

welcome back to the iit pal lectures on mathematics and today's lecture is going to be once again on the binomial theorem and its applications this is the sixth lecture for the binomial theorem and problems of the binomial theorem ok so we have done quite a few problems in the past we have solved some problems related to integral part fractional part we have done a variety of binomial expansions today we are going to do a few more and these are probably some of the harder problems ok

so among the harder problems we will start with some identities ok so we have done a few identities a long time back but ah this is once again going back to a few identities because these are the harder ones okay

so one popular identity is this  $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n}$  and

so on all the way till  $\binom{n}{n-1} + \binom{n}{n}$  and you have to show that this is equal to  $2^n$  all the way till  $\binom{n}{n}$  divided by  $n!$  times  $n!$

so this is a complicated one but if you rack your brains you are not going to find a straightforward way to do this expansion right but the answer lies somewhere else the answer actually lies in the very first class when we introduced the binomial theorem even before we introduced the binomial theorem

so the answer lies in pascal's triangle all right

so think of the following this point was  $1 \binom{n}{0}$  this is  $2 \binom{n}{0}$  to  $3 \binom{n}{0}$   $4 \binom{n}{0}$   $5 \binom{n}{0}$

so this is your placing you are giving a new light throwing a new light on the pascal's triangle and this is because we have already done five lectures in between pascal's triangle and today right i am sorry this is zero  $\binom{n}{0}$  actually i wont correct all of this let me just add one more dot over here that will be fine yeah

so this is one  $\binom{n}{0}$  the top one is zero  $\binom{n}{0}$  its not really that important this is one  $\binom{n}{1}$  two  $\binom{n}{2}$  three  $\binom{n}{3}$  four  $\binom{n}{4}$  five  $\binom{n}{5}$  and

so on right and what was the property of pascal's triangle the property was that if i want to compute any one of these dots all i have to do is add this one and this one ok look over here you see the resemblance you see why we are going to pascal's triangle ok

so for example  $5 \binom{n}{2}$  this is  $5 \binom{n}{2}$  is equal to four  $\binom{n}{1}$  plus four  $\binom{n}{2}$  ok so in that case can you tell me what is  $\binom{n}{0} + \binom{n}{1}$  now we are talking about  $n \binom{n}{0}$  right

so  $n \binom{n}{0}$

so if this is my  $n \binom{n}{0}$  and if this is  $n \binom{n}{1}$  then these 2 are going to add and give me the next row in the pascal's triangle which is going to be  $n+1 \binom{n}{1}$  ok then  $n \binom{n}{1}$  is going to add with the next number which is  $n \binom{n}{2}$  and that is going to give me  $n+1 \binom{n}{2}$  all right

so this complicated thing is nothing but  $n \binom{n}{0} + n \binom{n}{1} + n \binom{n}{2} + n \binom{n}{3} + \dots + n \binom{n}{n}$  is  $n+1 \binom{n}{1} + n \binom{n}{2} + n \binom{n}{3} + \dots + n \binom{n}{n}$  and

so on all the way sorry is the multiply till  $n+1 \binom{n}{n-1} + n \binom{n}{n}$  right that is equal to  $n+1 \binom{n}{n}$  is this fine

so far

so good and if you have gone this far the remaining part is trivial ok

so if you have gone this far the rest of it is very easy all you have to do is break it up  $n+1 \binom{n}{1}$  is factorial  $n+1$  which happens to be  $n+1$  times factorial  $n$  divided by in the denominator you have got factorial 1 and you have got factorial  $n$  but instead of writing  $n!$  i am going to write factorial  $n-1$  times  $n$  and then  $n+1 \binom{n}{2}$  is again  $n+1$  times factorial  $n$  divided by factorial two times factorial oh no yeah ah factorial  $n$

minus one and instead of writing factorial  $n$  minus one you could write it as factorial  $n$  minus 2 multiplied by  $n$  minus 1 and

so on and

so forth right all the way till the end at the end you have  $n$  plus 1  $C_n$  which is  $n$  plus 1 times  $n$  factorial divided by  $n$  factorial and 1 factorial

so instead of 1 factorial I am going to write  $0$  factorial times 1.

so there are  $n$  terms you have got  $n$  plus 1  $n$  plus 1 out of all these  $n$  terms

so all of these are going to become  $n$  plus 1 whole power  $n$  and then the way I have written it smartly right this portion is  $n$   $C_1$  right this portion is  $n$   $C_2$  this portion is next one you will get  $n$   $C_3$  and

so on all the way till  $n$   $C_n$  ok

so you have got all of those  $C_1$   $C_2$  all the way till  $C_n$  and then here you have got an  $n$  into  $n$  minus one into all the way down to one

so you have got  $n$  factorial and  $C_0$  is super flow  $C_0$  is equal to one

so that proves the identity right think about it you can try not using Pascal's theorem and not use this insight you will be pretty stuck with this problem okay

so this was one example of a hard problem

so here you have to think a little bit a little laterally right you have to go back to Pascal's theorem to figure out the sum of two coefficients two consecutive coefficients all right another example ok how are you going to do this this actually is not that hard how will you do this one any guesses  $n$  into  $n$  plus one by two if that is the answer right if you look at the answer the answer is  $n$  into  $n$  plus one by two that's a very familiar answer there is a clue there what is the clue the clue is that this should look like may be maybe this is going to look like  $1 + 2 + 3 + 4 + \dots$  all the way till  $n$  maybe ok and if that is the case then we are done maybe that's not it but if that is the case then we are done ok let's check let us try it out  $C_1$  by  $C_0$  is it one yes it is but  $C_2$  by  $C_1$  is not one is it no  $C_2$  by  $C_1$  is not one  $n$   $C_2$  divided by  $n$   $C_1$  ok that is not one  $C_1$  by  $C_0$  is also not one  $C_1$  is  $n$  yeah sorry correction  $C_1$  is  $n$

so if this is  $n$  look at it the numerator is going from  $C_1$  to  $C_2$  to  $C_3$  it is increasing the denominator is also increasing okay

so it could be it just could be that the whole thing is dropping all the way from  $n$  to one

so maybe it's a backward sequence  $C_1$  by  $C_0$  is  $n$  maybe this is  $n$  minus one this  $n$  minus two all the way till over here where maybe it is one could that be the case let's see right let's see

so if you look at  $C_1$  if I look at let's say  $C_r$  by  $C_{r-1}$  what is that equal to oh by the way this is not  $C$  this is  $r$  times  $C_r$  by  $C_{r-1}$  that's the  $r$ th term in this series what is  $C_r$   $C_r$  is factorial  $n$  by factorial  $r$  by factorial  $n$  minus  $r$  and what is  $C_{r-1}$   $C_{r-1}$  is factorial  $n$  by factorial  $r$  minus 1 by factorial  $n$  minus  $r$  plus 1 and then of course  $n$  factorial  $n$  factorial cancels out  $r$  times  $r$  minus 1 factorial by  $r$  factorial cancels out and all you are left with is  $n$  minus  $r$  plus 1 factorial by  $n$  minus  $r$  factorial and  $n$  minus  $r$  plus 1 is just one more than  $n$  minus  $r$  which means this is equal to  $n$  minus  $r$  plus 1

so this is the  $r$ th term the  $r$ th term evaluates to  $n$  minus  $r$  plus one

so which term is this this is one term right  $C_r$  right  $C_1$  by  $C_0$  times  $C_1$  by  $C_0$

so the 1 term is going to be  $n$  minus 1 plus 1 which is  $n$

so this term is evaluating to  $n$  the next term is the 2th term  $n$  minus 2 plus 1 which is  $n$  minus 1.

the third term  $r$  equal to three  $n$  minus three plus one the fourth term will be

$n$  minus three all the way till the  $n$ th term  $n$  minus  $n$  plus one

so this is a decreasing sequence

so it is as if one plus two backwards if you look you get one plus two plus three all the way till  $n$  if you look forwards it's a decreasing sequence and this obviously is equal to  $n$  into  $n$  plus one by two ok

so this is a trick question it looks tricky it is not that tricky ah let us try another one ok

so look earlier we had done a problem with which was which looked like if you recollect we had done a problem earlier which look like this  $c_1 + 2c_2 + 3c_3$  all the way till  $c_n$  is equal to  $n$  into  $2$  power  $n$  minus one earlier we had done something like this how did we do it you remember how we did it we looked at one plus  $x$  whole power  $n$  and then we did a differentiation of  $1 + x$  whole power  $n$  right that gave me  $n x$  power  $n$  minus 1  $n$  into  $1 + x$  whole power  $n$  minus 1 and then i plugged in  $x$  equal to 1.

so this was really  $n$  into  $1 + x$  whole power  $n$  minus 1 with  $x$  equal to  $x$  equal to 1 okay that is how we had arrived at this result but look here the coefficients were increasing right the coefficient of  $c_2$  was increasing  $2c_2$   $3c_3$  and

so on it reminded you of a differentiation here you are getting by 2 by 3 by 4 and what does this remind you of yes what does it remind you of it reminds you of integration the opposite of differentiation ok

so the clue over here is that you have to integrate not differentiate and then maybe later on you can plug in  $x$  equal to one or we will see what needs to be plugged in later on but the clue is you have to integrate okay now integration is always a little complicated right why is it complicated because you have an arbitrary constant whenever you talk about integration integral of  $x dx$  is  $x$  squared by 2 plus an arbitrary constant and that arbitrary constant is something very irritating right why is it annoying because if i have two sides and i integrate let's say i have some equation some identity right i just multiply both sides by  $dx$  and do an integral will the two answers necessarily be equal no why because there could be some arbitrary constant here there could be some other arbitrary constant here which means that these two integrals need not be exactly equal to each other ok

so this is not correct ok

so that is a little bit of a problem

so even before we start this right the clue was integration but when you whenever you have to do integrals you have to remember that if you just integrate two sides of an identity you do not always get an identity ok ah it is a little problematic what do we do how do you correct for these constants how do you fix the ok i want you  $c_1 + c_2$  sorry  $k_1 + k_2$  ok how do you get rid of the arbitrary constant you have to use a definite integral and not an indefinite integral ok that will fix the problem

so let us try  $1 + x$  whole power  $n$  and what is this equal to this is equal to  $c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$  all the way till  $c_n x^n$  this is something that we know and now what we are going to do is we are going to integrate both sides using a definite integral

so let's say i integrate  $1 + x$  whole power  $n dx$  from  $x$  equal to 0 to  $x$  equal to  $p$  then the right hand side also i am going to integrate definite integra integration ok

so far

so good this is equal a definite integral it works fine indefinite integral does not work fine ok

so let us do the definite integral of the left hand side right how do you do it

you first do an indefinite integral  $1 + x$  whole power  $n + 1$  by  $n + 1$  is that it how do you check if your integral is correct by doing a differentiation if you differentiate this do i get that one back you check you differentiate  $n + 1$  times  $1 + x$  whole power  $n$  divided by  $n + 1$

so it nicely cancels out

so indeed the integral of this is therefore equal to this but this integral you have to take it from  $0$  to  $p$  and if you plug in  $0$  if you plug in  $p$  you get one plus  $p$  ok ah we do not really need  $p$  right we we can plug in  $p$  equal to one let us not do  $p$

so let us work with  $p$  equal to one

so we are going to integrate from zero to one zero to one if i plug in  $x$  equal to one what do i get two power  $n + 1$  by  $n + 1$  and if i plug in  $x$  equal to  $0$  i get  $1$  by  $n + 1$  okay and why i did this was because of the question the questions look like this  $2$  power  $n + 1$  minus  $1$  the whole divided by  $n + 1$  one look over here two power  $n + 1$  by  $n + 1$  minus one by  $n + 1$

so i have arrived at the question right that is why i plugged in  $p$  equal to one over there ok

so the left hand side evaluates to my question very nice now all i have to do is work out the right hand side this is what i have ok i expanded the sum and of course that works out to  $c_0$  times  $x$  right and you plug in  $x$  equal to  $1$   $x$  equal to  $0$  you get just  $1$ .

$c_1$  times  $x$  squared by  $2$  plug in  $x$  equal to  $0$  you get zero  $x$  equal to one you get one  $c_2$  times  $x$  cube by three ok straight forward  $c_0$  plus  $c_1$  by  $2$   $c_2$  by  $3$   $c_3$  by  $4$   $c_n$  by  $n + 1$  that is exactly what you have got ok the key over here was you have to do a definite integral not an indefinite integral because indefinite integral means you are in for good amount of trouble and its not always correct all right similar question we are going to do one more and then we are going to try to solve some some more je problems and you look at the right hand side you know that first of all you have to do an integral divided by  $n + 1$  then you have to do a definite integral because there is a minus one by  $n + 1$  and thirdly you are not going to plug in the limit of  $1$  right it looks like this is going to be some other limit okay when you did it last time you had  $1 + x$   $n + 1$  whole power  $n + 1$  by  $n + 1$  when you plugged in a  $1$  over here you got  $2$  power  $n + 1$  here you have got three power  $n + 1$  which means that you are going to start from zero that will give you the second term and end at its one plus  $x$  you want three

so  $x$  has to be  $2$  right

so the limits all you have to do is change the limits you have to go from  $0$  to  $2$  and you are done because that is going to give you the right hand side ok just check just check that that is indeed the case  $1 + x$  whole power  $n$   $dx$  is equal to  $1 + x$  whole power  $n + 1$  by  $n + 1$  you can do the derivative to check that this indeed gives you that and then you place the limits of  $0$  and  $2$  when you plug in  $2$  you get  $3$  power  $n + 1$  by  $n + 1$  when you plug in  $0$  you get  $1$  by  $n + 1$

so this is correct right this is correct now all you have to do is break it up

so  $1 + x$  whole power  $n$  is  $c_0$  plus  $c_1 x$  plus  $c_2 x$  squared all the way till  $c_n x$  power  $n$  and each of those you are going to raise it from zero to two  $d x$  ok and what is that going to give you  $c_0 d x$  is  $c_0$  times  $x$  all right use the limits  $2$  and  $0$  you get  $2 c_0 x$   $c_1 x$  is going to give you  $c_1 x$  squared by two and use the limits zero and two you get two squared  $c_1$  by two  $c_2 x$  squared you integrate you get  $c_2 x$  cube by three  $c_2 x$  cube by  $3$  you raise it from  $0$  to  $2$  those are the limits  $0$  will give you  $0$   $2$  will give you  $2$  cube by  $3$ .

exactly in the same form as we need and how are we doing this how did we get our result we got our result just using some clues the denominator increasing

denominator gave me a clue that i need to do an integral the minus over here gave me a clue that i need to do a definite integral right

so those were all my clues ok

so this problem is done very good let's try uh another one this is also an integral right the denominator is increasing but look its the sign is alternating

so maybe you wont do  $1 + x^n$  you have to do  $1 - x^n$  you have to do an integral of that and look at this it is just  $\frac{1}{n+1}$  if you do an integral you have to do a definite integral right because just doing an integral is meaningless right the integral of this side indefinite integrals wont match up

so you have to do a definite integral but if i do a definite integral and i plug in  $x = 1$  then i get a  $\theta$  which means that if i plug in  $x = 0$  i am left with  $1 + 1$ .

so if you do work this out you get the right hand side and the rest i am going to leave to you because ok you need not do  $1 - x$  you can even do  $x - 1$ .

lets scratch it out let us do integral  $x^n dx$  from zero to one if i plug in one if i plug in 1 over here i get a  $\theta$ .

so  $\theta$  minus if i plug in  $\theta$  then i get  $-1 + 1 = 0$  ok and then depending on  $n$  odd or even i either get the plus or the minus of the answer right now you have to work this out i am going to leave this result to you this is not very hard to solve right you just have to plug in you have to expand and integrate do the definite integral ok let us try another one this was a je problem some time back ok

so 50.

so these are all we are we are talking about 50 c r calling this c r in this particular problem this is the abbreviation

so contextual you can figure it out c 50 is his last term

so it is got to be  $50 C 50$  ok

so this is your question find the coefficient of  $x^{49}$  in this expansion right and you have got how many terms 1 2 3 4 all the way till 50.

so you have got 50 terms  $x^{49}$  what does that mean that means if i pick this  $x$  then i have got to pick sorry i have to pick 49 x's

so if i don't pick this  $x$  then i have to pick  $x$  everywhere else if i don't pick this  $x$  then i have to pick  $x$  everywhere else which means that the answer is going to be  $\sum_{k=0}^{50} (-1)^k \binom{50}{k} x^{50-k}$   $\binom{50}{0} x^{50} - \binom{50}{1} x^{49} + \binom{50}{2} x^{48} - \dots + \binom{50}{49} x - \binom{50}{50}$  okay that is going to be your that's going to be the coefficient of  $x^{49}$  this whole thing and how do i do it  $\binom{50}{1}$  this is something again once again this is familiar right we just did something similar how did we do it we actually broke it up  $\binom{50}{1} = \frac{50!}{1!49!}$  we broke up  $50!$  as  $50 \times 49!$

so here we are talking about factorial fifty  $n$  is fifty

so the  $r$ th term in this in this series if i pick the  $r$ th term let us say this one first term second term third term right third term starts with  $3^2$

so the  $r$ th term will start with  $r^2$  and then third term has  $3^3$

so this will be  $r^2$  by  $r^2$

so this will be  $\frac{50!}{r!(50-r)!}$   $r!$  is factorial  $r$  and factorial  $50 - r$  and  $\binom{50}{r} = \frac{50!}{r!(50-r)!}$  and then these are going to cancel  $50!$  will cancel out  $r!$  minus

$1$  factorial will cancel out with  $r$  factorial and  $r$  squared one  $r$  will go away and then  $50$  minus  $r$  plus one  
 so this is larger  $50$  minus  $r$  is smaller  
 so this will completely cancel out with the factorial  
 so this whole thing is equal to  $r$  times  $50$  minus  $r$  plus  $1$  okay  
 so that is the  $r$  term our term is  $r$  into  $50$  minus  $r$  plus  $1$  now you just have to find the sum of everything with a minus sign i i have ignored the minus sign all the terms are minus  
 so at the end we'll put the minus sign in all right  
 so the sum of all of these are all of these terms where each term is  $r$  into  $50$  minus  $r$  plus  $1$  for example if  $r$  is  $1$  this is equal to  $1$  into  $50$  minus  $1$  plus  $1$   $50$  plus if  $r$  is  $2$   $2$  into  $50$  minus  $2$  plus  $1$   $49$  plus  $3$  into  $48$  plus  $4$  into  $47$  plus dot dot dot all the way till  $50$  into one ok that is what it is this is the sum that you have to do and you have to sum it from  $r$  equal to one to  $r$  equal to  $50$  and then you can break this up you can break it up as  $51$  first of all  $50$  minus  $r$  plus  $1$  is nothing but  $51$   $51$   $r$  minus  $r$  squared and you can take the  $51$  outside and sigma of  $r$   $r$  equal to one to fifty is nothing but  $n$  into  $n$  plus one by two  
 so  $50$  into  $50$  one by two and sigma of  $r$  squared  $r$  equal to one to fifty is nothing but  $n$  into  $n$  plus one into two  $n$  plus one  
 so  $50$  into  $50$  one into two into  $50$  plus one  
 so hundred one by six  
 so this is your answer now you just have to compute this out right ah you can do it i believe the answer is twenty two thousand one hundred but then eventually you have to place a negative sign okay  
 so the answer the final answer is minus  $22100$  okay let us do one more all right this is also a j e problem from some years back find the coefficient of  $t$  to the power  $24$  in  $1$  plus  $t$  squared whole power  $12$  times  $1$  plus  $t$  whole power  $12$   $t$  power  $12$  times  $1$  plus  $t$  to the power  $24$  notice that here this  $12$  is inside the brackets this  $24$  is inside the bracket which means for these you are not really looking at any binomial expansion the only binomial expansion is here ok then notice that you are looking for  $t$  power  $24$   
 so if you look over here this  $1$  plus  $t$  power  $24$   
 so if you select  $t$  power  $24$  if i select this term then what will i have to select from the other terms from here i have to pick one and from here i will have to pick one in all the twelve terms right  
 so i have to pick one everywhere  
 so if i pick this  $t$  power  $24$  all everywhere else i have to pick a one  
 so that is one possibility i am looking for  $t$  power twenty four now if i pick this one and not the  $t$  power twenty four then i am free to pick  $t$  power twelve here or not right maybe i will pick maybe i want to lets check  
 so if i pick one over here now that is the second possibility i can pick one from here and then i am looking for the entire  $t$  part  $24$  coming from here  
 so one over here let us pick one over here  
 so i have already done  $p$  power  $24$  times  $1$  times  $1$  right that is done now i have picked one over here then if i pick one over here i can either pick one or i can pick  $t$  part  $12$  let's pick one if i do pick one then i have to pick all of  $t$  power  $24$  from the first term and the only way to make  $t$  power  $24$  in the first term is to pick  $t$  squared every time right there is only one way of doing that  
 so i get a  $1$  over there then i pick a  $1$  from this last term the next possibility is i pick  $t$  power  $12$ .  
 so if i pick  $t$  power  $12$  over here i only need  $t$  power  $12$  out of this  $1$  plus  $t$  squared whole power  $12$ .

so what is the coefficient of  $t^{12}$  in  $(1+t^2)^{12}$ ?  
The coefficient is  $\binom{12}{6}$  because you are picking  $t^2$  six times if you pick  $t^2$  six times

so it is  $(1+t^2)^{12}$  such products right out of those 12 products if you pick 6  $t^2$ 's and 6 ones then you end up with  $t^{12}$  right you're looking for  $t^{12}$  out of this  $t^{12}$  you have taken

so this is your answer all right

so this is just a reasoning i think these are reasoning based problems which are quite common in your exams in your competitive exams let us try another one ok

so this is the question you have to find out what is the answer right you have what is the sum of all of this all of these pairs and this kind of looks like something that we have done before although it is very deceptive its not the same right

so earlier we had done something of this format this is what we had done a long time back but ah notice is a minus a minus and these minuses are going to throw a spanner into the works right and you will have to solve this all over again ok its not this this is not what you are looking for because if this had been what you are looking for you will get  $60$  something but that is not it right there are minus signs here these are all plus how will you work this out and the way to work this out is to look back at the way we worked that one out because it looks kind of similar right

so because it looks similar we can use the same approach to work with this even though the answer is not the same ok what was our approach over there our approach was we had looked at  $(x+y)^{2n}$  right and some coefficient of  $x^m y^n$  something else ok we were looking at  $(x+y)^{2n}$  in that case ok

so here because of the minus signs what will you do will you work on  $(x+y)^m$  or something else what do you want to do minus

so you are going to work with  $(x-y)^m$  ok thats a good start

so lets work with  $(x-y)^{2n}$  ok and what we are going to do is actually not quite this ok we can work with  $(x-y)^n$  and what we are going to do is we are going to break this up into  $(x-y)^n$  and  $(x-y)^n$  and there is of course a minus 1 do you want to do something like that ok

so maybe you can do something like this forget this minus 1 whole power n for now forget it right but let's see if this at all pans out

so you are going to work out the coefficients of this right what will you get you will get some  $\binom{2n}{n}$  something as your coefficient but what will you get on this you will get  $\binom{n}{n}$  something  $\binom{n}{n}$  something right times a minus right what you going to find is that this alternating sign is not going to come you won't be able to work out the alternating signs ok we need something little smarter ok what if we do what if we try  $(x-y)^n (x+y)^n$  what if we try that and by n over here i mean thirty ok if you try something like this  $(x-y)^{30}$  and lets look at the which coefficient you want to look at here when we were working with this we were looking at  $x^n y^r$  in this right this is you are looking at  $x^n y^r$

so n in this case is 30 r we have picked is 10

so maybe you want to look at coefficient of  $x^{40}$  maybe okay

so what will what will happen how will you generate  $x^{40}$  you can pick  $x^{30}$  from the first one and remember we always flip the second one

so you pick  $x^{30}$  from the first one and  $x^{10}$  from the second one ok

that is one possibility another possibility is  $x^{29}$  over here and  $y$  and  $y$  power 1 and  $x$  power  $y$  power 19  $x$  power 11 then you can do  $x^{28} y^2$   $y$  power 18  $x$  bar 12 right and all of these

so this coefficient is  ${}^{30}C_0$  this is  ${}^{30}C_{10}$

so so no  $y$ 's have been selected here 10  $x$ 's have been selected then the next time i select 1  $y$  and therefore i get a minus sign and here i select 11  $x$ 's which gives me  ${}^{30}C_{11}$  then the next time i select two  $y$ 's which gives me a plus sign and  ${}^{30}C_2$  and over here i select  ${}^{30}C_{12}$  okay

so do you see that we have constructed our expression

so our expression will be constructed in this way ok

so this is our question

so our question has an answer the answer is

so if you are asked to evaluate something like this you construct it in this format and therefore your answer is the coefficient of  $y^{20} x^{40}$  in this expansion the expansion is  $(x - y)^{30}$  into  $y + x$  whole power 30 and you are looking for the coefficient of  $y^{20} x^{40}$ .

now there is an easier way to evaluate this and that is  $x^2 - y^2$  whole power 30 correct and how many  $x^2$  do i need to get  $x^{40}$  for t i need twenty  $x^2$ 's and ten  $y^2$ 's right

so the answer is  ${}^{30}C_{20}$  also ten ok great

so ah this was also an interesting problem we have solved quite a few problems thank you very much you