

hello everyone and welcome back to ah the mathematics lecture on binomial theorem and its applications this is the third lecture in the sequence and brief recap very brief recap of what exactly is the binomial theorem sorry i am doing this again and again and this was the statement of the theorem right in the last class we found out that these two are equal these two are equal the sequence goes like this comes back like this and they are mirror images of each other whether you write it as  $x + y$  or  $y + x$  the answer should be the same okay that's one thing we noticed the other thing that we noticed is that if i plug in  $x$  equal to  $y$  equal to 1 you can plug in  $x$  equal to 1  $y$  equal to 1 you get  $2^n$  power  $n$  on this side but on this side you get  $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n}$  right

so that's the second result that the sum of all of these is  $2^n$  right of course you also know that this equal to this this equal to this this equal to that right

so if  $n$  was odd then my total sequence half of the sequence would have been equal to the other half of the sequence the sum of them is  $2^n$

so each of the sets is  $2^{n-1}$  if  $n$  was odd but all right forget  $n$  equal to odd what if you plug in  $x$  equal to 1 and  $y$  equal to minus 1 if you plug in  $x$  equal to 1 and  $y$  equal to minus 1 your net result is 0 and you get  $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \dots$  etc etc right this is also a result and then you put some of them on the left hand side some others on the right hand side the sum of the even terms is equal to the sum of the odd terms  $\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \dots$

so on okay

so this was another result that we saw then we did something else and we found out the coefficient of  $x$  plus 1 by  $x$  whole to the power  $2n$  and we found that in this expansion there is only one term which does not have an  $x$  there is only one term and that is the middle one and the middle coefficient is  $\binom{2n}{n}$  and then what did we do we broke this up in a different way we said maybe we can write it as  $(x + 1)^n (x + 1)^n$  right and what is the coefficient of the term which does not have an  $x$  in it

so you expand 1 you expand the other and then you find out that  $\binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \dots + \binom{n}{n}^2$

so on that's got to be equal to  $2^n \binom{n}{n}$  you can generalize this a little bit further and we got the above result  $\binom{n}{0} \binom{n}{r} + \binom{n}{1} \binom{n}{r+1} + \dots + \binom{n}{r} \binom{n}{n}$

so on that is equal to  $2^n \binom{n}{r}$

so this is also a result that we got and then we were doing some problems which do not necessarily use these results in a straightforward way but we were doing some ah just a few problems okay one problem one more problem that i have in my kitty and let us try this ok this problem happened to have been in one of the j e's ok j e problem how do you solve this this actually not very hard a minus  $b$  whole power  $n$  break it up the sum of fifth and sixth terms what is the first term you choose all  $n$ 's and all  $a$ 's and no  $b$ 's do not choose any  $b$ 's plus  $\binom{n}{1} a^{n-1} b$  that is the second term right then comes the third term fourth term fifth term what's the fifth term going to be  $\binom{n}{5} a^{n-5} b^5$

so the first term is  $\binom{n}{0} a^n$ .

so the fifth term is going to be  $\binom{n}{4} a^{n-4} b^4$  and therefore you are going to choose  $n - 4$  is and how many  $b$ 's 4  $b$ 's that's why  $n$  choose 4.

you are choosing four  $b$ 's

so that is the fifth term and what is the sixth term oh mistake mistake mistake mistake i forgot the minus

so you are not doing a plus  $b$  you are doing a plus minus  $b$

so this  $b$  comes with a minus if i just have a  $b$  it comes with a minus  $b$  squared

comes with a plus b cube comes with a minus b power 4 has a plus in front and then b power 5 is the sixth term

so this is the fifth and the sixth term is minus  $nc^5 a$  to the power  $n$  minus 5 times  $b$  to the power 5 that's my sixth term and the question says that the sum of the fifth and sixth is 0 which means these two

so that gives me  $nc^4 a$  power  $n$  minus 4  $b$  power 4 is equal to  $nc^5 a$  power  $n$  minus 5  $b$  power 5 all right and then simplify and what is  $nc^4$  and what is  $nc^5$  five and then once again simplify  $n$  factorial goes away 4 factorial is 1 times 2 times 3 times 4 and 5 factorial is 4 factorial times 5  $n$  minus 5 factorial  $n$  minus 4 factorial is bigger than  $n$  minus 5  $n$  minus 4 factorial is  $n$  minus 5 factorial times  $n$  minus 4.

which leads me to  $a$  by  $b$  is equal to  $n$  minus 4 by 5 okay that was a quick question for you quite straight forward all right let's do another one

so this is the problem statement that one plus  $x$  plus  $x$  squared whole to the power  $n$  is equal to  $a^0$  plus  $a^1 x$  plus  $a^2 x$  squared all of those terms are there and you know the largest term is going to be  $x$  power  $2n$  ok  $x$  part  $2n$  is the largest term and for  $x$  bar  $2n$  he is used a coefficient of  $a^{2n}$  in the question he is saying that this is given to you and then

so this is my question one how will you do it how will you do this oh come on this is very straight forward just like our last last the way we had done this remember we plugged in  $x$  equal to 1  $y$  equal to 1 and all of these disappeared we are left only with  $nc^0$  this is one this is one

so what will you do just plug in  $x$  equal to one if you plug in  $x$  equal to one a naught plus  $a^1$  plus  $a^2$  all the way till  $a^{2n}$

so what is the answer one plus 1 plus 1 3 whole power  $n$  ok

so that is a very straightforward question question 2.

what is this sum  $a^0$  minus  $a^1$  plus  $a^2$  minus  $a^3$  plus  $a^4$

so this term has to be a minus this term has to be a plus what does it remind you of  $x$  is equal to  $x$  is equal to minus 1 will do the job right  $a^0$  minus  $a^1$  plus  $a^2$  minus  $a^3$  and

so on and

so forth ok

so just plug in  $x$  equal to minus 1 and if you plug in  $x$  equal to minus 1 1 minus 1 is 0 plus minus one squared is one

so the net result is  $a^0$  whole power  $n$

so your answer is one ok very interesting all right then i have got a third question for you how will you do this let us check you have to plug in some  $x$  right

so what are you going to plug in you want to plug in  $x$  equal to ok

so one possibility is that you look at this and multiply it with a 1 by  $x$  ok this is one possibility because this the first one is going to expand out and what will you get here you will get  $a^0$  plus  $a^1 x$  plus  $a^2 x$  squared plus

so on all the way till  $a^n x$  power  $2n$  okay but the second one is going to expand out and here instead of  $x$  i am writing minus 1 by  $x$  instead of  $x$  right

so if i write minus 1 by  $x$  instead of  $x$  i get  $a^0$  minus  $a^1$  by  $x$  plus  $a^2$  by  $x$  squared minus and then plus  $a^{2n}$  by  $x$  power  $2n$  okay that's the second term okay and if you look at in in the product of these two expressions you are taking the product of these two expressions what is the term that is independent of  $x$  no  $x$

so if i multiply this  $a^0$  with any random term i will have some  $x$  in that random term unless i multiply it with this this first one

so if i multiply these two then i get something independent of  $x$  okay  $a^1 x$  if you multiply  $a^1 x$  with  $a^0$  you have got  $x$  if you multiply  $a^1 x$  with  $a^2$  by  $x$  squared you are ending up with 1 by  $x$  any other term you have you will

still have an x inside the only situation where a 1 x will multiply with something and you won't get an x is if you multiply with minus a 1 by x so you multiply these two right that will give you minus a 1 squared how about a 2 x squared if you multiply it with a naught x squared remains multiply with a 1 by x x remains anything else there is some x component in it except if you multiply with a 2 by x squared ok and

so on and

so forth

so in the in this expansion the term independent of x is the question ok that is the only term which is independent of x okay how else do you construct that

so i found out that the term independent of x is this now i have to multiply the 2 in some other way and find the term independent of x what's the other way the other way is i pre multiply and then do whole power n let's pre-multiply

so just by the way this is guesswork is intelligent guess work

so this is where your practice will pay the more you practice the better you will be at guessing what should be the way to go about solving this problem otherwise you will be stuck right you have to intelligently guess this construction okay

so let's try this out 1 plus x plus x squared whole power n into 1 minus 1 by x plus 1 by x squared whole power n and in this i am looking for the term that is independent of x just to remind you

so what we are going to do is we are going to take n out ok and you know how to do this multiplication

so you can do term by term or you can do a plus b times a minus b in which case you get 1 plus x squared oh no you can't do that

so let us do term by term i have a 1 minus 1 by x plus 1 by x squared

so i multiplied 1 with all these 3 terms then i take the x and multiply with all these three terms and finally i take the x squared and multiply with these three terms and then there's a lot of cancelling out this 1 by x cancels out with the next one the x cancels out with the minus x and there's a minus 1 that cancels out with a plus 1 although one of those two one of those three remains one by x squared plus 1 plus x squared whole to the power n okay recollect the question 1 plus x plus x squared whole to the power n was given as a naught plus a one x plus a two x square plus one a two n x bar two n

so if this is what is given what is going to be one by x squared plus one plus x squared whole power n it's a little hard to predict what it is going to be unless we take things common outside

so let's take 1 by x power 2 n common outside and then what remains is a 1 plus x squared plus x power 4 whole power n and i have got it in the same format right the format is the same which means that this is equal to 1 by x power two n times a naught plus a one the format where x is equal to x square

so a one x squared plus a two x power four plus dot dot dot a two n x power 4 n ok and what am i looking for i am looking for the term that is independent of x which means that somewhere inside this there is a term with x power 2 n and that term will be independent of x because of this outside 1 by x power 2 n okay what is that term that's the nth term right a 0 x power 0 a 1 x power x square 1 times 2 a 2 x power two times two

so what is going to be x power two times n the coefficient is going to be a n fine

so the term independent of x is nothing but a n and my question was

so the term independent of x in this expansion is a naught squared minus a 1 squared plus a 2 squared minus a 3 squared and i did it in some other way i first multiplied then raised to the power as opposed to raising to the power and then multiplying right and what did i get i got this as a n ok the clue over

here is this intelligent guess work i am sorry this guesswork cannot be taught it comes only with practice the more you practice the better you will be able to make these guesses ok

so let us try yet another one  $1 + x$  whole to the power 14 whole raised to the power 14 okay and the question is that if the  $r$ th  $r + 1$   $r + 2$  with terms are in arithmetic progression you know what arithmetic progression is i hope you know what arithmetic progression is okay

so  $r$ th  $r + 1$   $r + 2$  terms are in arithmetic progression then what is  $r$  it is a straight forward question let us try it out ap what it means is that that the sum of these two terms if these three are in ap then  $r$  term plus  $r + 2$  term will be two times  $r + 1$  eighth term or in other words  $r + 1$  term minus  $r$ th term will be equal to  $r + 2$  with term minus  $r + 1$  ether right

so what is the  $r$ th term what is the first term the first term is one right you choose all 14 ones no  $x$ 's that gives you the first term the second term is you choose 13 ones and one  $x$  the third term is you choose twelve ones and two  $x$ 's right

so the  $r$ th one is you choose how many ones the  $r$ th term you are going to choose  $14 - r + 1$  once and how many  $x$ 's you are going to choose  $r - 1$   $x$ 's

so the term is going to be  $14 - r + 1$  choose  $r - 1$   $1$  to the power  $14 - r + 1$  is just  $1$  times  $x$  power  $r - 1$

so this is the  $r$ th term okay likewise what is the  $r + 1$  term it's the next stop fourteen  $c_r \times x$  power  $r$  and what is the  $r + 2$  term fourteen queues  $r + 1$   $x$  power  $r + 1$  ok

so he says that these three terms are in arithmetic progression what is fourteen  $c_r - 1$  factorial fourteen by factorial  $r - 1$  and factorial  $13 - r$  no  $15 - r$  okay

so this is the  $14 - r + 1$ .

this is the  $r + 1$  term and this sum is equal to two times fourteen factorial by fourteen minus  $r$  factorial  $r$  factorial  $x$  power  $r$

so this is pretty much the problem statement and we have to find out what is  $x$  in terms of sorry what is  $r$  in terms of  $x$  obviously

so one thing you can do is you can scratch out the  $14$  factorial from everywhere the next thing you can do is you can scratch out  $x$  power  $r - 1$  from all over the place okay next what you can do is which is the least one  $r - 1$   $r + 1$  or  $r$  the smallest is  $r - 1$

so you can scratch out  $r - 1$  factorial all over the place and here you will get  $r + 1$  times  $r$  and here you will get  $r$  and then further what do you want to scratch out which is the smallest over here  $15 - r$  or  $14 - r$  or  $13 - r$   $13 - r$  is the smallest

so you can scratch it out completely and over here you get fourteen minus  $r$  and over here you get fourteen minus  $r$  into fifteen minus  $r$  and now things are simpler to work with

so you have got a much simpler much more straight forward relationship right

so my relationship now is  $1$  by  $14 - r$  times  $15 - r$  plus from the second term i have got  $x$  squared by  $r$  into  $r + 1$  and from the third term i have got  $2x$  by  $r$  into  $14 - r$  and now you have to solve for  $r$  ok

so hopefully you can do it multiply by  $14 - r$  maybe okay i i am going to leave you to solve the rest of this because this is a straightforward application of algebra

so we are not going to pursue this any further

so these are the kind of questions that you are going to find typically coming up when we discuss the binomial theorem ok now we're going to generalize a little bit generalize a little bit because we have a lot more to do

so i go back over here  $x + y$  whole power  $n$  is  $\binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \dots$  ok the generalization is that  $n$  this formula right now this formula is for  $n$  as an integer positive integer it turns out that the same formula will work even when  $n$  is not an integer all right even when  $n$  is not an integer something similar will work not the same but something similar let's rewrite it  $1 + x$  whole power  $n$

so this is going to be equal to  $\binom{n}{0}$  and what is  $\binom{n}{0}$  factorial  $n$  by factorial  $0$  factorial  $n$  plus  $\binom{n}{1}$  times  $x$  which is factorial  $n$  factorial  $1$  factorial  $n - 1$  times  $x$  plus  $\binom{n}{2}$  times  $x$  squared ok

so this is a 1 and what is this factorial  $n$  by factorial  $n - 1$  is just an  $n$  factorial  $n$  by factorial  $2$  factorial  $n - 2$  is nothing but  $n$  into  $n - 1$  by this is  $2$  into  $1$  the third term is going to be  $n$  into  $n - 1$  into  $n - 2$  times by factorial three ok

so i am just rewriting the binomial theorem its just a rewrite

so far now it

so happens that if you plug it in this form if you put it in this form this is  $1 + x$  whole power  $n$  this form of the binomial theorem is valid even when  $n$  is not an integer as long as

so this is there is one condition right as long as you play by this rule that as long as i am going to keep a magnitude of  $x$  to be less than 1 as long as you play by this rule then this statement is valid even when  $n$  is not an integer  $n$  can be anything  $n$  can be a fraction  $n$  can be positive  $n$  can be negative whatever you want you plug in there it's going to work

so this is a generalization of the binomial theorem of course you are going to say what is the proof of this the proof of this is going to come from taylor series you are hopefully going to study taylor series when you study calculus

so we are not going to do the taylor series we are going to use this as a generalization of the binomial theorem okay one possible use i don't know what's the level of calculus that you have studied

so far but let's assume that you have at least studied till limits right

so one possible use is ah you might have studied this relationship  $\lim_{x \rightarrow 0} e^x - 1 = x$  you would have studied this one look i mean we have already covered the basics of binomial theorem right we are now going into more serious applications we are extending the binomial theorem and the reason why we are extending the binomial theorem is because we want to solve a variety of problems and to be able to solve a variety of problems you have to be open minded you have to throw away the boundaries between algebra and arithmetic and calculus and geometry all these boundaries are broken you are now trying to solve a variety of problems okay

so some calculus you possibly have already studied and we are going to borrow from the calculus and this is one result that you might know already  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$  this is one of the standard relationships limit relationships that you would have studied ok

so if you have studied this you can rearrange this right you can rearrange this as you multiply  $x$  on both sides

so  $e^x - 1 = x$  limit  $x$  tends to zero is equal to limit  $x$  tends to zero of  $x$  ok or in other words you add one on both sides ok and then you take to the power one by  $x$  on both sides ok

so this is ah standard definition of  $e$

so a standard definition of  $e$  is that  $e$  is  $\lim_{x \rightarrow 0} (1 + x)^{1/x}$

so you make  $x$  smaller and smaller and smaller keep computing this and you will converge towards the value of  $e$  the answer for  $e$  of course is two point seven

one eight two eight one eight two eight four eight whatever it is right ok its like e is a number like pi right

so this number is defined as e and e is very popular when it comes to taking powers  $e^x$  powers and taking logarithms i could also have written it in this format  $\lim_{y \rightarrow \infty} (1 + \frac{x}{y})^y$

so y is a very large number  $1 + \frac{x}{y}$  whole power y

so i have replaced x with  $\frac{x}{y}$  in this formulation ok suppose you want to compute  $e^x$  to the power x ok  $(1 + \frac{x}{y})^y$  to the power x y any problem no problem  $\lim_{y \rightarrow \infty} (1 + \frac{x}{y})^y$  how will you work this out  $1 + \frac{x}{y}$  remember  $1 + \frac{x}{y}$  satisfies right i am not calling it x anymore i have called it y

so  $1 + \frac{x}{y}$  is certainly smaller than 1 okay if  $\frac{x}{y}$  because  $\frac{x}{y}$  is tending to infinity

so  $1 + \frac{x}{y}$  is certainly smaller than 1 it's very small ok

so it satisfies that relationship which means that one plus something which is very small smaller than one raised to some power to solve this i can use my binomial theorem or rather the extension the generalization of the binomial theorem right it is conceivable that i just plug in whatever is the value of n x etc into this and work it out ok

so what is my n in this case

so let us write it as  $(1 + \frac{x}{y})^y$  i have put capital to the power n is  $1 + \frac{x}{y}$  blah blah blah

so this n has to be x y and capital x has to be  $1 + \frac{x}{y}$  and after that i just plug this in and of course  $\lim_{y \rightarrow \infty} (1 + \frac{x}{y})^y$  remains

so the first is one next is n times x where n is x y and x is  $1 + \frac{x}{y}$

so this becomes just an x plus n x y times n minus 1 by two x squared x is one by y plus ok

so far

so good and now we are almost there we just declare the  $\lim_{y \rightarrow \infty} (1 + \frac{x}{y})^y$  is very large

so you have got x times y times x times y minus 1 the whole thing by y squared so this is x squared y squared by y squared which is x squared minus x y by y squared which is very small

so this entire term is going to behave as if this minus 1 portion was not really there because y is very large right likewise this minus 1 portion this minus two portion are going to be negligibly small because they will have to be divided by y squared y cube right each will be divided by a y right

so net result is going to be sorry i need a position where you can see it

so the net result is going to be the second term was x the third term is going to be x y x y by y squared

so x squared by 2 factorial plus x y x y minus 1 x y minus 2 by 3 factorial y cube

so these terms go away y's go away i get x cube by 3 factorial and then i will have x bar 4 by 4 factorial and

so on and

so forth and of course  $\lim_{y \rightarrow \infty} (1 + \frac{x}{y})^y$  is irrelevant because there is no y inside this

so i do not have to bother about this

so this whole thing is e to the power x

so we arrive at a series  $e^x$  is  $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$

so on and

so forth e power minus x just plug in x equal to minus x of course you get  $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots$

so on and

so forth right and what is e itself just plug in x equal to 1 if i plug in x equal to 1 what do i get 1 plus 1 plus 1 by 2 factorial plus 1 by three factorial plus one by four factorial and

so on and

so forth okay you work it out two point five two point seven seven right and then

so on and

so forth ok

so ah before wrapping up our lecture for today let us just do a few more problems and there are a lot of identities with the binomial theorem

so i am going to try to solve a few of them

so one of them is to show prove that

so this is the question the question is prove that  $c_0 + 2c_1 + 3c_2 + \dots + nc_{n-1}$  plus all the way till  $nc_n$  is equal to  $n$  times  $2^{n-1}$

so this is the question will you be able to do it how will you do it

so earlier what did we do earlier the ah one was  $c_0$  we started from  $c_0$  over there plus  $c_1$  plus  $c_2$  plus  $c_3$  all the way till  $c_n$  and what did we get how did we do this we did  $1 + x^n$  right and then plugged in  $x$  equal to 1 right if you think of  $1 + x^n$  plug in  $x$  equal to 1 you get this as your expansion right this has your expansion and the answer is  $1 + x^n$  equal to 1

so  $2^n$  ok

so this is similar but not quite you do not even start from  $c_0$  right look  $c_0$  is gone right you start from  $c_1$  and then  $2c_2 + 3c_3 + \dots$  all the way till  $nc_n$  and look at the answer  $n$  to the power  $2^{n-1}$  does it kind of remind you of something does it remind you of lets say  $d$  by  $dx$  of  $x^n$  is  $n x^{n-1}$  does it remind you of that it does doesn't it it reminds you of  $d$  by  $dx$  of something to the power  $n$  which is equal to  $n x^{n-1}$

so that's the clue right

so a lot of times these clues pop up here and there and you have to take that  $q$  and you have to realize that this problem can easily be solved with the help of one differentiation

so what are we going to do we know that  $1 + x^n$  is equal to  $c_0 + c_1 x + c_2 x^2 + \dots + c_n x^n$  ok this is something we know and do a differentiation why why am i doing a differentiation because i had a clue over here that's the only reason why okay right now i am just going to do a differentiation and see what comes out

so i do a differentiation let us differentiate both both sides

so i get  $n$  times  $1 + x^n$  minus 1 times the derivative of what's inside which is nothing but  $0 + 1$  right

so that's just a 1.

so i differentiated the left hand side now if i differentiate the right hand side differentiate  $c_0$  you get nothing again nothing  $c_1 x$  you get  $c_1$   $c_2 x^2$  you get  $2c_2 x$  next one you will get  $3c_3 x^2$  and so on and

so forth all the way till  $n$  times  $c_n$  times  $x^{n-1}$  and now guess what what you have to plug in  $x$  equal to look at the answer  $x$  equal to one

so plug in  $x$  equal to 1 you get  $2^{n-1}$  times  $n$  and that is equal to  $c_1 + 2c_2 + 3c_3 + \dots + nc_n$  all right

so was that nice if you had not done the differentiation you would have been

roaming around with this problem ok let us try another one

so this is what i have and eventually you have to show that this is equal to  $n$  plus  $2 \cdot 2^{\text{power } n} \text{ minus one}$

so there are actually ah two nice ways of doing this ok the first one what what do you think this is very easy right this is equal to  $c_0$  plus  $c_1$  plus  $c_2$  plus so on plus  $c_{n-1}$  plus  $c_n$  right and you know that the you know what is the first one the first one was  $2^{\text{power } n}$  and the second one was just what we computed just now and there you go okay

so you could do it like this but a lot of times these neat simplifications do not pop up in our head at the right time at an exam you are going to look at this and then if this does not occur to you or you have forgotten this this expansion the second expansion then it does not work out

so this is this is a good shortcut this is a very nice way of doing things but unfortunately this might not be the first way you think of trying to solve this problem

so so let us try to solve it in one more way

so maybe you can look at it in this fashion

so this is what you have and can i write it backwards ok

so lets say this is  $k$

so this is also equal to  $k$  and then what you have to realize is that  $c_0$  is the same as  $c_n$  remember  $c_0$  is the same as  $c_n$   $c_1$  is the same as  $c_{n-1}$   $c_2$  is the same as  $c_{n-2}$   $c_n$  is the same as  $c_0$ .

so when you add these two you get  $2k$  on the right hand side and on the left hand side you get  $c_0$  times  $n$  plus  $2$  plus  $c_1$  times  $n$  plus two plus  $c_2$  times  $n$  plus two and

so on all the way till  $n$  plus two times  $c_n$  therefore  $k$  is equal to  $n$  plus  $2$  by  $2$  times  $c_0$  plus  $c_1$  plus  $c_2$  all the way till  $c_n$  and this is something that you know this is equal to  $n$  plus  $2$  tau by  $2$  times  $2^{\text{power } n}$

so you have got the same answer ok

so let's stop this lecture here we solved a variety of problems we will solve even more problems in the next class thank you you