hello everyone this is the first lecture on the binomial theorem and the binomial theorem is related to the function a plus b raised to n now this is going to be the core subject matter of this chapter how do we evaluate a plus b whole to the power n and we are also going to look at different problems based on the whatever we learned in this subject

so to start with this is something that you all know we are going to start with a plus b whole squared and this is something that you all know you have all studied this this is a squared plus 2 times a b plus b square you all know why this is true next you can do a plus b the whole cube and this is also something that you all know a cube plus three a squared b plus three a b squared plus b cube now there are two more things that you know i didn't write and one is a plus b whole power one

so that is

so lets ah there is no space over here a plus b whole power one and thats equal to a plus b and there is one more non trivial result although very easy and that is a plus b whole power zero and what is that what something to the power 0 it is equal to 1 great

so these are things that you already know now i am sure some of you who are listening to this lecture know more than this you also know a plus b whole power 4 a plus b whole power 5 6 and

so on and

so forth you might know some of these things however is there a pattern now lot of people have thought about this you might have thought about this yourself and you might already know the answer and there is a pattern the pattern is as follows

so this pattern is called the pascal's triangle

so you start with a one

so the triangle goes along two edges right and you can terminate the triangle wherever you want

so you write once on these two sides of the triangle now you start with one this is a plus b whole power zero and then a plus b whole power one has 1 times a plus 1 times b right

so that is your second answer

so the pascal's triangle is going to give you the coefficients of each of these individual terms the third one is a plus b whole squared and what you do is from these two ones you add them and you get a two then the next one is a plus b whole cube ok

so a plus b whole square this is one you have a two over here and you have a one i hope its clear and then a plus b whole cube what you do is you need to find the intermediate terms you get a 3 over here you get a 3 over here 2 plus 1 is 3 right

so it is 1 3 3 and 1 and you look over back at a plus b whole cube its one times a cube plus three a square b plus three a b squared plus one times b cube

so you have got the coefficients all right and then you can predict that if all this is correct then hopefully one and three will give me a four over here and three and three will give me a six over here three and one should give me a 4 and this should be related to a plus b whole power 4 and then a plus b whole power 5 will be 1 5 10 10 5 1.

and then a plus b whole part 6 will be 1 6 15 20 15 6 and 1 and

so on and

so forth

so this is fairly straight forward this is called pascal's triangle and guess what who came up with this triangle for the first time it was pascal but there are there are many other people who came up with this there are even reports of

this being known to ancient hindu mathematicians there are several i mean if you ask me its not terribly hard to figure this out even as a school as a school student in class 5 class 6 you might have figured this out yourself too without the help of pascal or anybody else ok

so this is not something very hard what's hard is what if i ask you what is a plus b whole to the power 95 what will you do are you going to start writing this triangle and you know make it go 95 steps and then give me the answer is that what you are going to do or are you going to do something smarter than that and this has bothered many people and indeed mathematicians have figured out how to do something in a much smarter way and the idea behind that is as follows the idea is if i plan to do a plus b and lets not do 95 lets do something smaller lets do seven okay i am going to take that as an example

so let us say you want to do a plus b whole to the 7 how will you do it

so one way is to do a plus b times a plus b whole to the power six but i wont do that i will do a plus b times a plus b and i am going to write this seven times okay i have written this up seven times and now what is your job your job is to do these seven multiplications and instead of doing seven multiplications one at a time let's try and do all of them at the same time in one shot how are we going to do it

so the first thing you can do is we will take a from here

so we will take all the a's together

so we will spread it spread all the factors out and let us all let us take all the a's

so a times a times a times a times a times a and that happens to be equal to a power 7 that is the first term that is the easiest one next what we are going to do is just for the first term and only for the first term we are going to keep the b we are going to use b only for the first term and then all the others let us use a

so what will that give me that will give me a power 6 times b now just like you picked b only from the first term and the remaining were all all a's you could have also picked b from the second one and the remaining others could have been is right

so you could have chosen any one of the seven b's right any one of the seven b's could have been chosen

so you do this seven times am i right you do it seven times you do this is the first time you do it the next time you pick this b and remaining all a's the third time you pick this one as b and remaining all a's fourth time you pick this one as b and all other a's and

so on and

so forth like this you can pick b seven times and seven times you will get the product as a power 6 times b

so therefore you end up with the second term as 7 a power 6 times b now what are you going to do next what we are going to do next is we are going to say that lets have this as b this is b and the remaining 5 will keep us a's right

so what are we going to get we are going to get a power 5 times b squared now you could have picked just these two as b you could have picked any 2 as b any 2 out of the 7

so you could have chosen any 2 out of the 7 terms as b right and you add them add all of these terms one after the other what do you get what is that process called you have seven elements you have got seven elements and you are being asked to pick any two and how many different ways can you do such a thing how many different ways can you pick these seven elements

so that has a shortcut in short that is called 7 c 2 remember this is from your combinatorics right

so 7 c 2 how many different combinations of how many combinations of two choices can you pick out of seven possibilities what is seven c two just by the way seven c two is seven factorial seven divided by factorial two divided by factorial five right and factorial seven is one into two into three into four into five into sixteen to 7 right factorial 5 cancels out the first 5 terms
so these 2 partially cancel out and you are left with 6 into 7 over here and factorial 2 is just 2
so this 7 c 2 is nothing but 21 next what you can do is
so this this completes a power 5 times b squared next you say that i am going to have three such terms as b and remaining four i will keep them hanging around as a's if you do such a thing what will you get you will get a power four b cube right and how many possible ways can you do this how many different ways can you choose three b's out of seven possibilities right the answer is 7 c 3 and what is 7 c 3 7 c 3 is 5 into 6 into 7 by one into two into three and that gives me thirty five and then next what you can do is you pick four b's and the remaining ones you keep them as a's
so you end up with a cube b power four and how many different ways can you do this the answer is of course 7 c 4 now 7 c 4 is factorial 7 by factorial 4 by factorial 3 right and factorial 7 by factorial 4 is again 5 into 6 into 7 by factorial 3 is 1 into 2 into 3.

so you get the same answer as before is that a surprise is that a surprise that you got these two as the same
so how many ways can you pick four b's out of seven is the same as how many ways can you pick three a's out of seven right and in the last exercise you did how many ways can you pick three b's out of seven possibilities
so seven the if you have a pool of seven how many ways can you pick three and the complement of that is how many ways can you pick 7 sorry pick 4
so they are supposed to give you they should give you the same answer and we have got the same answer for both all right and then what is going to be next
so this is not yet done next you can say that i will keep 5 terms as b and 2 of them as a and that will give me a squared b power 5 right and how many ways can you pick 5 out of 7 the answer is 7 c 5 and 7 c 5 happens to be equal to 7 c 2 and that is once again equal to 21 and then lastly not lastly i am sorry there are two more terms left you can have six b's and only one a
so which one of them is going to be a there are seven possibilities clearly right
so how many ways can you pick six b's out of seven that seven c six and seven c six happens to be equal to seven c one which is equal to seven
so this answer is easy and lastly you say that i am going to collect all the b's together and that leaves me with b to the power seven
so this is your answer and guess what we did not have to rely on pascal we came up with the answer straight away right and in short this is what the entire chapter is about the entire chapter is about the binomial theorem what we just worked out was actually i will state the theorem theorem states that a plus b whole to the power n is nothing but a to the power n plus n c 1 a to the power n minus 1 b plus n c 2 a to the power n minus 2 b squared plus n c 3 a to the power n minus 3 b cube and
so on and
so forth all the way till n c n minus 1 a b power n minus 1 plus b power n and it
so happens you can in fact write this as n c 0 right 1 is nothing but n c 0 how many ways can you choose none of n things there is only one way you can choose nothing right this is n c n

so in the language of mathematics when there is something complicated that looks like this we try to compress all of this in mathematical shorthand and the mathematical shorthand looks like this this is a sum of many terms

so a sum is usually expressed as the capital greek alphabet sigma and inside this sum you you have what you have n c and then something it could be k right imagine the kth term actually the we are going to use the k yeah the kth term is fine right

so if you look at the kth term any general term out of this let us say the third term

so instead of 3 you write k

so you get n c k a to the power n minus k b to the power k right

so k can be equal to 3 k can be equal to 0 1 2 3 4 all the way till n

so k goes from 0 all the way till n

so this is the mathematical shorthand notation

so sigma means a gigantic summation right sum of all of these terms and this is how we compress such a complicated formula into shorthand ok

so this is pretty much it this is the binomial theorem now of course knowing just the theorem is not good enough we are we need to look at a lot of its applications

so our first let us say corollary is going to be a minus b to the power let us say 7 let's do 7 because we did 7 last time

so this is what we had done a plus b whole to the 7 and now i am asking you if you know how to do a plus b whole to the power 7 can you also tell me what is a minus b whole to the power 7 and the answer is quite straight forward all you have to do is in the old expression you replace every b with a minus b right you you have the old expression a plus b whole power seven is equal to a power seven plus seven a power six times b plus seven c two a a power five times b squared etcetera etcetera now in this old expression for every b you write minus b and you get a minus b whole power seven

so what is that that is equal to a power seven and then minus seven a power six b and then over here you have got a b squared

so b squared means you replace b with minus b you end up with a plus b and then you have got seven c three times a power four times b cube right you replace b with a minus b you get minus b cube and then you have 35 a cube b power 4 you replace b with a minus b you still end up with b power 4 and then next you have 21 times a squared b power 5 what is this going to be this is going to be minus 21 a squared b power 5 thank you and then you had ah 7 times a times b power 6 you replace b with minus b you still end up with b power 6.

and lastly you had b power seven you replace b with a minus b what will you get you will get a minus b power seven great

so this was our first corollary if you know how to do a plus b whole power n then there should not be any reason why you can't do a minus b whole power n and i won't get into the formula it's just an extension of this all right let's do one quick application one quick application is something that you possibly have not studied

so you have studied simple interest in banks right banks give you simple interest but in real life ah interest is not that simple

so you deposit let us say thousand rupees in the bank and the bank is going to give you six percent interest now if you keep this for one year then the bank gives you six percent

so let us do an interest problem right this is an extension to arithmetic let us look at this application right i am picking this interest as an application

so you keep money in the bank thousand rupees six percent interest six percent interest per annum now of course if you keep it for one year then thousand

rupees six percent interest will give you sixty extra rupees
so you end up with one thousand and sixty rupees right
so it becomes
so if you keep a principle p then at the end of one year you get one point zero six times p right what happens at the end of two years at the end of two years you do not start from p you do not get 12 percent you start from 1.
06 p and what do you get you get 1.
06 into 1.
06 p right this is something that you did not learn in school or maybe some of you did ah but this is what happens in real life in real life you don't get six percent year after year on the same principle in real life the principle grows
so you keep principle p today after one year it becomes 1.
06 times p and now that is your principle
so the next year around you will get 6 percent on that new principle right as your interest
so you end up with 1.
06 times 1.
06 times p what happens if you keep this money locked into the bank for 20 years what is going to happen you are going to get i mean in 1 year its 1.
06 p in 2 years its 1.
06 square p in 20 years it is going to be 1.
06 whole power 20 times p this is how much your money has become if you keep the money in the bank for 20 years now how will you do this of course one easy way to work this out 1.
06 times 20 is to to the power 20 is to punch in the numbers in a calculator and work it out right but that is not what we are going to do in a mathematics class in the mathematics class we are going to break this up into 1 and 0.
06 and then we are going to do whole power 20 right and what will you get you will get 1 power 20 always remember you do not have to memorize the formula this is the grand formula right but no memorization needed at all don't try to memorize this i i in fact i i am teaching this class but i do not i i do not memorize this right i worked it out right in front of you from scratch
so i do not memorize this neither should you no need to memorize this if you are asked to do 1 plus 0.
06 whole power 20 you should be able to do it from first principles right how will you do it you're going to break it up into several pieces 1 plus 0.
06 1 plus 0.
06 1 plus 0.
06 1 plus 0.
06 many times 20 times right and then you choose the ones you multiply the ones together then the next step you multiply some ones and some point sixes and
so on and
so forth
so what we are going to do is we are going to break this up 20 times i am not going to write it of course because writing something 20 times seems extremely foolish right i do not want to write the same thing 20 times i am not doing handwriting practice
so i wont do it you also should not bother writing the same thing 20 times but the understanding what should be inside your head is that you are breaking this up 20 times
so you do 1 plus 0.
06 whole power 20.
the first step is to pick one from all 20 of those terms
so you get 1 power 20 which is nothing but one in the next step you pick point

o six from any one of those twenty terms and the remaining all should be ones
 so you get point o six times one to the power nineteen one to the power nineteen is obviously one
 so you get point o six only and how many possible ways can you pick the 0.
06 there are 20 c 1 right which is nothing but 20.
 all right
 so this is the first term then what you do next you are going to pick any two of the 0.
06s remaining 18 should be once right how do you pick two of
 so you do twenty c two there are twenty c two different possibilities of different ways of picking point zero six right and the remaining will all be once
 so into 1 power 18 which i am not going to write then what is going to be the next term the next term is going to be 20 c 3 times point o six whole cube and then the next term is going to be twenty c four point o six whole power four and
 so on and
 so forth right many many terms but look at this lets quickly try to look at this lets try to evaluate some numbers because i am doing arithmetic
 so this is a 1 20 times point o six what is two times point o six its point one two and point one two times ten is one point two ok very nice twenty c how do you work out 20 c 2 20 c 2 is nothing but factorial 20 divided by factorial 18
 so 18 out of those 20 terms cancel out all that you are left with is 20 into 19 divided by factorial 2 which is 2.

 so this is equal to one hundred ninety all right and just i'll keep this result also on the side how do you do twenty c three twenty c three is nothing but factorial 20 divided by factorial 17
 so 17 terms have cancelled out and then what remains 20 into 19 into 18 that's what remains and then the whole thing divided by factorial 3 which is 2 into 3 right and this gives you 190 times six whatever that is right what is that one one four zero is it okay let us just keep this on the side and let us also keep 20 c4 on the side i just would like these pre computed in advance 20 c4 would be 20 into 19 into 7 into 18 into 17 divided by 2 into 3 into 4 which is nothing but 20 c 3 into another 17 on top and 4 in the bottom ok great and 20 c 5 would be nothing but twenty c four into sixteen in the top and five in the bottom right
 so just letting you know and we are going to keep these handy for our nice calculation all right
 so this is the calculation that we are working on 1 plus 0.
06 whole power 20 and our result is coming out to 1 plus 1.
2 plus 20 c2 which was 190 into 0.
06 into 0.
06 ok and then the next term is going to be 190 into six into six yes into point o six into point o six into point o six ok and then the next term will be
 so it is the same 1140 into 17 by 4.
 into point o six whole power four and
 so on
 so this is what we want to do right looks very complicated but look at the end of the day if you are a very serious mathematics student then you are going to work it out very accurately now if you are not that serious about it if you are a person who is going to work in the bank and who is going to only work till you know rupees and by say right not beyond he is not going to give you a fraction of 1 pisa then you do not have to work
 so accurately

so let us just see how accurate how many how many of these terms do we need if you think about it 0.
06 times 0.
06 what is this this is point zero zero three six
 so this is becoming a small term then the next term is becoming even smaller right the next term is point o six times point o o three six right
 so what is that point zero zero zero two sorry
 so its become tiny point zero zero zero two one six ok the coefficient is also growing this term is becoming larger but the pace at which it is becoming larger is not necessarily the pace at which the other term is becoming smaller and smaller right
 so you probably do not need to compute too many terms over here
 so the first term is fine 1 the next term is 1.
2 which is very significant the third term is 190 times 0.
0036 how much is that
 so 19 times zero three six right if you approximate nineteen as twenty its twenty times point zero three six right two times point three six is point 0.
72
 so this is approximately 0.
7 something 0.
72 i i do not want to i do not have a calculator on me i just want to get an idea of what the number is going to look like
 so it is something like 0.
7 then the third term the coefficient has gone up by a factor of 6 whereas the the other term 0.
06 that term has gone down by a by a factor 0.
06 right it has become smaller whereas the coefficient has become larger right
 so 6 into 0.
06 is by the amount by which it has grown 6 into 0.
06 is nothing but 0.
36
 so you can expect this entire term to be something like 0.
36 times 0.
7
 so it has actually become smaller right how much smaller it will become 0.
7 times point three six is something like point two two i do not want to work out the whole thing it is becoming smaller the fourth term the coefficient has grown by 17 by 4 but the the exponent because of the exponent you have shrunk right
 so it is 17 by 4 times 0.
06 right what what is that going to be 17 times 0.
015 right
 so that is approximately zero eight five
 so this is now much smaller than the previous one
 so you can if if you say that its point one then this has shrunk even further and then the next term you do not have to bother too much about right these are becoming smaller and smaller but what do you see over here you see that these two terms are not insignificant at all they are quite significant 0.
7 and 0.
22 okay
 so you started with the principle p thousand rupees you ended up with thousand rupees plus 1200 rupees plus 700 rupees plus 220 rupees plus 20 rupees right and the the further and further terms are going to become smaller and smaller if you had worked only with simple interest you would have got only this first term

this is all you would have got if you had worked with simple interest from your school right twenty years you have kept it in the bank all you get back is your principal and thousand two hundred rupees as interest right but for twenty years that's very small

so if you work with compound this is called compound interest if you work with compound interest you get some more you get another 700 another 220 another 20 and

so on and

so forth and all of this is the result of your binomial theorem ok

so this is one small application that we did all right in in your ncrt book you will find problems like this which one is greater you might have a problem like this and what is the answer you look at 1.

01 you break it up into 1 and 0.

01 the first term is going to be 1 to the power 1000 the second term is going to be 1000 times 1 to the power 999 times 0.

01 1 to the power 999 is 1.

so this breaks up into 1 plus 1000 into point one plus thousand c two into point one squared plus thousand c three into point one cube and

so on and

so forth now this is clearly i am sorry this is point o one yeah now this is clearly a one and thousand into point o one is nothing but ten

so that was how i framed the problem

so i took the first two terms i added the first two terms in my mind and i said which one is greater is 11 larger or 1.

01 times 1000 larger and your answer is going to be 1.

01 times to the power 1000 is larger than 11.

so there are there are quite a few problems in the exercises of your ncrt book that look like this

so this is a trivial application ok let us do maybe ah one more or a couple more and see how much we will see how much time we have

so this is an easy one 1 minus 2 x whole power 5 and how will you do it you want to take the first term which is 1 to the power 5 you'll pick that 5 times the next time around you'll pick the first term four times and the second term once right and how many ways can you do this you can do this five different ways

so you get five times 1 to the power 4 which i am not going to write times minus 2x and then the third term is going to be you pick one three times

so it is one cube and you take minus two x twice

so minus 2 x the whole squared which is plus 4 x squared right and how many different ways can you do this you can pick this 5 c 2 times 5 c 2 is 5 into 4 divided by 2 which is 10 then the next term you are going to pick 1 only 2 times and 2 x 3 times

so if you pick 1 2 times that gives you a 1 and into minus 2 x into minus two x into minus two x three times

so that is minus eight x cube and how many different ways can you do this out of the 5 sets you are going to pick 3.

so 5 c 3 now 5 c 3 is nothing but 5 c 2 they are the same thing

so you still get a 10 over here and then you want to pick 1 only once and 2 x 4 times and how many ways you can do that you can do that five different ways

so you are going to pick 2x 4 times means you are going to do 2x whole power 4 that's 16 x power 4 and then finally we want to take all the two x's and multiply them together

so you get minus 2 x whole power 5 and then you expand this and write it out as 1 minus 10 x plus 40 x squared minus 80 x cube plus 5 times 16 is 80 x power 4 minus 32 x power 5 and you are done let us try couple more how will you do this x plus 1 by x whole power 6.

so in the first chance you pick x six times you get x power six you never pick one by x ok all right then in the next chance you pick x five times and one by x just once
so its x power 5 times 1 by x which makes it x power 4 and how many different ways can you do this 6.

so it's six x power four then the next time around you pick x four times and one by x twice
so x four times means x power four one by x twice means one by x squared x power four times one by x squared gives you x squared and how many different ways can you do this six choose 2 6 c 2
so that 6 into 5 by 2 then the next time around you pick x 3 times 1 by x 3 times x three times one by x three times makes it one right no exponent at all
so one and how many different ways can you choose three x's out of this six c 3 that 6 into 5 into 4 divided by 3 into 2 into 1
so that is equal to 20 and then the next time you pick 4 sorry you pick x only twice and one by x four times right and that is again six c two times and next you pick x only once and one by x five times and that gives you one by x power four and how many different ways can you do that you can do that six different ways and lastly you pick only one by x
so you are left with 1 by x power 6 ok
so what what i am trying to do is i am trying to give you some practice
so that you do not have to memorize that formula at all right this formula is not needed although it is there in the book this expression this complicated thing should be in your mind you do not have to memorize it right you can work it out just from first principles each and every time and i think in a lot of ways that is easier to do than to remember because there are already
so many things that you have to remember you do not have to remember this extra information ok especially when you can work it out every time it is fast enough with a little practice you will see that it just comes naturally
so let us try one more
so this is also there in the book 2 by x minus x by two whole power five somewhat similar but let us do it
so first time you pick all two by x's
so you get two by x whole power five the next time you pick one of the minus x by twos and four of the two by x's right
so you get a minus because you have picked minus x by 2 only once
so you end up with minus and x by 2 into 2 by x whole power 4 is nothing but 2 by x whole cube and how many different ways can you do this five and the next time you pick two by x just three times and x by 2 minus x by 2 2 times
so when you are picking minus x by 2 2 times it becomes a plus and x by 2 whole squared times 2 by x whole cube leaves you with just 2 by x and how many different ways can you do it 5 c 2 which is 10 and then the next time you pick 2 by x twice and x by 2 3 times minus x by 2 3 times
so you end up with a minus and x by two goes away with two by x but you have done this three times you have done this two times
so you are left with minus x by two and how many ways can you do this you can do this 5 choose 3 5 choose 3 is nothing but 10.
it's the same as 5 choose 2.

and then you pick 2 by x just once x by 2 4 times

so that leaves you with minus x by 2 now you have chosen minus x by 2 4 times

so it becomes a plus x by two whole cube right and how many different ways can you do this five and lastly you do not pick 2 by x at all you pick only minus x by 2 5 times

so minus to the power 5 gives you a minus and then x by 2 whole power 5

so that is your answer ok

so we are going to wrap up our first lecture on the binomial theorem here and net what did we figure out we figured out the most important thing this is our binomial theorem but what's more important is the way we arrived at the binomial theorem the way we arrived at it was by breaking it up into several pieces a plus b whole power 7 was broken up into seven pieces and then you keep picking right the first time you pick all a's you get a power seven the next time you pick only one b and six is you get a power six times b and how many different ways can you pick one b you can pick it from this one you can pick it from this one or this or this or this or this or this

so there are seven different ways to do that then the next time you pick five is and two b's right

so you get a power five times b squared and how many different ways can you do that seven c two and then seven c three times a power four b cube seven c four times a cube b power four seven c five a square b power 5 and

so on and

so forth

so this is in a nutshell the gist of the entire chapter all the problems revolve around this expression the way you made you broke this up and found the result ok thank you for your attention and hope to see you