welcome to the second problem solving session on binomial expansions we resume solving some more problems on binomial expansions let us look at this question for two positive integers m and n we consider the expansion of 1 plus x to the power m into 1 minus x to the power n suppose that in the expansion the coefficients of x and x square are 3 and minus 6 respectively we have to find out what is the value of m for this purpose we shall first write down what is 1 plus x to the power m into 1 minus x to the power n this is equal to summation over k k runs from 0 to up to m m c k into x to the power k into sum over r r runs from 0 to up to n n c r minus 1 to the power r into x to the power r it is evident that the coefficient of x is m c 0 into minus n c 1 plus m c 1 into n c 0 that is minus n plus m and we are given that this is equal to 3

so we have one equation in n and m here now we write down the coefficient of x square this is m c 0 into n c 2 plus m c 1 into minus n c 1 plus m c 2 into n c 0

so we can note that this is equal to n into n minus 1 divided by 2 minus mn plus m into m minus 1 divided by 2 now it is given that this is equal to minus 6

so we are getting another equation in m and n let us simplify this equation after simplifying we get n squared minus n plus m square minus m minus 2 m n is equal to minus 12

so we get m minus n whole square minus m plus n is equal to minus 12 now from the previous equation we know that m minus n is equal to 3

so therefore we can substitute this value there

so we get 9 minus m plus n is equal to minus 12 that means m plus n is equal to 21 now we have m plus n is equal to 21 and we also have m minus n is equal to 3

so from here we get that 2m is equal to 24 that means m is equal to 12

so we have found out the value of m and we see that the third option is the correct answer here in this question we are asked to find out the coefficient of t to the power 24 in the binomial expansion of 1 plus t square to the power 12 into 1 plus t to the power 12 into 1 plus t to the power 24 to solve this question let us first write the binomial expansion of this 1 plus t square to the power 12 into 1 plus t to the power 12 into 1 plus t to the power 24 is equal to sum running over k and k runs from 0 to up to 12 12 c k t to the power 24 minus 2k and then we write the next two products 1 plus t to the power 12 into 1 plus t to the power 24 and this is equal to sum running from k is equal to 0 to up to 12 12 c k into t to the power 24 minus 2 k plus t to the power 36 minus 2 k plus t to the power 48 minus 2 k plus t to the power 60 minus 2 k we have to find out the coefficient of t to the power 24 in this expansion now note that in this expansion from this part we'll get the coefficient of t to the power 24 for k is equal to 0 and from this part we will get 4 k is equal to 6 and from this part we'll get for k is equal to 12 and from this part we will not get any coefficient of t to the power 24

so we get the coefficient of t to the power 24 in this expansion is 12 c 0 plus 12 c 6 plus 12 c 12 now 12 c 0 is 1 this is 12 c 6 and 12 c 12 is also 1

so our answer is 2 plus 12 c 6

so we see that here the third option is the correct answer

so this is the coefficient of t to the power 24 in the given expansion in this question we are asked to find out the coefficient of x to the power 11 in the expansion of 1 plus x square whole to the power 4 into 1 plus x cube whole to the power 7 into 1 plus x to the power 4 whole to the power 12 we will write down the binomial expansion of each of the factors

so 1 plus x square whole to the power 4 is equal to sum from k is equal to 0 to up to 4 4 c k x to the power 2 k the second one 1 plus x cube whole to the power 7 is equal to sum let's say from r is equal to 0 to up to 7 7 c r x to the power 3 r and the last one 1 plus x to the power 4 whole to the power 12 is equal to

sum from ace is equal to 0 to up to 12 12 c s x to the power 4 s now we can take product of all the three sums let us first note that in the first sum the exponents of x are 0 2 4 6 and 8.

and in the second sum the exponents are 0 3 6 9 and then next one is 12

so since we want to find the coefficient of x to the power 11

so we can discard that

so for us the possible choices of the exponents are this and in the last one the possible choices for the exponents are 0 4 8 and then the next one is 12

so that cannot be a possible choice

so here we consider 0 4 and 8.

let us call the first set as a the second set as b and the third set as c now we will see the possible choices of writing 11 as sum of three integers one coming from the set a one coming from the set b and one from the set c we can now note that it can be done in the following ways

so here we have 11 now from the set a let us first take 0

so if we take 0 from the first set a and then we can take 3 from the set b and 8 from the set c 0 3 and 8

so 11 is equal to 0 plus 3 plus 8 now we can note that for 11 fixed here and 0 fixed here there are no more possible choices in b and c other than 3 and 8.

now similarly we get that 11 is equal to 2 plus 9 plus 0 and 11 is equal to 4 plus 3 plus 4 and 11 is equal to 8 plus 3 plus 0 we can then note that there are no more possibilities left

so we have only 4 possibilities of writing 11 as sum of three integers each one coming from the set a b and c respectively therefore the coefficient of x to the power eleven in the given expansion is 4 c 0 into 7 c 1 into 12 c 2 plus 4 c 1 into 7 c 3 into 12 c 0 plus 4 c 2 into 7 c 1 into 12 c 1 plus 4 c 4 into 7 c c1 into 12c 0 now if we compute the values we can see that this value is equal to 462 this value is 140 this value is 5 0 4 and the last one is 7

so summing all these 4 integers we get this is equal to 1 1 1 3 therefore we get the coefficient of x to the power 11 in the given expansion is 1 1 1 3 and

so the third option is the correct answer we now look at the following question for some integers n and r such that 0 is less than or equal to r and r is strictly less than n we have a real number k

so that n minus 1 c r is equal to k square minus 3 into n c r plus 1 we have to find out among these given four intervals in which k can lie we use the information that n minus 1 c r is equal to k square minus 3 into nc r plus 1 that means n minus 1 factorial divided by r factorial into n minus 1 minus r factorial is equal to k square minus 3 into n factorial divided by r plus 1 factorial into n minus r minus 1 factorial

so we get r plus 1 divided by n is equal to k square minus 3 now note that we are given 0 is less than or equal to r and r is strictly less than n

so putting the values r is equal to 0 and r is equal to n minus 1 we get 1 by n is less than or equal to r plus 1 divided by n and this is less than or equal to 1 that means we have 1 by n less than or equal to k square minus 3 and this is less than or equal to 1 we write it here again 0 is strictly less than 1 by n which is less than or equal to k square minus 3 which is less than or equal to 1

so from here we can conclude that k square is less than or equal to 4 and also k squared is bigger than or equal to 1 by n plus 3 that means k square is strictly bigger than 3 now k square is less than or equal to 4 that implies minus 2 less than or equal to k and k is less than or equal to 2 and k square is strictly bigger than 3 that implies k is strictly bigger than square root of 3 or k is strictly less than minus square root of 3 to have a better understanding let us draw the real line this point is 0 this point is minus 1 this point is plus 1 this point is minus 2 this point is plus 2 minus square root of 3 will be

somewhere here and square root of 3 will be somewhere here
 so we now know that k from the first condition lies in this region and from the second condition that is this one we get that k lies in either in this region or in this region therefore we have the possible range of k to be closed interval minus 2 to open minus square root of 3 union open interval square root of 3 to closed interval 2.
 now we look at the options given to us we can see here that first second and third options are not correct and fourth option is correct that is k can lie in this interval open square root of 3 to closed 2 as this interval open square root of 3 to close to is a subset of the interval which we have found out we have the following question here we have been asked to find out the remainder of 8 to the power 2020 minus 62 to the power 2021 when we divide it by 9 to solve this problem we write 8 as 9 minus 1 and we write 62 as 63 minus 1 we know that 63 is 7 into 9
 so if we write 8 and 62 like this then it becomes simpler to find out the remainder of the given number when we divide the number by 9.
 our number is 8 to the power 2020 minus 62 to the power 2021
 so this one we have written as 9 minus 1 to the power 2020 minus 63 minus 1 to the power 2021 now we write the binomial expansion of this part and then of this part
 so this is equal to sum running from k is equal to 0 up to 2020 2020 ck 9 to the power 2020 minus k into minus 1 to the power k and this one is sum running from let's say r is equal to 0 to up to 2021 2021 cr 63 to the power 2021 minus r into minus 1 to the power r as we can clearly see that in the first sum each term of the sum is divisible by 9 except the term corresponding to k is equal to 2020 and in the second sum each term is divisible by 9 except the term corresponding to r is equal to 2021
 so therefore the remainder is 2020 c 2020 minus 2021 c 2021 into minus 1 that is 2 therefore we get that the first option is the correct answer here let us now look at this question for two non-zero numbers a and b consider the binomial expansion of a minus b whole to the power n for an integer n which is bigger than or equal to five we are given that the sum of the fifth and the sixth terms in this expansion is zero we have to then find out the ratio a by b we first write down the binomial expansion of a minus b whole to the power n this is equal to sum running from k is equal to 0 up to n n c k a to the power n minus k into minus 1 to the power k into b to the power k let us denote the fifth term by t5 and the sixth term by t6 we write down from this expansion what is t5 and what is t6
 so t5 is nc4 a to the power n minus 4 into b to the power 4 and t 6 is minus n c 5 a to the power n minus 5 into b to the power 5 we are given that t 5 plus t 6 is equal to 0
 so therefore we have n c 4 into a to the power n minus 4 into b to the power 4 minus n c 5 into a to the power n minus 5 into b to the power 5 is equal to 0 now let us take n c 4 into a to the power n minus 5 into b to the power 4 common then we get a minus n minus 4 into b divided by 5 is equal to 0 we are given that a and b are non-zero
 so we have a minus n minus 4 into b divided by 5 is equal to 0
 so therefore we get a by b is equal to n minus 4 divided by 5
 so we have got the ratio a by b
 so we see that the second option is the correct answer here now we look at the following question it is given to us that for a positive integer n the coefficients of three consecutive terms in the expansion of 1 plus x whole to the power n plus 5 are having ratio 5 is to 10 is to 14 we shall find out the value of n first we write down the binomial expansion of 1 plus x whole to the

power n plus 5 this is equal to sum running from t is equal to 0 to up to n plus 5 n plus 5 c t into x to the power t now from this binomial expansion it is very evident that three consecutive terms will be having coefficients in the following form n plus 5 c r minus 1 n plus 5 c r n plus 5 c r plus 1 for some r which is strictly bigger than 0 and is strictly less than n plus 5 now as we know that these coefficients are having ratio 5 is to 10 is to 14 we can write n plus 5 c r minus 1 is equal to 5 k n plus 5 c r is equal to 10 k and n plus 5 c r plus 1 is equal to 14 k for some k therefore we have n plus 5 factorial divided by r minus 1 factorial into n plus 6 minus r factorial is equal to 5 k in plus 5 factorial divided by r factorial into n plus 5 minus r factorial is equal to 10 k and n plus 5 factorial divided by r plus 1 factorial into n plus 4 minus r factorial is equal to 14 k now let us divide this equation by this equation we get r divided by n minus r plus 6 is equal to 5 divided by 10

so therefore we get 2r is equal to n minus r plus 6 that means we get 3r minus n is equal to 6

so let us have this equation next we divide this equation by this equation we get r plus 1 divided by n minus r plus 5 is equal to 10 by 14 that means 7 r plus 7 is equal to 5n minus 5r plus 25

so we get 12r minus 5n is equal to 18.

so we have another equation in r and n now from these two we can find out the value of n

so we have 3r minus n is equal to 6 if we multiply this equation by 4 then we get 12 r minus 4 in is equal to 24 then we subtract the equation 12 r minus 5 n is equal to 18 from here we get n is equal to 6

so therefore we have found out the value of n from here in this question we are given m to be the smallest positive integer

so that the coefficient of x square in the sum 1 plus x whole square plus 1 plus x whole cube plus

so on and

so forth 1 plus x whole to the power 49 plus 1 plus mx whole to the power 50 is 51 c 3 into 3 n plus 1 we shall find out the value of n let us first write the sum 1 plus x whole square plus 1 plus x whole cube plus

so on and

so forth up to 1 plus x whole to the power 49 as 1 plus x whole square into 1 plus 1 plus x plus

so on and

so forth up to the last term is 1 plus x whole to the power 47 now note that this inside part is a geometric series therefore we can write the whole thing as 1 plus x whole square into 1 plus x whole to the power 48 minus 1 divided by 1 plus x minus 1 that is equal to x

so this is equal to 1 plus x whole to the power 50 minus 1 plus x whole square divided by x

so our given sum turns out to be 1 plus x whole to the power 50 minus 1 plus x whole square divided by x plus 1 plus mx whole to the power 50 now let us write down the coefficient of x square in this sum

so the coefficient of x square will have contribution from the coefficient of x cube from the sum 1 plus x whole to the power 50 and from the coefficient of x square of the sum 1 plus mx whole to the power 50 and from the sum 1 plus x whole square we will not get any contribution to the coefficient of x square in the given sum in the question as in the denominator we have here x

so we write the coefficient of x square in the given sum is equal to 50 c 3 plus m square into 50 c 2 now this is given to be equal to 51 c 3 into 3 n plus 1

so we get $3n + 1$ is equal to $\frac{50c_3}{51c_3}$ plus m squared into $\frac{50c_2}{51c_3}$

so finally we have $3n + 1$ is equal to $\frac{48}{51}$ plus m square into $\frac{3}{51}$ that is 150 $3n + 51$ is equal to $48 + 3m^2$

so therefore $3m^2 - 3$ is equal to $153n$

so $m^2 - 1$ is equal to $51n$ now we know that m is the smallest positive integer which satisfies this equation taking m is equal to 1 we can see that we get n is equal to 0.

and for m is equal to 1 and n is equal to 0 this equation holds true

so we can take the choice of m to be 1

so the value of n is equal to zero

so in the given question we find out that the value of n is equal to zero we now look at the following question for integers m and n with n is bigger than or equal to m we want to show that $nc_m + (n-1)c_m + $ up to mc_m is equal to $(n+1)c_{m+1}$ using that we shall show $nc_m + 2$ into $(n-1)c_m + 3$ into $(n-2)c_m + $ up to $(n-m+1)$ into mc_m is equal to $(n+2)c_{m+2}$ let us begin with the first part of the problem we write this expression in the reverse direction that is $mc_m + (m+1)c_m + (m+2)c_m + $

so on and

so forth $(n-1)c_m + nc_m$ now note that we can write mc_m as $(m+1)c_{m+1}$ therefore from these two terms we get $(m+2)c_{m+1}$ and then the next term is $(m+2)c_m$ again from these two terms we get $(m+3)c_{m+1}$ note that here the term is $(m+3)c_m$

so if we keep on repeating this process we get this whole expression is equal to $(n-1)c_{m+1} + (n-1)c_m + nc_m$ again from these two terms we get nc_{m+1} and lastly from these two terms we get the whole expression is equal to $(n+1)c_{m+1}$ and note that this is what we wanted to show that this expression is equal to $(n+1)c_{m+1}$ now we start the second part of the problem we write the expression $nc_m + 2$ into $(n-1)c_m + 3$ into $(n-2)c_m + $ up to $(n-m+1)mc_m$ as $nc_m + (n-1)c_m + (n-2)c_m + $ up to $mc_m + (n-1)c_m + (n-2)c_m + $ up to $mc_m + (m+1)c_m + mc_m + mc_m$ now let us again go back to the first part of the problem we are going to use this part now using that we can write this one is equal to $(n+1)c_{m+1}$ this one is equal to nc_{m+1}.

this one is equal to $(m+2)c_{m+1}$.

and let us write this term mc_m as $(m+1)c_{m+1}$ again we use the first part of the problem and we obtain this whole expression is equal to $(n+2)c_{m+2}$ and now let us note that this is what we want it to show this solves our question number 19.

this is our question number 20.

we are given here two statements the first statement is sum from r is equal to 0 to up to n $(r+1)$ into nc_r is equal to $(n+2)$ into 2^{n-1} and the second statement is sum over r r is equal to 0 to up to n $(r+1)$ into nc_r into x^r is equal to $(1+x)^n + nx$ into $(1+x)^{n-1}$ we have to find out whether these statements are true or not and in case both of them are true we shall figure out whether statement 2 is a correct explanation for statement 1 or not note that if we put x is equal to 1 in statement 2 lhs is equal to sum from r is equal to 0 to up to n $(r+1)$ into nc_r and rhs is equal to $2^n + n$ into 2^{n-1} which is equal to 2^{n-1} into $2 + n$ therefore statement two implies statement one

so we will start verifying whether statement 2 is correct or not let us begin with the binomial expansion of $(1+x)^n$ this is equal to

sum running from r is equal to 0 up to n n c r x to the power r now multiplying x to both sides we get x into 1 plus x whole to the power n is equal to sum from r is equal to 0 to n n c r x to the power r plus 1 note that this is a polynomial identity

so we can take its derivative by taking derivative we get 1 plus x to the power n plus n into x into 1 plus x whole to the power n minus 1 is equal to sum from r is equal to 0 up to n r plus 1 into ncr into x to the power r now note that this is our statement 2

so statement 2 is true therefore among these four options given the option two is correct this is our last question of the problem-solving session on binomial expansions in this question we are given three sums s1 s2 and s3 we are also given two statements regarding the values of these three sums we shall figure out whether statement 1 and statement 2 are correct or not in case these two statements are correct we shall find out whether statement 2 is a correct justification of statement 1 or not first we note that s1 plus s2 is equal to s3 therefore we begin by verifying whether statement 2 is correct or not let us start with the binomial expansion of 1 plus x whole to the power 10 we know that this is equal to sum running from j is equal to 0 to up to 10 10 c j into x to the power j we take derivative on both sides of this polynomial equation we get 10 into 1 plus x to the power 9 is equal to sum from j is equal to 0 to up to 10 10 cj into j into x to the power j minus 1 now we put x is equal to 1 in both sides of this equation we get left hand side is equal to 10 into 2 to the power 9 and right hand side is equal to sum from j is equal to 1 to up to 10 j into 10 c j now we can note that this sum from j is equal to 1 2 up to 10 j into 10 cj is nothing but the sum s2 therefore we have found out the value of the sum is 2 which is 10 into 2 to the power 9 in statement 2 the value of s2 is given to be 10 into 2 to the power 8 therefore statement 2 is false now from the given options it is very evident that option 4 is the only possible correct answer but for the sake of completeness let us find out the values of s1 and s3 we take derivative of this polynomial equality now we get 10 into 9 into 1 plus x whole to the power a is equal to sum from j is equal to 0 to up to 10 j into j minus 1 into 10 c j into x to the power j minus 2 we then put x is equal to 1 in both sides of this equation we get lhs is equal to 90 into 2 to the power 8 which we write as 2 to the power 9 into 45 and rhs is equal to sum from j is equal to 2 2 up to 10 j into j minus 1 into 10 c j one can note that this is nothing but the sum s1

so therefore we have found out that the value of the sum is 1 is equal to 2 to the power 9 into 45 and as we know that s 3 is equal to s1 plus s2 therefore the value of s3 is equal to 2 to the power 9 into 45 plus 2 to the power 9 into 10 which is equal to 2 to the power 9 into 55

so the value of s1 and the value of s3 given here are correct and therefore statement 1 is true and statement 2 is false i end this session here with this we conclude the problem solving session on binomial expansions you