

this is our second and the last problem-solving session on arithmetic geometric and harmonic progressions we resume this session with problem number eight now we have the following question let  $a$ ,  $b$  and  $c$  be in an arithmetic progression and  $a^2$ ,  $b^2$  and  $c^2$  be in a geometric progression if  $a$  is strictly less than  $b$  and  $b$  is strictly less than  $c$  with  $a + b + c = 3\sqrt{2}$  then we shall find out the value of  $a$  since we know that  $a$ ,  $b$  and  $c$  are in an arithmetic progression we can take  $a$  as  $p - r$ ,  $b$  as  $p$  and  $c$  as  $p + r$  for sum  $p$  and  $r$  now we are given that  $a + b + c = 3\sqrt{2}$

so therefore we are getting  $p - r + p + p + r = 3\sqrt{2}$   
so from here we are getting that  $p = \sqrt{2}$  note that this is the value of  $b$  also if we can now find out the value of  $r$  then we know what is  $a$  because  $a$  is nothing but  $p - r$

so  $\frac{1}{2} - r$

so we need to only find out now the value of  $r$  in the question we are given that  $a^2$ ,  $b^2$  and  $c^2$  are in a geometric progression

so we have  $(p - r)^2$ ,  $p^2$  and  $(p + r)^2$  are in geometric progression

so we can write  $(p - r)^2$  into  $(p + r)^2$  is equal to  $p^2$  to the power 4 therefore we get  $(p - r)^2$  whole square is equal to  $p^2$  to the power 4 hence  $p^2$  to the power 4 minus  $2 p^2 r^2$  plus  $r^2$  to the power 4 is equal to  $p^2$  to the power 4 that means  $r^2$  into  $r^2$  minus  $2 p^2 r^2$  is equal to  $0$  note that  $a$ ,  $b$  and  $c$  are distinct numbers  $a$  is strictly less than  $b$  and  $b$  is strictly less than  $c$

so here we can conclude that  $r$  cannot be equal to  $0$  therefore we must have  $r^2$  is equal to  $2 p^2$  that means we have  $r^2 = 2$  so  $r = \pm \sqrt{2}$

so for  $a$  we are getting two possibilities  $\sqrt{2} + \sqrt{2}$  or  $\sqrt{2} - \sqrt{2}$

now recall that we are given  $a$  is strictly less than  $b$  and  $b$  is strictly less than  $c$  and the value of  $b$  we know already is  $\sqrt{2}$  therefore this choice of  $a$  is not possible and hence  $a = \sqrt{2} - \sqrt{2}$

therefore here the third option is the correct answer this is our question number 9 if the sum of the first 10 terms of an arithmetic progression is  $c n^2$  then we shall find out the sum of the squares of those terms note that the sum of first  $n - 1$  terms of this progression is  $c(n - 1)^2$  because the sum of the first  $n$  terms is  $c n^2$  therefore we know what is the  $n$ th term the  $n$ th term is nothing but the sum of first  $n$  terms minus the sum of first  $n - 1$  terms

so we have the  $n$ th term is  $c n^2 - c(n - 1)^2$  which is  $c(2n - 1)$

so we know what is the  $n$ th term of this arithmetic progression that is  $c(2n - 1)$  let us call  $r$ th term of this progression by  $a_r$  our job is to find out sum over  $r$   $a_r^2$  running from 1 to up to  $n$   $a_r^2 = c^2(2r - 1)^2$  now we know that this is nothing but we can take  $c^2$  out and inside we have  $(2r - 1)^2$   
so this is  $4 c^2$  into sum running over  $r$  from 1 to up to  $n$   $(r^2 - 4r + 1)$  inside we have  $r^2 + c^2$  into sum over  $r$  are running from 1 to up to  $n$  inside we have 1.

so if we sum this up we get  $4 c^2$  into  $n(n + 1)(2n + 1)$  divided by 6 minus  $4 c^2$  into  $n(n + 1)$  divided by 2 plus  $c^2 n$

so we take  $c^2$  in divided by 3 common from this expression and we get inside  $2n^2 + 2n + 1 - 2n - 2 + 3$  simplifying that we get  $c^2 n$  divided by 3 into  $4n^2 + 4n + 2$

minus six n minus three that is  $c^2 n$  divided by 3 into  $4n^2 - 1$   
 so now we know what is the sum of the squares of the first 10 terms of this arithmetic progression the third option we can see is correct here we consider the following question for  $1 \leq i \leq 10$  let  $b_i$  be strictly bigger than 1  $\log_2 b_i$  is in an arithmetic progression with the common difference  $\log_2 2$  and  $a_1 = 2$  up to  $a_{10}$  be in an arithmetic progression

so that  $a_1$  is equal to  $b_1$  and  $a_{10}$  is equal to  $b_{10}$  now consider the sum of  $a_1$  to up to  $a_{10}$  to be  $s$  and the sum of  $b_1$  to up to  $b_{10}$  to be  $t$  then we shall find out among the four options given here which one is the correct answer since  $\log_2 b_1, \log_2 b_2, \dots, \log_2 b_{10}$  is in an arithmetic progression with the common difference  $\log_2 2$  we can write  $\log_2 b_2$  is equal to  $\log_2 b_1 + \log_2 2$  that is  $\log_2 b_2$  is equal to  $\log_2 (2b_1)$  now taking exponential on both sides we get  $b_2$  is equal to  $2b_1$  next let us look at  $\log_2 b_3$

so  $\log_2 b_3$  is equal to  $\log_2 b_2 + \log_2 2$  now we know that  $\log_2 b_2$  is equal to  $\log_2 (2b_1)$

so we are getting that  $\log_2 b_3$  is equal to  $\log_2 (2^2 b_1)$  again we take exponential on both sides and we get  $b_3$  is equal to  $2^2 b_1$  if we proceed in this way we would obtain  $b_i$  is equal to  $2^{i-1} b_1$  for all  $2 \leq i \leq 10$

so we are getting one form of  $b_i$  for all  $i$  bigger than or equal to 2 in terms of  $b_1$  next let us write what is  $t$  we know that  $t$  is  $b_1 + b_2 + b_3 + \dots + b_{10}$

so on and

so forth up to  $b_{10}$

so  $t$  is equal to  $b_1 + 2b_1 + 2^2 b_1 + \dots + 2^9 b_1$

so on and

so forth up to  $2^{10} b_1$  let us take  $b_1$  out and we get  $1 + 2 + 2^2 + \dots + 2^9$

so on and

so forth up to  $2^{10} b_1$  now note that this is a geometric series and this is equal to  $2^{11} b_1 - b_1$  therefore we get  $t$  is equal to  $b_1 (2^{11} - 1)$  next let us try to write  $s$  in a simpler form we know that  $s$  is equal to  $a_1 + a_2 + a_3 + \dots + a_{10}$

so on and

so forth up to  $a_{10}$  we can write  $a_2$  as  $a_1 + d$ ,  $a_3$  as  $a_1 + 2d$  and continuing this way we have the last term  $a_{10}$  as  $a_1 + 9d$  where  $d$  is the common difference of the arithmetic progression  $a_1, a_2, \dots, a_{10}$  now if we collect all the events together we get  $10a_1$  and from the remaining terms we take  $d$  common and we get  $1 + 2 + \dots + 9$

so on and

so forth up to  $50$  again we can see that this is an arithmetic progression and this is equal to  $50 \times 51$  divided by 2 let us now find out what is  $d$  for that we recall in the question we had  $a_{10}$  is equal to  $b_{10}$  and  $a_1$  is equal to  $b_1$

so  $a_{10}$  is equal to  $a_1 + 9d$  as  $a_{10}$  is equal to  $b_{10}$  and  $a_1$  is equal to  $b_1$  we get  $b_{10}$  is equal to  $b_1 + 9d$

so  $d$  is equal to  $(b_{10} - b_1) / 9$  now we know that  $b_{10}$  is equal to  $2^{10} b_1$

so we are getting  $d$  is equal to  $(2^{10} b_1 - b_1) / 9$  now if we substitute this value of  $d$  here and as  $a_1$  is equal to  $b_1$  we get  $s$  is equal to  $51b_1 + 2^{10} b_1 - b_1$  we take  $51b_1$  out and inside we write  $2^{10} + 1$  divided by 2.

to compare with  $t$  we write this one as  $(51b_1 + 2^{10} b_1 - b_1) / 2$  we already got  $t$  is equal to  $b_1 (2^{11} - 1)$  therefore we can conclude that  $s$  is strictly bigger than  $t$

so coming back to the question option 3 and option 4 are not correct now we are going to see among  $a_{101}$  and  $b_{101}$  which one is bigger we know that  $b_{101}$  is equal to 2 to the power 100 into  $b_1$  and  $a_{101}$  is equal to  $a_1$  plus 100  $d$  and we have already got the value of  $d$  which is 2 to the power 50 minus 1 divided by 50 into  $b_1$  therefore we have here  $a_{101}$  is equal to  $b_1$  plus we are substituting  $b_1$  in place of  $a_1$  and this is here 2 to the power 51 minus 2 into  $b_1$

so basically we are getting 2 to the power 51 minus 1 into  $b_1$

so clearly  $b_{101}$  is strictly bigger than  $a_{101}$  therefore the second option is correct and the first option is incorrect this is our 11th question let  $a_1, a_2, a_3$  and

so on and

so forth be an infinite harmonic progression with the first term being 5 and the 20th term being 25 we shall find out the least positive integer in for which  $a_n$  is negative we write the  $n$ th term  $a_n$  as  $\frac{1}{b + n - 1}$  into  $d$  for sum  $b$  and  $d$  now note that  $a_1$  is equal to  $\frac{1}{b}$  as  $a_1$  is given to be 5 therefore we get  $b$  is equal to  $\frac{1}{5}$  also note that  $a_{20}$  is equal to  $\frac{1}{b + 19}$  therefore  $b + 19$  is equal to  $\frac{1}{a_{20}}$  which is equal to  $\frac{1}{25}$  and we know  $b$  is equal to  $\frac{1}{5}$  therefore  $d$  is equal to  $\frac{-4}{25}$ .

now our job is to find out the least positive integer  $n$

so that  $a_n$  is strictly less than 0 as  $a_n$  is  $\frac{1}{b + n - 1}$  into  $d$  we shall find out the least positive integer in for which  $b + n - 1$  into  $d$  is strictly less than 0 we know  $b$  is equal to  $\frac{1}{5}$  and we also know the value of  $d$

so we use this inequality to find out the range of  $n$  for which  $a_n$  is negative now substituting the value of  $b$  and  $d$  here we get  $\frac{1}{5} + n - 1$  into  $\frac{-4}{25}$  is strictly less than 0 this implies  $1 + n - 1$  into  $\frac{-4}{25}$  is strictly less than 0 this implies  $n - 1$  shall be strictly bigger than  $5$  into  $19$  divided by  $4$  that is  $n$  shall be strictly bigger than  $5$  into  $19$  divided by  $4$  plus 1 and this is equal to  $99$  divided by  $4$

so therefore  $n$  has to be bigger than or equal to 25

so that  $a_n$  is strictly less than zero therefore 25 is the least positive integer for which  $a_n$  is negative hence we get option 4 is correct in this question we have four distinct numbers  $a_1, a_2, a_3$  and  $a_4$  which are in a geometric progression we have  $b_1$  is equal to  $a_1$  and  $b_i$  is equal to  $b_{i-1} + a_i$  for all  $i$  is equal to 2, 3 and 4 then we have two statements the first statement is the numbers  $b_1, b_2, b_3$  and  $b_4$  are neither in arithmetic progression nor in geometric progression and second statement is the numbers  $b_1, b_2, b_3$  and  $b_4$  are in harmonic progression we have to check whether statement 1 and statement 2 are correct or not and in case both the statements are true we have to check whether statement 2 is a correct justification of statement 1 or not we have  $b_1$  is equal to  $a_1$  and  $b_2$  is equal to  $b_1 + a_2$  as  $b_1$  is equal to  $a_1$  we get  $b_2$  is equal to  $a_1 + a_2$  now  $b_3$  is equal to  $b_2 + a_3$

so  $b_3$  is equal to  $a_1 + a_2 + a_3$  and  $b_4$  is equal to  $b_3 + a_4$

so  $b_4$  is equal to  $a_1 + a_2 + a_3 + a_4$  note that from here we get  $b_2 - b_1$  is equal to  $a_2$  and  $b_3 - b_2$  is equal to  $a_3$  we are given that  $a_i$ 's are all distinct

so  $a_2$  is not equal to  $a_3$  that means  $b_2 - b_1$  is not equal to  $b_3 - b_2$

so we have  $2b_2$  is not equal to  $b_1 + b_3$  this implies  $b_1, b_2, b_3$  are not in arithmetic progression

so  $a_1, a_2, a_3$  and  $a_4$  are not in arithmetic progression next we say whether  $b_1, b_2, b_3$  and  $b_4$  are in geometric progression or not for that let us write  $a_i$  as  $a_1$  into  $r$  to the power  $i - 1$  for all  $i$  is equal to 2, 3 and 4 as we are given that  $a_1, a_2, a_3$  and  $a_4$  are in a geometric progression here this  $r$  is the common ratio of the geometric progression  $a_1, a_2, a_3, a_4$  let us make a small note here

that  $a_1$  is not equal to 0 and  $r$  is not equal to 0 because if  $a_1$  is equal to 0 or  $r$  is equal to 0 then it contradicts the fact that  $a_i$ 's are all distinct using this we obtain  $b_2$  is equal to  $a_1$  plus  $a_1 r$

so this is nothing but  $a_1$  into  $1 + r + r^2$  is equal to  $a_1$  plus  $a_1 r$  plus  $a_1 r^2$  which is same as  $a_1$  into  $1 + r + r^2$  and  $b_4$  is equal to  $a_1$  plus  $a_1 r$  plus  $a_1 r^2$  plus  $a_1 r^3$  that is  $b_4$  is equal to  $a_1$  into  $1 + r + r^2 + r^3$  now if  $b_1, b_2, b_3$  and  $b_4$  are in a geometric progression then we must have  $b_1$  into  $b_3$  is equal to  $b_2$  square let me write it here if  $b_1, b_2, b_3$  are in a geometric progression

so we shall have  $a_1$  into  $a_1$  into  $1 + r + r^2$  is equal to  $a_1$  square into  $1 + r$  whole square now as  $a_1$  is non-zero we can cancel  $a_1$  from both sides and we get  $1 + r + r^2$  has to be equal to  $1 + r$  whole square that is  $1 + 2r + r^2$  recall that we already got  $r$  is not equal to zero we can easily note that for  $r$  not equal to zero this equality does not hold

so in this case we do not get  $b_1$  into  $b_3$  is equal to  $b_2$  square that is we have  $b_1$  into  $b_3$  is not equal to  $b_2$  square

so  $b_1, b_2, b_3$  and  $b_4$  are not in geometric progression

so here our statement 1 is true

so we can immediately strike off option 1.

next we check whether statement 2 is correct or not for that we have to check whether  $b_1, b_2, b_3$  and  $b_4$  are in harmonic progression or not if  $b_1, b_2, b_3$  and  $b_4$  are in harmonic progression then we must have  $\frac{1}{b_1} + \frac{1}{b_3}$  is equal to  $\frac{2}{b_2}$  that is we shall have  $\frac{1}{a_1(1+r+r^2)} + \frac{1}{a_1(1+r+r^2+r^3)}$  is equal to  $\frac{2}{a_1(1+r+r^2)}$  that is we shall have  $\frac{1+r+r^2+r^3}{1+r+r^2+r^3} + \frac{1}{1+r+r^2+r^3}$  is equal to  $\frac{2}{1+r+r^2}$  that is  $\frac{1+r+r^2+r^3+1}{1+r+r^2+r^3}$  shall be equal to  $\frac{2}{1+r+r^2}$  simplifying this we get  $2+r+r^2+r^3+2r+r^2+r^3$  shall be equal to  $2+2r+2r^2$  that is we shall have  $r$  into  $1+r+r^2$  is equal to 0 that is we shall have  $r$  is equal to 0 or  $r$  is equal to plus minus  $i$  now recall that we have already made this remark that  $r$  is not equal to 0 as  $a_1, a_2, a_3$  and  $a_4$  all of them are distinct and if  $r$  is equal to plus minus  $i$  then note that  $b_4$  is equal to  $a_1$  into  $1+r+r^2+r^3$  is equal to 0 therefore  $b_1, b_2, b_3$  and  $b_4$  cannot be in harmonic progression

so we see that statement 2 is false and already we have seen that statement 1 is true

so option 4 is the only correct answer this solves our question number 12.

here is the question number 13 let  $a_1, a_2$  up to  $a_{100}$  be in an arithmetic progression with  $a_1$  being 3 and let us call sum over  $i$   $i$  running from 1 to up to  $p$   $a_i$  as  $S_p$  for all  $1 \leq p \leq 100$  for any integer  $n$  with  $1 \leq n \leq 20$  we take  $m$  to be  $5n$  then if  $S_m$  by  $S_n$  does not depend on  $n$  we shall find out the value of  $a_2$  first note that to find out the value of  $a_2$  it is enough to find out the value of the common difference of this arithmetic progression let us call  $d$  to be the common difference of this arithmetic progression now  $a_2$  is nothing but  $a_1$  plus  $d$

so as we know what is  $a_1$  if we can find out what is  $d$  we will know what is  $a_2$  we have  $S_p$  is equal to sum over  $i$   $i$  running from 1 to up to  $p$   $a_i$

so this is equal to  $p$  into  $a_1$  plus  $\frac{p-1}{2}$  plus

so on and

so forth up to  $p-1$  into  $d$  we know that  $S_1$  is equal to 3

so we substitute it here

so  $S_p$  is equal to  $3p$  plus  $\frac{p-1}{2}$  into  $d$  let us take  $t$  by 2 common then we get inside  $6$  plus  $p-1$  into  $d$  now we write what is  $S_m$  by  $S_n$  where  $m$  is equal to  $5n$

so  $m$  by  $s$   $n$  is equal to  $s$   $5$   $n$  by  $s$   $n$  and this is equal to  $5$  in by  $2$  into  $6$  plus  $5$  in minus  $1$  into  $d$  divided by  $n$  by  $2$  into  $6$  plus  $n$  minus  $1$  into  $d$  cancelling  $n$  by  $2$  from the denominator and the numerator we get  $30$  plus  $25$   $n$  minus  $5$  into  $d$  divided by  $6$  plus  $n$  minus  $1$  into  $d$  in the question we are told that  $s$   $m$  by  $s$   $n$  does not depend on  $n$  that assures that  $s$   $m$  by  $s$   $n$  is a constant let us call that constant as  $c$

so we have  $30$  plus  $25$   $n$  minus  $5$  into  $d$  is equal to  $c$  into  $6$  plus  $n$  minus  $1$  into  $d$  to emphasize we have  $sm$  by  $sn$  is a constant which we have called here by  $c$  because even if  $n$  changes the ratio  $s$   $m$  by  $s$   $n$  does not change now from this equation we get  $30$  plus  $25$   $n$   $d$  minus  $5$   $d$  is equal to  $6$   $c$  plus  $c$   $n$   $d$  minus  $c$   $d$

so  $25$  minus  $c$  into  $n$   $d$  is equal to  $6$   $c$  minus  $c$   $d$  minus  $30$  plus  $5$   $d$  therefore if  $25$  minus  $c$  into  $d$  is non-zero then we get a definite value of  $n$  from this equation but this equation is true for all  $n$  bigger than or equal to  $1$  and less than or equal to  $20$

so we must have  $25$  minus  $c$  into  $d$  is equal to zero from here we get either  $d$  is  $0$  or  $c$  is equal to  $25$  now if  $d$  is equal to  $0$  then we get a  $2$  is equal to a  $1$  plus  $0$

so a  $2$  is equal to  $3$  and if  $c$  is equal to  $25$  then  $30$  plus  $25$   $nd$  minus  $5d$  divided by  $6$  plus  $nd$  minus  $d$  is equal to  $25$  therefore  $30$  plus  $25$   $nd$  minus  $5$   $d$  is equal to  $150$  plus  $25$   $nd$  minus  $25$   $d$  therefore  $20$   $d$  is equal to  $150$  minus  $30$  which is equal to  $120$

so from here we obtain  $d$  is equal to  $6$

so in this case we get  $a_2$  is equal to  $a_1$  plus  $6$

so  $a_2$  is equal to  $9$  hence  $a_2$  has  $2$  possible values namely  $3$  and  $9$  this solves our question number  $13$ .

this is our  $14$ th question let  $a$   $b$  and  $c$  be three positive integers such that  $b$  by  $a$  is an integer if  $a$   $b$  and  $c$  are in a geometric progression and the arithmetic mean of  $a$   $b$  and  $c$  is  $b$  plus  $2$  then we shall find out the value of  $a$  square plus  $a$  minus  $14$  divided by  $a$  plus  $1$  since  $a$   $b$  and  $c$  are in a geometric progression we write  $a$  as  $b$  by  $r$  and  $c$  as  $p$  into  $r$  where  $r$  is the common ratio we are given that  $b$  by  $a$  is an integer in fact we know that  $v$  by  $a$  is a positive integer as  $b$  and  $a$  both of them are positive note that from here we obtain  $r$  is equal to  $b$  by  $a$  therefore we can conclude that the common ratio  $r$  is a positive integer we denote the set of positive integers by said in the subscript strictly bigger than  $0$  we know the arithmetic mean of  $a$   $b$  and  $c$  is  $b$  plus  $2$  therefore  $a$  plus  $b$  plus  $c$  divided by  $3$  is equal to  $b$  plus  $2$  that means  $a$  minus  $2$   $b$  plus  $c$  is equal to  $6$  we substitute  $a$  is equal to  $b$  by  $r$  and  $c$  is equal to  $b$   $r$  in this equation we then get  $b$  by  $r$  minus  $2$   $b$  plus  $v$   $r$  is equal to  $6$  that is  $b$  minus  $2$   $b$   $r$  plus  $b$   $r$  square is equal to  $6$   $r$  we can write it as  $r$  squared minus  $2$   $r$  plus  $1$  into  $b$  is equal to  $6$   $r$

so  $p$  by  $r$  into  $r$  minus  $1$  whole square is equal to  $6$  we could divide by  $r$  because we know  $r$  is non-zero now recall that  $b$  by  $r$  is equal to  $a$  therefore we have  $a$  into  $r$  minus  $1$  whole square is equal to  $6$ .

we also know that  $a$  and  $r$  both of them are positive integers note that the only possible positive integer solution of this equation is  $r$  is equal to  $2$  and  $a$  is equal to  $6$

so we substitute  $a$  is equal to  $6$  in this expression  $a$  square plus  $a$  minus  $14$  divided by  $a$  plus  $1$  and we get this is  $36$  plus  $6$  minus  $14$  divided by  $7$  that is  $28$  by  $7$

so we get the value of this expression is  $4$

so this solves our question number  $14$  let us now look at the following question suppose that all the terms of an arithmetic progression are positive integers if the ratio of the sum of the first seven terms to the sum of the first  $11$  terms is  $6$  is to  $11$  and the seventh term of the progression lies between  $130$  and  $140$

then what is the common difference of this arithmetic progression let us denote the common difference of this arithmetic progression by  $d$  and  $r$  term of this progression by  $a_r$  therefore we have  $a_r = a_1 + (r-1)d$  for all  $r$  bigger than or equal to 1 we are given that  $a_r$  belongs to the set  $Z$  strictly bigger than  $0$  that means  $a_r$  is a positive integer for all  $r$  bigger than or equal to 1 therefore we can conclude that  $d$  is also a positive integer because  $d$  is nothing but  $a_2 - a_1$  and we know this is a positive integer and also this is a positive integer all right in the question we are also given that  $\sum_{i=1}^7 a_i = 63$  and  $\sum_{i=1}^{11} a_i = 110$  now note that  $\sum_{i=1}^7 a_i = 7a_1 + 21d$

so on and

so forth up to 6 into  $d$  note that this is equal to  $6a_1 + 15d$  therefore this sum turns out to be  $7a_1 + 21d = 63$  next we consider the sum over  $i$   $i$  running from 1 to up to 11  $a_i$  this is equal to  $11a_1 + 55d = 110$

so on and

so forth up to 10 into  $d$

so this is  $11a_1 + 55d = 110$  divided by 2 into  $d$

so this sum turns out to be  $11a_1 + 55d = 110$  therefore from this equation we get  $7a_1 + 21d = 63$  divided by  $11a_1 + 55d = 110$  is equal to  $63/110$  that means  $7a_1 + 21d = 63$  that is  $a_1 = 9 - 3d$  we are also given that  $130 < a_7$  and  $a_7$  is strictly less than  $140$  now  $a_7 = 1 + 6d$  and as  $a_1 = 9 - 3d$  we get  $a_7 = 1 + 6d = 9 - 3d + 7d$

so therefore we have  $130 < 1 + 6d$  and  $d$  is strictly less than  $140/7 = 20$

so we get this inequality  $26 < 3d$  and  $d$  is strictly less than  $28/3$  we can write it like this  $9 < d < 28/3$  as we know  $d$  is an integer we get  $d = 10$

so the common difference of this arithmetic progression is  $9$ .

with this we conclude our problem solving session on arithmetic geometric and harmonic progressions you