made formula for sum of first n terms of an ap it is given by n by 2 times 2 into first term plus n minus 1 into common difference which we denoted by d thus s1 receives this formula see that s2 is given to be sum of terms of this series occurring in odd places for clarity i should say s2 is sum of terms of this series in odd places that is sum of first terms of this series first few terms of this series in old places therefore s 2 is a 1 plus a 3 plus a 5 plus etc plus since n is given to be ode the sum under consideration ends up with a n

so s2 is sum of terms occurring in odd places let us note that the sequence involved in s2 namely a1 a3 f5 and

so on is again an arithmetic progression to establish this note that a 3 minus a 1 can be written as a 3 minus a 2 plus a2 minus a1 using the fact that a1 a2 a3

so on is an ap a3 minus a2 is the common difference d similar is the case with a 2 minus a 1 therefore a 3 minus a 1 is 2d similarly f5 minus a3 is f5 minus a4 plus f4 minus a 3 with a little manipulation similar to what we did above we get it 2d proceeding like this we can see that the difference between successive terms of the progression a1 a3 a5 and

so on remains the same which is 2d hence a1 a3 f5

so on is an arithmetic progression with first term a1 common difference 2d next the question is how many terms are there in this arithmetic progression a1 a3 a5 and

so on a n it's not hard to see that the number of terms of this summation is n plus 1 by 2 thus s 2 being sum of n plus 1 by 2 terms of an ap with common difference equal to 2d receives the formula s2 equal to plus n plus 1 by 2 minus 1 times the common difference n plus 1 by 2 is the number of summons involved in this summation and 2 a 1 were a 1 is the first term of that ap and 2 d is the common difference as we observed simplifying this s 2 equal to n plus 1 by 4 2 a 1 plus n minus 1 times d thus we have the formula for s1 and s2 involved in the question we can complete the solution by just finding the ratio s 1 by s 2

namely n by 2 times 2 a 1 plus n minus 1 times d which is s 1 divided with n plus 1 by 4 times 2 a 1 plus n minus 1 times d which on further simplification is 2 n by n plus 1 this is the ratio of sum of first n terms with sum of terms occurring in the old places this completes the solution similarly let me remind you that a gp is a sequence where the ratio of two successive terms remains a constant this constant is referred to as common ratio a typical gp with first term a and common ratio r can be represented or listed as a a r a r square and so on let us recall that nth term of this gp is given by the formula a into r power n minus 1 we shall denote the nth term by a n or by t n further the sum of first n terms of this g p has the formula s n is equal to a into r power n minus 1 by r minus 1.

if r is not equal to 1 if r is equal to 1 the gp reduces to a constant sequence a a a and

so on hence sum of first n terms will be n times a further let us recall that sum of an infinite g p namely a plus a r plus a r square plus etcetera is a by 1 minus r if mod r is less than 1 if the common ratio has absolute value lying between 0 and 1 then the corresponding geometric series is convergent that is summable and its sum receives the formula a by 1 minus r in the other cases that is mode r greater than or equal to 1 the series a plus a r plus a r square plus etcetera is not convergent we cannot talk about it sum having recalled this let us try to tackle certain problems on sequence and series more specifically on the concept of ap and gp here is your first problem pf term of an ap is one by q and q at the term of the same ap is one by p prove that sum of first p q terms is 1 by 2 times pq plus 1 it's also given that p is not equal to q this is your question observe that the problem involves a p recalled the formulae for nth term of an ap and sum of first n terms of an ap for the solution let a b the first term and d b the common difference recall that for an ap with first term a and common difference d nth term is given by the formula a plus n minus 1 into d further recall that sum of n terms which we shall denote by sn is given by the formula n by 2 times 2 a plus n minus 1 into d or n by 2 times first term plus last term in the concerned sum having noted this formula the information given in the question translates to a plus p minus 1 into d which is the ph term is equal to 1 by q similarly the qth term which is given by the formula a plus q minus 1 into d is given to be 1 by p let us designate these two equations as 1 and 2.

recall that for the required sum what we need is the first term a and the common difference d let us try to find this first term and common difference from these two equations you have two equations and two unknowns namely a the first term and d the common difference let us subtract the second equation from first which will yield p minus 1 minus q minus 1 times d equal to 1 by q minus 1 by p which boils down to p minus q times d equal to p minus q by q p that on simplification is d equal to 1 by qp and this gives the common difference for our ap using this and one of these two equation let us isolate the first term note that the first equation gives a equal to 1 by q minus p minus 1 into d that is equal to 1 by q minus p minus 1 times d which we found out to be 1 by qp this simplifies to the one by q gets nullified yielding a the first term equal to 1 by p q thus our ap has first term 1 by pq and common difference 1 by pq using these two information s p q the sum of first p q terms is given by p q by 2 into 2 times a the first term which is 1 by p q plus p q minus one times d which is again one by p q this is obtained by plugging n equal to p q a equal to 1 by p q and d equal to 1 by p q in the formula for sum of first n terms of arithmetic progression this simplifies to p q by 2 2 by p q expanding this bracket it will be one minus one by p cube which on further simplification provides p q by 2 2 by p q minus 1 by p q gives 1 by p q plus one let us simplify further p cube by

two this is one plus p q by p q now p q gets cancelled and one by pq by 2 which establishes the required solution let us move further to the next problem a b and c are three consecutive terms of a gp further a power 1 by x equal to b power 1 by y equal to c power 1 by z prove that x y z are in a p you can observe that the problem concerned consecutive terms of a gp and ap here recall that three numbers m n and p are in gp implies the middle term n equal to root of the product of the other two terms in other words the middle term is geometric mean of the other two terms similarly recall that three terms are in ap means the middle term is arithmetic mean of the other two terms now the solution is immediate given a power 1 by x equal to b power 1 by y equal to c power 1 by z let us assume these equal quantities to be k thus a power one by x is k b power one by y is k and c power one by z is also k taking x the power on both side this would imply a equal to k power x similarly one would get b equal to k power y and c equal to k power is it note that since abc are in gp the middle term b is equal to the geometric mean of other two terms which same as saying b square is equal to ac that is b is k power y square is equal to product of a and c k power x into k power is it that is k power 2 y equal to k power x plus z this follows by law of indices this equality implies 2 y equal to x plus is it which reads y equal to x plus z by 2 that is y is arithmetic mean of x and yz which is same as saying x y and z are three consecutive terms of an arithmetic progression this fact we have established in our theory lectures let us continue with our next problem the next problem reads as follows let m and n be positive reals assume that arithmetic mean of m and n is capital a and geometric mean of m comma n is capital g then show that the quadratic whose roots are m and n is x square minus 2 a x plus g square equal to 0 as in the previous problem this concerned with three consecutive terms of an arithmetic progression and geometric progression let us solve it three numbers a b c are in ap if b equal to a plus c by 2 and a plus c by 2 is called arithmetic mean of a and c similarly abc are in gp implies that b square equal to ac that is b is square root of ac and square root of ac is referred to as geometric mean of a and c we are given that arithmetic mean of m and n is a that is m plus n by 2 is a which gives m plus n is equal to 2a similarly we are given that geometric mean of m and n is g that is square root of m and n is g which implies the product m n is equal to g square thus we have m plus n equal to 2a and mn is equal to g square let us keep it aside now a quadratic with roots m comma n is given by x minus m into x minus n is equal to 0 expanding we get x square minus m plus n times x plus mn is equal to 0.

 which is a well-known fact a quadratic with roots m and n are given by x square minus sum of the roots into x plus product of the roots equal to zero with the given information we have m plus n is two a and m n equal to g square let us move on question reads as follows if a is arithmetic mean and g is geometric mean of two positive numbers then show that numbers are a plus or minus root of a square minus g square it's very similar to the previous problem and it concerns arithmetic mean and geometric mean of two positive numbers let us recall again that arithmetic mean of two numbers a and b is a plus b by two and geometric mean of two numbers a and b is square root of a b further recall that arithmetic mean of two numbers is always greater than or equal to geometric mean and both coincides if the numbers are equal since a the arithmetic mean is greater than or equal to g the geometric mean root of a square minus g square is a real number you are talking about square root of a non-negative number
 so it makes sense let us call the numbers as amended let the numbers be positive numbers b m and n then what is given to us is arithmetic mean of m and n thus m plus n by 2 is given to be capital a which gives the sum of two unknown numbers which we are seeking for as 2a geometric mean of m and n which is root

mn is given to be g that gives product of the unknown numbers m and n which we are searching for is g square thus the problem reduces to finding two numbers whose sum is 2a and the product is g square which you might be familiar in quadratic equations however let us apply the details given m plus n is equal to 2a and m n is equal to g square with these two we have to find m and n let us recall that m minus n the whole square is m square minus 2 m n plus n square which can be thought of as m plus n the whole square minus 4 mn thus given sum and the product of two numbers we can find the difference of those two numbers plugging the value available to us this is 4 a square minus 4 g square

 so far our observation is that therefore a minus n is equal to plus or minus root of four times a square minus g square that is plus or minus 2 root of a square minus g square this is m minus n now we have m plus n and m minus n from which we can isolate m and n let us add these two to obtain two m is equal to two a plus or minus two times root of a square minus g square isolating m this gives m is equal to a plus or minus root of a square minus g square thus there are two possible values of m taking one particular value of m namely a plus root of a square minus g square you using one of this equation we can get n secondly we can take the other possible value of m and find out n we will get that the possible values of n are same a plus or minus root of a square minus g square let me leave the details to you let us continue given f is a function satisfying f of x plus y equal to f of x into f of y for every x y in n and natural numbers let f evaluated at one is three if sum f of x x is equal to 1 to n is 120 find value of n as such the problem does not seem to be connected with a p gp etcetera however note that it involves a summation sum x equal to 1 to n f of x equal to 120 which gives in expanded form f of 1 plus f of 2 plus f of 3 plus etcetera plus f of n is equal to 120 the question is concerned with a series let us proceed to solve it note that f is a function satisfying f of x plus y is equal to f of x into f of y for every x and y natural numbers with this f 2 can be calculated as f of 1 plus 1 which coincides with f of 1 into f of 1 that is f of two is equal to f of one square similarly f of 3 will be f of 2 plus 1 by the property of f given f of 2 plus 1 is f of 2 into f of 1 f of 2 is calculated to be f of 1 square

 so finally we get f of 3 is f of 1 q and

 so on continuing like this f evaluated at n is f evaluated at 1 taken nth power this is the observation from the property of f now we are given that summation x is equal to 1 to n f of x equal to 120 this is given that is f of 1 plus f of 2 plus etcetera up to f of n is 120 that is f of 1 plus f of two is f of one square plus etcetera plus f of n is f of one power n is one twenty the value of f at 1 is given to be 3 substituting this value 3 plus 3 square plus etc up to 3 power n is 120.

 thus collecting all the information given in the question we end up with this equation 3 plus 3 square plus etc plus 3 power n equal to 120 we have to get n out of this equation note that the finite sum occurring in the left hand side of this equation is sum of terms of a geometric progression do you see that the terms are 3 3 square 3 cube and

 so on hence it is a geometric progression with first term 3 and common ratio 3 the left hand side represents sum of terms of a gp with first term as 3 and common ratio as 3.

 let us recall the formula to sum first in terms of a gp it is sn is equal to a into r power n minus 1 by r minus 1 this is sum 2 first n terms of a gp using this we have the left hand side of the equation as 3 into 3 power n minus 1 by 3 minus 1 this is equal to 120 therefore 3 into 3 power n minus 1 is equal to 120 into 2 which is 240

 so 3 power n minus 1 is equal to 240 by 3 which is 80

so 3 power n is equal to 81 thus 3 power n is equal to 81 which i can express in terms of power of 3 81 is 9 into 9 which is 3 multiplied 4 times and that yields n equal to 4 which solves the question let us continue find some of terms of the following sequence the given sequence is 7 77 777 777 and
 so on up to n terms one could easily observe that the given sequence namely 7 77 777 and
 so on is neither in arithmetic progression nor in geometric progression
 so as such we cannot use the ready-made formula available for sum of n terms of an ap or gp however we shall tackle this problem as follows let us denote the sum required to be sn seven plus seventy seven plus seven seven d seven plus etc up to n terms remember for an infinite sum we need the notion of convergence that is whether it is summable or not in fact if you consider this sum with infinite terms then we cannot have a finite value because the nth term grows arbitrarily as n becomes larger however we are not asked to find the sum up to infinity we have to sum only first n terms as i pointed out earlier the difficulty here is that apparently it's neither an arithmetic progression nor a geometric progression whose sum we are familiar with let us pull a 7 out
 so that it will be 7 into 1 plus 11 plus 1 1 1 plus etcetera when i say etc here i mean sum up to n terms only however the sum inside bracket is not sum of terms of an ap or a gp
 so the issue remains let us write it as let us multiply by an ion and a divide to nullify the effects thus the given sum takes the form 7 by 9 into 9 plus 99 plus etcetera the whole point is to introduce an ap or a gp here with that in mind let us write 9 as 10 minus 1 99 as 100 minus 1 and
 so on when i say
 so on we mean only up to n terms now this is 7 by 9 into 10 plus 100 plus 1000 plus etc up to n terms and minus 1 minus 1 etc added n times which amounts to minus n now you can see that a gp has appeared ten plus hundred plus thousand plus etcetera corresponds to a geometric progression with the first term as ten and the common ratio as ten therefore the sum of first n terms of that g p is seven pi nine into for that g p is concerned it is a times r power n r is ten here minus one by r minus 1 and then the second term minus n this is the required formula for sum
 so you see though the given sum neither corresponds to a gp or an ap it is convertible in some way or the other to a gp which facilitated us to solve this problem we shall continue with more problems in the next lecture too thank you you