welcome to the second lecture on sequence and series from the first lecture it should be clear that by a sequence a n n is equal to 1 to infinity written in expanded form as a 1 a 2 a 3

so on a n

so on we actually mean a function f from n to r we are talking about real sequence and various ways to describe a sequence including the recursive formula were a particular term of a sequence is expressed in terms of one or more of its previous term now let us try to represent a sequence using graph commonly a sequence is represented by graphs in two ways consider the sequence a n n is equal to one to infinity one way to represent this using a graph is to mark a few terms of this sequence on a real line let us say a 1 a 2 a 3 and

so on a specific example will make it more clear consider the sequence a n n is equal to 1 to infinity where a n is given by root n nth term is given by root n to represent this using graph we plot the real axis here is 0 there is 1 2 3 4 5 and

so on the first term of this given sequence namely a n with a n as root n is 1

so this is a 1 a 2 is going to be root 2 it's less than 2 greater than 1

so somewhere here a 3 is going to be root 3 which is greater than root 2

so it is somewhere here and a 4 is root 4 which is 2 a 5 is root 5 greater than 2 but less than 3 this is the graph of the given sequence root n let us see another example consider sequence b n n is equal to 1 to infinity where the nth term b n is given by 1 by n to graph this let us have the real line see that b one will be one one by one b two is one by two

so this is b one b two is one by two which is halfway between zero and one b three is one by three which is less than one by 2

so somewhere here b 4 is 1 by 4 which is half b between 0 and half

so this is b4 and

so on you can see that the terms of the sequence coming nearer and nearer to zero this is one way to represent a sequence by graph another way is as follows recall that a sequence is a function and hence we can graph the corresponding function to explain it with specific example consider the sequence a n n is equal to 1 to infinity where a n is given by root n we are going to consider the corresponding function f from n to r given by f of n is equal to root n and we are going to consider the graph of this function

so for that we consider the axis with n plotted along the x axis and a n plotted along the y axis corresponding to 1 the value of the function is root 1 which is 1

so the point is 1 1 corresponding to 2 let me mark the points on y axis which designates a n corresponding to 2 the value of the function is root 2

so we plot 2 comma root 2 2 is here root 2 is in between 1 and 2.

so this is 2 comma root 2 and corresponding to 3 the value of the function is root 3 which is less than 2 but greater than root 2

so somewhere here 3 root 3 corresponding to 4 the value of the function is root 4 which is 2

so we plot 4 2 and

so on and these isolated points provides the graph of the function corresponding to the sequence a n is equal to root n this is another way to graph a sequence having said this let us practice with some examples to graphs the basic concept of sequence i am going to give some problems write first five terms of the sequence given by the formula a n the nth term of the sequence is equal to n into n plus 2 i should admit that it's bit confusing to say first five terms of the sequence because we have remarked that a sequence can be treated as a function from n to r in that case the corresponding list will be a1

a2 a3 and

so on or it may be treated as a function from a subset of n to r in that case it is not necessary that the sequence starts with a 1 it may be for instance a 6 a 7 a 8 and

so on but unless otherwise specified let us assume that the sequence start with n is equal to 1 that is the list consists of a 1 a 2 and

so on with that agreement we will find first 5 terms of the given sequence a 1 the first term is 1 into 1 plus 2 obtained by just plugging n is equal to 1 is 3 a 2 is obtained by plugging n is equal to 2 this 2 into 2 plus 2 which is 2 into 4 which is 8 a3 is obtained by substituting n is equal to 3

so 3 multiplied with 3 plus 2 which amounts to 3 multiplied with 5 which is 50 a 4 is given by 4 multiplied with the number corresponding to 4 plus 2 which is 4 into 6 which is 24 and a 5 is obtained by plugging n equal to 5 in the formula phi into 5 plus 2 which is 5 into 7 which amounts to 35 the list consists of 3 8 15 24 and 35 these are the first 5 terms if you want to write it as a sequence it will be 3 8 15 24 35 etc etc and the number occurring in nth place will be n into n plus 2 etc and it is always suggested to write if possible the term in the nth place as a function of n instead of just listing first few terms what i mean is for a sequence a n corresponding to this problem it is always suggested to list it as 3 8 15 24 35 etc and then the nth place element n into n plus 2 etc instead of just writing 3 8 15 24 35 etc the reason from the first few terms the pattern may not be always recognizable let us proceed with the example find first four terms of the sequence given by the formula a n is equal to n into n square plus phi by 4.

of course this is just numerical but let us do it step by step all the details first term corresponding to this sequence namely a 1 is 1 into 1 square plus 5 by 4 which on calculation reduces to 1 into 6 by 4 which can be simplified into 3 by 2.

second term a2 is equal to 2 into 2 square plus 5 by 4 which on calculation reduces to 2 into 4 plus 5 by 4 which reduces to 2 into 9 by 4 by cancellation it reduces to 9 by 2 let us proceed third term a3 will be 3 into 3 square plus 5 by 4 which on calculation reduces to 3 into 3 square is 9 9 plus 5 by 4 which is 3 into 9 plus 5 is 14 by 4 which reduces to 3 into 7 by 2 which is 21 by 2 last term to be found is the fourth term since the question demands you to find first four terms is 4 into 4 square plus 5 by 4 and this on calculation reduces to 4 into 4 square is 16 plus 5 by 4 which can be written as 4 into 21 by 4 and which gives a whole number twenty one

so these are the first four terms if you list the first four terms alone it will be three by two nine by two twenty one by two twenty one hope i have not made any calculation mistake it is a good exercise to recheck next example demands you to find ninth term of the sequence sequence a n n is equal to 1 to infinity where a n is given by minus 1 power n minus one into n cube the ninth term namely a nine can be obtained by plugging n equal to 9 in the general expression

so a nine will be minus 1 power 9 minus 1 into 9 cube which is equal to minus 1 power 8 into 9 cube which is minus 1 power 8 amounts to 1 and 9 cube is 81 into 9 which is 7 2 9 that solves the previous problem in the next example you are asked to find first three terms of the sequence sequence a n n is equal to one to infinity were a n is described by the formula a n is equal to a n minus 1 minus 1 for n greater than 2 and a 1 and a 2 are two i would suggest you to recall the rabbit problem example were instead of writing a n in terms of n we write a n in terms of its previous terms describing a sequence such a manner that the nth term is expressed in terms of previous terms is called recurrence relation or it is called recursive definition of the sequence and here you are

given with the recursive definition a n equal to a n minus 1 minus 1 and the recursion starts with the terms a 1 and a 2 which is given to be 2.
 in fact a 1 is 2 and a 2 is 2 which is given this is how you can start the recursion for n greater than 2 a n is defined as the previous term minus 1 therefore a 3 will be put n equal to 2 in the recursive relation a 3 will be a 2 minus 1 and a 2 is given to be 2 2 minus 1 which is 1
 so you should observe that unlike in the case where sequence is described with a formula in terms of n in the recursive definition to get a particular term we may have to find the previous term and then plug that previous term and
 so on let us continue with one more example find first to four terms of the sequence a n n is equal to one to infinity were a n is described using a 1 is equal to 3 and a n is equal to 3 into n minus 1 for n greater than or equal to 2 observe that in this example as well the sequence is described with a recurrence relation first term is given to be 3 the second term a2 using the recurrence relation is 3 times 1 which is 3 into 3 3 square third term a 3 is equal to 3 into a 2 and a 2 we have previously found which is 3 into 3 square which is 3 cube a 4 is 3 times a 3 this is by the recursive definition which is equal to 3 times a3 we have found in the previous step and which amounts to three power four these are the four terms asked to found but then in this example let us see whether we can find a n in terms of n remember that the recurrence relation given a n is 3 into a n minus 1 using the recurrence relation a n minus 1 is 3 into a n minus 2 and a n minus 2 is 3 into a n minus 3 and
 so on applying this in succession we will see that it will be a 1 times observe that 1 can be thought of as n minus n minus 1
 so that the power of 3 with which a 1 to be multiplied is 3 power n minus 1 see the pattern when we have a n minus 1 power of 3 is 1 when we have a n minus 2 power of 3 is 2 and
 so on
 so when we have a 1 that is a n minus n minus 1 the power of 3 is n minus 1 a 1 is given to be 3
 so this is 3 power n minus 1 into 3 which is 3 power m in this example even though as such the sequence is given in terms of a recursive definition using that recursive definition successively we could write a n in terms of n this is called closed form expression for a the nth term a n is expressed as a function of n
 so that given any n we can directly find what will be the term corresponding to the given n without finding the previous terms this way in which a n is expressed solely in terms of n is called closed form expression this example illustrate that there are cases where though the sequence is expressed originally in terms a recursive definition or a recurrence relation ultimately we can come up with a closed form expression for the same sequence this is called solving of recurrence relation of course there is a systematic theory to solve given recurrence relation we are not going into the details of it but this example is expected to shed light that there are cases where a recurrence relation given for a sequence can be solved to obtain the nth term of a sequence in terms of n there are cases where the recurrence relation is preferred rather than formula in terms of n having said this let me continue with few more examples but this time with a different intention consider the sequence a n n is equal to 1 to infinity where the nth term a n is given in terms of n using the expression a n is equal to 1 by n to be explicit let me write few terms of this sequence first term is 1 second term is 1 by 2 third term is 1 by 3 fourth term is 1 by 4
 so on and
 so forth recall the same thing can be expressed graphically in fact in two ways

let me use the first way of marking the terms in the real axis this is the real axis with natural numbers or more specifically non negative integers indicated the first term is 1

 so this is a 1 second term is 1 by 2 it is midway between 0 and 1 this is a 2 1 by 2 and a 3 is 1 by 3 somewhere here a 4 is 1 by 4 it is midway between 0 and a2

 so this is a4 and

 so on do you observe either from the list of elements in the sequence or from the graph that we have plotted that as we progress towards the end of the sequence one one by two one by 3 1 by 4 etcetera etcetera the terms get closer and closer to 0 because as you progress towards end of the sequence n increases number corresponding to the place first place second place third place and

 so on increases that is n increases the number occurring in nth place is 1 by n

 so as n increases 1 by n keeps on decreasing and it eventually comes near to 0

 so the observation in regard to this example is that as we progress towards the end of the sequence in other words as we increase n the term a n becomes close to a fixed number zero that's clear from the graph a1 a2 a3 a4 that moves towards zero keeping this in mind let us proceed with another example consider the sequence 0 1 by 2 2 by 3 3 by 4 etc as i told you previously it is always suggestive if possible to write what is the term in nth place if you see the pattern it is n minus 1 by n and

 so on consider this sequence now let us do the similar exercise as in previous example that is let us observe what happens to this sequence as n becomes larger and larger one way to do it is draw the graph i mean represent the sequence using graph using the first method i suggested here is zero here is 1 let us say 2 and

 so on the first term is 0 this is a 1 second term is 1 by 2 which is midway between 0 and 1 this is 8 2 third term is 2 by 3 which is greater than 1 by 2 you can observe it but it is less than 1

 so somewhere here is a3 a4 is again less than 1 because it is 3 by 4 but it is greater than a3

 so somewhere here and

 so on you can observe more and more points f i a six and

 so on you can see that the terms as far as this example is concerned comes closer and closer to one it can also be observed in a different manner without relying on the plot let us write the term 0 1 by 2 2 by 3 and

 so on the nth term namely n minus 1 by n can be rewritten as 1 minus 1 by n isn't it now the terms are 0 the second term is actually 1 minus 1 by 2 third term is 1 minus 1 by 3 and

 so on nth term is 1 minus 1 by m now can you guess what happens when n becomes larger and larger as n becomes larger 1 by n comes close to 0

 so that 1 minus 1 by n those numbers will come near to 1.

 thus as far as this particular example is concerned as n increases or as you move towards the tail end of the sequence the terms of the sequence are coming close and close to 1.

 recall in the previous example namely the sequence 1 by n as n becomes larger and larger the terms were becoming very close to 0 and in this example as n becomes larger and larger the other way of saying it is as you progress towards the end of the sequence the terms gets closer and closer to 1.

 let us continue with another example namely the sequence root n n is equal to 1 to infinity to be explicit let us list few more terms 1 root 2 second term root 3 third term and

 so on nth term is root n and

 so on it's an infinite sequence let us do the exercise we have done previously

namely try to observe what happens when n becomes larger and larger remember
when n grows larger root n grows larger for instance root of hundred which is
ten is greater than root of two root of ten thousand is greater than root of 100
namely 10 and

so on thus when you progress towards the end of the sequence the terms are
becoming larger and larger and this growth is not controllable in the sense as
you go on you can increase the value of the terms thus unlike in the previous
example in this example we don't observe that as n becomes larger and larger the
terms of the sequence become close to some particular value if you graph it it
will be like first term is 1 second term is root 2 which is greater than 1 third
term is root 3 which is greater than 1 and also greater than root 2.

a 4 which is root 4 which is 2 and

so on

so whatever large number i give you can find a term in this sequence such that
that term is larger than number given by me for example suppose i say 100 you
can always find a term in this sequence greater than 100 for instance if you
find a 1001 it will be actually root of ten thousand and one and root of ten
thousand and one is greater than hundred

so given hundred i can find a term namely ten thousand one term which is
greater than hundred now suppose i give another number larger than hundred still
you can find a term am greater than the given number k in other words as you
progress the terms of the sequence are becoming larger and larger

so you do not find any fixed number to which the terms are becoming near now
let us proceed with another example consider the sequence 1 minus 1 1 minus 1
etc the nth term is minus 1 power n plus 1 first term is minus 1 square which is
1 second term is minus 1 power 3 which is minus 1 and

so on here as you progress towards the end of the sequence in other words when
you make n larger and larger what happens to the sequence is its bounces back
between 1 and -1 if n is a large number which is an ode integer then n plus 1
will be even and the term will become 1 and if n is a large number which is an
even integer then n plus 1 will become ode and hence the term will be minus 1
thus as n progresses the terms will be either 1 or minus 1 we cannot say it
definitely it depends upon what is the value of n

so we cannot find a number to which the terms of the sequence come closer
consolidating the previous examples that you see in the first example namely 1
by n what happened was as n increases the terms changes but then it becomes
nearer and nearer to zero in the second example as n increases the term comes
nearer and nearer to one in the third example as n increases the term becomes
larger and larger

so that we cannot say that that all terms will be ultimately near to some fixed
number in the last example though the terms are not becoming larger either it
will be 1 or minus 1 but still we cannot find a number

so that all the terms are coming close to this particular number these examples
should illustrate that there are cases where sequence behave in such a manner
that as n increases term become close to a number and there are instances where
as n increases the terms of the sequence will not remain near a fixed number to
distinguish these two cases we introduce the terms called convergent sequence
and divergent sequence informally a sequence is said to be convergent if as n
increases the nth term of the sequence a n comes near to a fixed number l let me
write informally a sequence a n n is equal to 1 to infinity is said to be
convergent if as n increases a n a ns become sufficiently close to a number l
keep the first example in mind namely the sequence 1 by n as n increases the
terms become close to 0.

remember that the number l may not be a term in the sequence for example in the

case of 1 by n terms are becoming closer and closer to 0 but no term is exactly 0.

such sequences for which there exists a number l such that as n increases all the terms of sequence a n comes near to l is called convergent this l is called limit of that sequence in notation we write it as limit n tending to infinity a n is equal to l l is called the limit of the sequence and such a sequence is called convergence if there is no number l for which as n increases a and come close to l then the sequence is called divergence those who recall limit of a function can understand that this is a particular case in the sense that sequence is also a function let us not go into the details of this convergence sequence etc but at least informally what's the meaning of a sequence to be convergent and what the meaning of limit should be clear we will not dwell on the precise definition of convergence how sequence conversion and the limit of a function which you studied etcetera are connected having known the informal definition of convergence let us practice with some more examples consider the sequence a n n is equal to 1 to infinity where a n is equal to 5 by n square of course here i used a bracket instead of a set notation i should say that a sequence can also be represented by using parenthesis like this instead of set notation we have used

so far that is instead of writing a sequence like this we can write in this manner as well in fact i feel this is more suggestive in the sense that it will not be confused with a set sequence is ordered whereas set there is no specific order of elements now let us consider the sequence a n where nth term a n is given by the formula phi by n square now i would like you to guess what happens to the terms of the sequence as n becomes larger and larger let us list a few terms first term is 5 second term is 5 by 2 square third term is 5 by 3 square fourth term is 5 by 4 square and

so on observe that the numerator is fixed to be 5 and denominator gets increased like 2 square 3 square 4 square etc the 100th term will be 5 by 100 square etc

so as you progress as n becomes larger and larger the terms 5 by a very very large number goes to 0.

so almost all terms after that will be zero not almost all term in fact every term will become closer and close to zero except finite number of terms okay in other words this can be represented by limit n tending to infinity 5 by n square is 0 or the sequence 5 by n square is convergent and 0 is its limit as a final example consider the sequence sequence a n n is equal to 1 to infinity where a n is given by 4 minus 7 n power 6 by n power 6 plus 3.

the question is observe what happens to the terms of the sequence when n becomes larger and larger or determine whether the given sequence is convergent or not as such by the given form it may not be straightforward to see what happens to a n as n tending to infinity but let us do some manipulation a n can be written as i am taking an n power 6 common from numerator

so it will be 4 by n power 6 minus 7 by n power 6 1 plus 3 by n power 6.

now n power 6 cancels out and nth term can be rewritten as 4 by n power 6 minus 7 by 1 plus 3 by n power 6.

now as n becomes larger and larger 4 by n power 6 becomes closer and closer to 0 because denominator n power 6 gets increase

so this goes to 0

so the numerator reduces to minus 7 as n becomes larger similarly the denominator 1 plus 3 by n power 6 the second summat namely 3 by n power 6 goes to 0 as n tending to infinity three by n power six become close to zero

so denominator becomes close to one plus zero

so as a result a n when you observe as n becomes larger and larger comes near to minus 7 thus the given sequence is convergent and the limit is minus -7 we will discuss more about sequence and then enter into series in the next class thank you you