

welcome you all to this first lecture on the topic sequence and series to begin with let me draw your attention to the fact that the two words namely sequence and series are used interchangeably in day-to-day life we don't make any distinction between the words sequence and series in day-to-day life for instance when we say a sequence of events or when we say a series of mathematics tests or when we say cricket test match series in these instances a sequence or a series is used to suggest succession of events or succession of objects by succession i mean a ordered list to sum up we don't want to make any distinction between the two words sequence and series in day-to-day life however in mathematics the two words sequence and series are used with separate technical meaning having said this a natural question at this juncture is what are the different technical meaning that we want to attach with the words sequence and the series or how do they differ further is there any relationship between these two words and these questions will be answered as the course progresses let us look at the word sequence in mathematics to begin with let me give a couple of examples first example i list even integers as you all know even integers to be specific even positive integers can be listed as 2 4 6 8 and

so on the end even integer will be $2n$

so on and

so forth as another example let us consider the process of dividing 10 by let us say 3.

let us list the successive quotient obtained when we divide 10 by 3 at different steps

so what we want to list is the quotient that we obtain in the division process while we perform the division in step by step process

so let us divide 10 by 3

so 3 continuing 3.

3 3.

33 and

so this is what i mean by listing successive quotient when we perform the division 10 by 3 stepwise as another example let us consider the

so called rabbit problem assume that a pair of rabbits say one male and one female is put in a field assume that after one month the rabbits get sexually matured and the female produces a new pair of rabbits again pair consists of male and female at the end of second month let us consider some idealistic circumstance let us say that the rabbits never die and let us say that each female rabbit produces a new pair of rabbits again male and female in every month from second month onwards is it clear

so this is the situation and the question is how many pairs of rabbits are there at the end of one year let us say this is the question the solution generation by generation produces a list of numbers the

so called historically called fibonacci numbers let us have a try and list some fibonacci numbers let us try to find how many pairs of rabbits are there at the end of one month two months three months and

so on let us eventually find out how many pairs of rabbits are there at the end of one year we started with one pair of rabbit one male and one female recall that at the end of one month the rabbits get mature but then it does not produce any new rabbit

so at the end of one month total pair of rabbits in the field is again one the end of second month the female rabbit produces a new pair of rabbit

so that totally there will be two pairs of rabbit at the end of second month now recall that a female produces a pair of rabbit every month from second month onwards

so at the end of third month there will be three pairs of rabbits one pair

newly produced by the original female that we put in the field now at the end of four months the female rabbit created at the end of second month will produce a new pair and hence there will be total of five pair rabbit at the end of four months you can continue this and try to list what will be the number of pairs of rabbit at the end of fifth month six month and

so on keeping in mind that the female rabbit produced two months ago will produce a new pair of rabbits this problem was originally posed by fibonacci and the numbers created which stands for the number of pairs at the end of one month two months and

so on is called fibonacci numbers let us continue with another example now the problem is related to a depositor and a bank assume that a bank pays interest at the rate of 10 percentage per annum further assume that a depositor invests rupees 1 let us say at the bank if the bank calculates simple interest then the question of what will be the amount the depositor get after one year can be obtained from the formula that you have studied earlier amount equal to principle plus interest recall the investment principle is rupees 1 in case of simple interest the formula is $p n r$ where p stands for the principle n stands for the number of years and r stands for rate of interest per annum in this case it is easy to calculate that the interest amounts to 1 into 1 into 1 by 10 namely 1 by 10 and hence the amount at the end of one year will be one plus one by ten now let us assume that the interest is computed as compound interest let us further assume that the interest is compounded twice a year in that case it will yield an amount equal to 1 plus the interest for half year namely 1 into 1 by 2 into 1 by 10 is equal to 1 plus 1 by 20 at the end of half year first half here this will be the amount for the second half for this amount the interest will be the amount into number of years it is half into rate and hence the interest for the amount yielded in first half will be this and total amount at the end of 1 year would be 1 plus 1 by 20 which is the amount after first half year plus 1 by 20 into 1 plus 1 by 20 which amounts to 1 plus 1 by 20 the whole square do you see it now let us assume that the bank calculate compound interest and compounded twice a year by a similar procedure you can see that the amount at the end of one year when interest is calculated in a compounded fashion twice an year at equal rest of course will be 1 plus 1 by 30 the whole cube it's not hard to calculate please have a try suppose we assume that the bank calculates interest compound fashion but compounded n times in any year at equal rest let us say amount after one year would be equal to 1 plus 1 by 10 n the whole power n now we list the amount we obtain if the bank calculates simple interest will be 1 by sorry 1 plus 1 by 10 amount obtained if the bank compounds interest half yearly will be 1 plus 1 by 20 the whole square the amount that depositor gets if the bank calculates compound interest compounded twice an year will be 1 plus 1 by 30 the whole cube etc if we assume bank compounds interest n times an year at equal rest then the amount obtained will be 1 plus 1 by 10 n the whole power n we can continue listing many examples but or what i want you to observe through this example is that what we deal with in the first example namely list of even integers in the second example namely the list of successive quotient obtained when 10 is divided by 3 step-by-step manner and in the rabbit problem which gives number of pairs of rabbits at the end of n month and

so on we deal with ordered list of numbers do you see it in all this example in one way or the other we deal with ordered list of numbers a sequence informally is an ordered list of numbers for instance let us have one 3 5 7 9 etc let us have 1 1 by 2 1 by 3 1 by 4 1 by 5 etc slightly more generally let us have the list $a_1 a_2 a_3$ etc a_n etc where a_i 's are numbers this is what informally a sequence is by this time whenever you hear a word sequence you should attach it with an ordered list of numbers when i say ordered it should be born in mind

that the first member in the list of $a_1 a_2 a_3$

so on a n is a_1 second number in the list is a_2 third number in the list is a_3 and

so on thus though it may not be apparently visible there is an input output arrangement with the place taking the role of input that is first place second place third place and

so on takes the roles of input and the numbers we list $a_1 a_2 a_3$ and

so on take the role of output in other words when we list $a_1 a_2 a_3$ and

so on and we emphasize that order is important what we mean is first place

number a_1 occurs second place number a_2 occurs third place number a_3 occurs and

so on could you see an input output arrangement here with inputs as the places 1 2 3 and

so on and output as the numbers we listed $a_1 a_2 a_3$

so on yes recall that an input output arrangement that is a rule which gives a unique output to each input is what we call mathematically as function

so associated with a sequence in fact there exists a function and what's the domain of this function the places which number occur in which place takes the role of the domain

so 1 2 3 etc constitute the domain and the numbers we list take the role of range hence a sequence can be more formally defined as follows a real sequence is a function f from set of natural numbers to the set of reals if we designate set of natural number by \mathbb{N} and set of reals by \mathbb{R} a sequence is a function f from \mathbb{N} to \mathbb{R} more precisely a real sequence is a function from \mathbb{N} to \mathbb{R} f of n is what we write as a_n when we list $a_1 a_2 a_3$ and

so on inherent with it there is a function which sends 1 to a_1 2 to a_2 3 to a_3 and

so on if that inherent function is named as f f of 1 is a_1 .

f of 2 is a_2 and

so on the number a_n we list in n th place of a sequence is actually the function evaluated at n to sum up sequence informally means an ordered list of numbers and more formally it is a function f from set of natural numbers to \mathbb{R} for instance in the example of lists of even natural numbers 2 4 6 8 the n th even natural number is $2n$ and

so on the associated function f can be written as f from \mathbb{N} to \mathbb{R} f of n is equal to $2a$ when you hear the word sequence you should immediately attach it with a function with the domain as set of natural numbers in fact the codomain can be different from \mathbb{R} it can be a general set yes but we will confine to the case of real sequence in the sense that the elements that we list are always real numbers having said this let me set some notation a sequence a_n can be represented or described by writing the rule that provides n th term a general term of the sequence by the way when we have a sequence $a_1 a_2 a_3$ etcetera a_n etcetera $a_1 a_2$ etcetera those numbers are called terms one way to describe a sequence is write a_n in terms of n for instance the sequence of even integers positive even integers can be described by writing a_n is equal to $2n$ n is equal to 1 2 3 etc this is one way of describing a sequence another way to describe a sequence is to list its terms and write a sequence as $a_1 a_2$

so on a_n and

so on in a compact manner this can be written also as set $\{a_n \mid n \text{ is equal to } 1 \text{ to infinity}\}$

so far we have seen two ways of describing a sequence one write a rule that provides you the n th term in terms of n or we can list the terms inside a set set $\{a_1 a_2 a_3\}$ and

so on a_n and

so on or in a compact manner it can be written as set $\{a_n \mid n \text{ equal to } 1 \text{ to}$

infinity if you recall the examples we started with more specifically in the rabbit problem you can see that we have listed some few terms of the sequence at the end of one month the total pair of rabbits available in the field is one at the end of two months total pair of rabbits available in the field is again 1 and then 2 3 and

so on you can observe that at a given stage say at the end of n month the number of rabbits will be the number of pairs of rabbits available at the end of previous month plus number of pairs of rabbits available two months before because each rabbit two months old can produce a new pair you can observe that a_n is equal to a_{n-1} plus a_{n-2} for every n greater than or equal to 2 and instead of writing or describing the number of pairs of rabbit in end of n months in terms of n most easiest way as far as this problem is concerned is to write the n th term in terms of previous terms such an expression which describes a sequence by writing a particular term using previous terms is called recurrence relation thus recurrence relation is another way to describe a sequence to sum up a sequence can either be described with the help of a function which provides the n th term in terms of n it can be listed like this inside the site notation or in some specific problems it will be easier to describe a particular term in terms of the previous terms and that's called recurrence relation at this gender i should make a remark concerning the notation of a sequence using set as i told previously a sequence can be described inside a set like $a_1 a_2 a_3$ and

so on a_n but it should be born in mind that a sequence is different from set for instance whereas in in set the order in which the elements occur is not important whereas in sequence the order in which the elements occurs it matters in other words sequence 2 4 6 8 etc is different from sequence 4 to let us say 8 6 etc for instance whereas as i said both are same another reason that i can give why sequence should be streeted different from set is the following consider the sequence a_n n is equal to 1 to infinity written in more expanded form as let us say 1 1 by 2 1 1 by 3 1 1 by 4

so on and

so forth more precisely it can be described as follows a_{2n-1} that is all terms first term third term fifth term and

so on is 1 for every n element of n and even terms second term

so on is of the form 1 by n the second term a_2 is 1 by 2 the fourth term a_4 is 1 by 3 and sixth term a_6 is 1 by 4 and

so on that is a the second term 2 into 1 is 1 by 2 the fourth term a 2 into 2 is 1 by 3 the sixth term a 2 into 3 is 1 by 4 and

so on

so a $2n$ if you see the pattern it will be 1 by n plus 1 you can cross check it so this is the sequence inherent with the list 1 1 by 2 1 1 by 3 1 1 by 4 and

so on the o terms are 1 and the even terms a_{2n} can be described with the help of 1 by n plus 1 for every n in n whereas underlying set here set a_n recalling that in the set we don't write the elements repeated just one one by two one by three one by four and

so on

so in a sequence the element can be repeated and in the set we do not write the same element repeatedly several times thus though we use the notation set a_n n equal to 1 to infinity for a sequence keep in mind that sequence is different from a set mainly in the sense that sequence is an ordered list whereas in a set we don't bother about order in which elements occur first by recalling that in the precise definition of a sequence a sequence is defined as a function with the domain as set of natural numbers that is a sequence is a function f from n to r recall this fact now consider the list for instance 12 14 16 18 and

so on can you recognize the pattern it is not hard to recognize the pattern and write a_n if it follows this pattern the n th term will be $10 + 2n$ am i right the first term is $10 + 2$ the second term is $10 + 2 \times 2$ the third term is $10 + 2 \times 3$ that is 16 and

so on

so the sequence 12 14 16 18 etc can be described as sequence a_n n is equal to one to infinity where a_n is expressed with the help of the rule i would like to draw your attention to the fact that the same sequence can also be expressed as b_n where n is equal to 6 to infinity b_n is equal to $2n - 6$ is $2 \times 6 - 6 = 6$ b_7 is $2 \times 7 - 6 = 8$ and

so on

so the same sequence can be described with b_n where b_n is given by the rule $2n - 6$ but now n starts from 6 and continues up to infinity

so this remark is to pronounce that though in the definition of a sequence we took function from n to r sometimes it is convenient to work with a subset of n starting from say n_0 instead of whole n we may work with the set $n_0, n_0 + 1$ and

so on in the previous example we started with b_6, b_7 and

so on this remark should shed light on the fact that a sequence can be defined as a function from some subset of natural numbers to r though it is customary to define it from set of natural numbers recall the notion of term of a sequence if you have the sequence a_1, a_2, a_3 and

so on the elements a_i are called terms there may be circumstances where we want to have sequence with finite number of terms in the previous list of examples that we have seen rabbit problem we were asked only to find how many pairs of rabbits are there at the end of one year

so our list will be fine as we have to deal only up to number of pairs of rabbits at the end of 12 months like this there are instances where we would like to have sequences with finite number of terms and such sequence with only finite number of terms are called finite sequence a sequence such as the list of or even integers consists of infinite number of terms and sequences with infinite number of terms are called infinite sequences we will be mainly concerned with infinite sequences that is a sequence having infinite number of terms going back to the formal definition since we have to incorporate also sequences with finite number of terms a sequence may be defined as a function from set of natural numbers to r or a finite subset of set of natural number 1 2 3 etcetera up to k to r let me write it here a sequence in fact a real sequence is defined as a function from set of natural numbers denoted by n to r or a function from a subset 1 2 3 etc up to k of n to r in the definition we added a function from a subset 1 2 3 etcetera up to k of n to r just to incorporate finite sequences at the end of this lecture you should be able to understand informal definition of sequence namely a sequence is ordered list of real numbers formal definition a sequence is a function from set of natural numbers to r or a subset of natural numbers to r various ways of describing sequence one is using the notation a_n n is equal to 1 to infinity and listing it as a_1, a_2, \dots or writing a_n in terms of n using a rule or with the help of recursive definition you should also know why a sequence is different from sets we shall proceed with more examples of a sequence and other notions regarding sequence in next few lectures thank you