so in everyday life we encounter ah various problems that involve counting ah for example if you have three types of vehicles ah for example you may have a car you may have a scooter and you may have a cycle then you can choose one of these to go to the office ah i am a teacher and there are 10 topics that i have covered in the class and i have to give five questions

so in how many ways i can choose the topics to make five questions ah you might have booked tickets in say a flight and then when you go to the counter then the seats are allocated

so there are several ways of allocating the seat for example you may get an aisle seat a window seat or a middle seat or a seat which is in the near the emergency exit ah similar type of allocation problem occurs when the seats are allocated for one person or for a family in train

so in general you are encountered with counting problems allocation problem arrangement problems in ah almost every walk of life for example a team of players has to be selected

so there are say 20 probables and the team suppose it is a cricket team

so ultimately you may have to select only ah 11 full players and one reserved player

so in how many ways you can choose these 12 players out of 20 players such that 11 are to play in the main team and one player in the reserve ah a similar kind of problem occurs we have to form a committee ah for taking certain decisions ah taking a taking some courses from a list of courses for example at the beginning of every semester the student has to choose say 5 courses out of a list of say 30 courses which are available

so in how many ways he can choose then again there may be restrictions on the choice for example two of them may have to be compulsory and three of them two of them may have to be elective courses one of them may have to be lab kind of thing

so this counting problems are encountered in almost every decision making process ah in day to day life ah let me tell briefly about the ah history of this topic ah the term permutation and combination

so that means the problems related to the counting ah they were introduced ah probably around 6th century bc by certain indian mathematicians in fact there are references in ancient texts ah for example there is a reference in sushrut sanhita let me just say it sushruta sanhita this is by sushruth he was ancient indian medical professional

so you can say one of the first you can say doctors and he mentions that if there are six different tests

so how many combinations of these tastes can be made to produce medicines and he gave the answer as 63 this is the number that sushrut sanita mentions now in modern terminology we can calculate it in modern terminology this is like i will explain to you a little later in modern terminology this can be calculated as follows that is if we choose a single test

so a medicine with a single test can be chosen in six ways if you consider a medicine which is a combination of two tests with a combination of two tests then that can be chosen in now you have total six possibilities

so the first one you can choose in six ways the second one you can choose in five ways however the order in which they are chosen is not important therefore you can divide it by two

so this number becomes fifteen similarly if we consider a medicine with a medicine with a combination of three tests then that can be chosen now again let us look at this six ways for choosing the first one five ways of choosing the second one and four ways of choosing the third one now once again these three things can be in any particular order that order does not make any difference

so we divide by three into two into one

so that is equal to 20 ways similarly if you consider a medicine with a combination of four tests then that can be chosen in six into five into 4 into 3 divided by 4 into 3 into 2 into 1 that is simply 15 ways a medicine with a combination of five tests can be chosen in six into five into four into three into two divided by five into four into three into two into one that is six ways and a medicine with all six

so that means only one way

so now if you look at the total number of ways that is equal to six plus fifteen plus twenty plus fifteen plus six plus one

so that you can see easily sixty three

so you can see here that how many combinations of six different tests can be made to produce medicine

so there are sixty three ways

so this type of calculation was known in ancient india ah then the second one is that around 3rd century bc a sanskrit scholar by name pingala he wrote chand sutra and he discussed the ways of determining the number of combinations of a given number of letters taken one at a time taken two at a time

so as you can understand from the name chand sutra means how to write various chandas with different type of combinations

so for example you have these many matras this many letters etcetera

so he used counting methods for considering various combinations of letters ah another reference is in jain mathematicians and they studied this topic under the name vikalp ah around 850 a d ah jain mathematician mahavir he has provided general formulae for permutation and combination around 1150 a d ah mathematician bhaskaracharya 2 ah

so in fact bhaskaracharya 2 is well known as one of the most important ancient indian mathematicians it is credited that he actually compiled all the results known till that time and also added large number of his own results

so he also ah con in his book that is famous book called lilavathi which was named after his daughter ah

so under this topic called unk under the topic unk pash he has given various counting methods and he has actually given you can say the modern formulae for permutation and combination of course he did not use that notations which are used today but he actually was able to provide the general method of counting these things ah there are other references ah in ancient china ah then greece then there are works in ancient arab and in israel which is modern israel that time

so you can say in hebrew literature there is some reference to the counting techniques the modern treatment of this subject is found in detail in the book arse conjectandi it was published in seventeen hundred thirteen and this is by swiss mathematician jacob bernoulli his timeline is 1654 to 1705 that means the book was published posthumously and other important contributions are are by tarta galia pascal the french mathematician format

so these are the famous mathematicians who actually originated the theory of probability also ah d mayor and from the bernalillo family itself james bernali leibniz and euler all of these european mathematicians contributed in great detail to various aspects of the subject you can say combinatorics which has permutation and combination as one of the most important ah components ah

so you can say that this subject is somewhat old and in your particular course of class 11 12 we tell you the basic principles of counting

so you can say actually there are several basic methods of counting what we actually consider so for example if i say that i have say two cars and three motorcycles and i want to choose one vehicle for transportation then how many ways i can choose

so naturally one will give straight away answer that there are two plus three options

so for example ah if i have some particular event for which there are say m methods or m ways there is another event for which there are n ways then the total number of methods for a or b will become m plus n

so this is the first counting principle which is actually you can say as a layman you can think about it because it is simply adding the number of possibilities

so in combinatorics we call it the addition principle the first one is the addition

so let me formally state this let there be m ways for an event a to occur and let there be n ways for another event b to occur if all ways are distinct then the number of ways for a r b to occur is m plus n ah we can express it in the modern language of sets also let me put it that way if we use the language of sets then let me put this symbol this is called cardinality of a ah denotes the number of elements in set a that is also called cardinality of set a ah then the addition principle states that if a and b are disjoint the cardinality of a is m and the cardinality of b is n then cardinality of a union b is equal to m plus n

so basically it means that the number of ways if you are considering several things can be simply accumulated that means you can add them now i have written here two events now you can immediately write for several number of events suppose i have three events suppose i have four events and if i have to consider the total number of ways for those events to occur then i have to simply add the number of ways for each of them

so this gives rise to the general addition principle let there be m one ways for an event e one two occur m two is for an event e two two occur and

so on mk weighs for an event e k to occur then the number of for either of e one e two e k to occur is m one plus m two plus m k let us consider a simple illustration here from say daily to mumbai one can travel by air that is eight flights are there by train and 12 trains are available and land and in land you here you can use long distance bus service or he can use a car then in how many ways one can travel from daily to mumbai

so naturally if you consider the event of travelling by air as event e one then the cardinality of e one is eight if i consider the event of travelling by train from daily to mumbai then the cardinality of e two will be twelve and if you consider e three to be the event of travelling by the land route then he can have the cardinality of e3 as two because long distance was service or car

so if we formally define e one is travelling by air then cardinality of e one is eight e two is travelling by train then the cardinality of e two is twelve and e three suppose i consider travelling by land then the cardinality of e three is two

so the total number of ways of travelling from daily to mumbai is

so this will be cardinality of e one union e two union e three which are all disjoint

so that is cardinality of e one plus cardinality of e two plus cardinality of e three

so that is equal to eight plus twelve plus two that is equal to twenty two

so there are total number of twenty two ways of travelling from daily to mumbai if these kind of options are available ah to illustrate one counting example generally we consider some geometric shapes in which there are nested triangles or nested squares or nested rectangles etcetera

so i will give one problem where we have nested squares for example we usually

consider a chess board which is eight by eight
so we can have any kind of such arrangement
so let us consider say how many squares are there in a five by five array
so let me just show you by diagram to make it very clear ah
so this is a five by five array and each of the cell is a square here ok
so if we consider the squares in the array can be counted as even the set of one by one squares e two is the set of say two by two squares e three the set of three by three squares e four is the set of four by four squares and e five is the set of five by five squares
so let us just look at it one by one square means each individual each individual cell is a square ok
so if we look at how many of these are there
so if we consider the cardinality of e one
so if it is a five by five array then total five square that is twenty five one by one squares are there if we consider two by two squares that means taking two at a time then it is becoming
so if you look at the method of counting here let us see we can consider this as a two by two square and then if i leave the first column and i move to the next one then i have another two by two here similarly if i leave the first two and go to the third and fourth then again this is a two by two then similarly i can leave the first three and i can go to the fourth and fifth then that is also a two by two
so there are actually four such squares because what has happened that initially there are five cells but when we are taking two at a time then we have to skip one because starting from the first one the two are counted and then we can slide down
so there are four such similarly if we consider the width of this
so it is occupying two rows now
so again we can slide down we can consider this here from the second and third row the same counting can be done that means i can consider the first one and the second one second column and the third column third column and the fourth column fourth column and the fifth column
so again it is four such two by two squares are there and again if we slide down we can consider third and fourth row
so again four such squares will be there and we consider fourth and fifth
so again there will be four such squares
so there are four such cases here
so the number of two by two squares it is actually five minus one square that is four square that is sixteen
so i have written five minus one just to illustrate that since i am taking two
so one less will be there now that gives a pattern here if we consider three by three squares
so suppose i consider three by three squares then it will become five minus two square that is nine because if i am considering three by three then i will consider first second third column and first second third row
so that will be one three by three square and then if we slide along the column that means i consider next time second third fourth or fourth fifth ah third fourth fifth column then there will be three such three by three squares in one two three rows same thing happens if i consider second third and fourth row are third fourth and fifth rows
so there will be total three into three
so i am writing it in the form five minus two square that is nine and in a very similar way then you get e four the number of four by four squares is five minus three square

so that will be simply four and the five by five square is five minus four square that is one the five by five square there is only one

so the total number is then becoming equal to that is equal to one plus four plus nine plus sixteen plus twenty five that is equal to fifty five

so in a five by five arrays if we can consider all the nested square cells then there are total such fifty five squares available ah let us generalize this

so like in a chess board you have eight square

so in general if i consider an n by n array then how many such ah squared cells will be there

so let me consider this how many squares are there in an n by n array

so if i consider e i to be the set of i by i squares where i can take values one to n then if we keep the same method of counting then the number of one by one squares is simply n square the number of two by two squares will be n minus one square the number of three by three squares will be n minus two square and

so on the number of n by n squares is one

so the total number of such squares you can easily see it is simply one plus two square plus three square plus and

so on up to n square it is sum of squares of first n natural numbers

so we actually know the formula for that

so the total number of squares in an n by n array is one plus two square plus three square plus and

so on n square ah

so you have done the formula for that it is actually n into n plus one into two n plus one by six ah you can check we actually solve the problem for five we got the answer fifty five

so if we consider here n is equal to five then it becomes five into six into eleven divided by six

so this six six cancels out you get five into eleven is equal to fifty five which was the answer for this one

so as an example we can consider in a chess board how many squares are there for example a chess board is an 8 by 8 square array

so the total number of squares will be 8 into 9 into seventeen by six

so that is equal to two hundred four

so in a chess board their total number of squares if you count that is equal to two hundred four

so ah you can consider this as a very simple illustration of the addition principle because what we are doing is that we are splitting the total event as a union of ah several events and then we count the number of possibilities for each of those events to occur and these events are disjoint

so in how many ways the total number of the complete event can occur that is simply adding all the possibilities

so this is the first counting principle in combinatorics ah the next important principle is the multiplication principle if there are m ways of an event a to occur and there are n ways for an event b to occur then the total number of ways for occurrence of event a followed by event b

so you notice the difference from the language in addition principle in the multiplication principle in the addition principle we are saying either event a occurs or the event b occurs etcetera

so the what is the total number of ways

so we simply add m plus n in this case both events a and b are occurring

so we just considered is something like this that firstly a occurs and then b occurs or firstly you can say b occurs r a occurs or you just say that both a and b occur in this case ah you will have the m and n multiplied rather than added ah you just think it like this ah i mentioned there are 22 ways of

travelling from delhi to mumbai
so suppose there are another say ah 20 ways of travelling from mumbai to chennai
so then what is the total number of ways of travelling from delhi to chennai via mumbai then in this case we can use any of the 22 ways in the first case and any of the 20 ways in the second case
so you can multiply
so it becomes four hundred and forty ways
so let me just give a brief ah illustration of this proof of this ah a theoretical proof of this multiplication principle
so we can use the language of the set theory using the terminology of set theory let a be the set consisting of m elements a one a two a n
so for an event a to occur there are m ways
so we describe it in this particular fashion ah that a is a set consisting of m distinct elements a one a two a m and similarly let us write the set b as consisting of elements b 1 b 2 b n then the possible number of ways of occurrence of event a followed by event b
so you can describe it in the form of ordered pairs
so for example you can say a one b one what does it mean that means for event a to occur we choose the method a one similarly for the event b to occur the occurrences by b1 ah like i can say for travelling from delhi to mumbai we chose a flight
so maybe the first flight and for travelling from mumbai to chennai again we choose the first slide
so it is a one b one now you can consider other options here it can be the first flight and here it can be the second flight and
so on here it is say first flight and here it is some other method for example it could be by a ship also and then you can have the second flight from delhi to mumbai then b 1 a 2 b 2 and
so on a 2 b n and
so on finally here you can have the last method that is travelling by car and here you can have the first flight and
so on
so by this arrangement you can see that the total number of elements is m into n because we are able to arrange all the elements in an m by n array
so the total number of ways is m into n ah once again we can write the formula using ah the cardinality principle here if we want to write like this ah let cardinality of a be m the cardinality of b b n ah then what are these elements actually a one b one a one b two and
so on these can be considered as elements of the cartesian product a cross b ok
so a cross b is actually a one b one a one b two and
so on which we also write like this it is an element x y such that x belongs to a y belongs to b
so then cardinality of a cross b is nothing but cardinality of a into cardinality of b ah we can consider this as a compound event that means when one event is occurring and then it is followed by another event
so ah it can be considered as a compound event
so the number of ways of counting the possibilities of compound event is nothing but you multiply for the individual events which are involved there ah you can easily generalize again this to more than two events also
so we have general multiplication principle
so ah let there be m one ways for and event even to occur m two ways for event e two two occur and
so on mk weighs for event e k to occur then the total number of ways in which

events e one e two and
so on e k occur in this order is m 1 m 2 m k this is the product here one point which you should note here is that for example in this case i considered firstly the event a and then the event b ah and then i am writing the number to be mn ah suppose you exchange the order suppose firstly i say event b and then the event a that means i am just redesignating my events now if we apply the same logic the answer will be nm now this is not surprising because if you consider multiplication multiplication is commutative therefore m n and n m they are same therefore it does not make any difference
so therefore in the general multiplication principle when i am writing events e one e two e k they occur in this particular order that means i say firstly e 1 occurs then e 2 occurs and
so on and then finally e k occurs then the number of possibilities is m one into m two into mk now since multiplication is commutative you will receive the same answer if i conduct the events in any other order for example i may firstly say e3 occurs and then maybe e7 occurs and then e1 occurs
so in any other order if i write is still the number of elements or the number of ways will be the same this is because if i consider the cartesian product a cross b r b cross a it has the same number of elements the order of the elements may be different because if i say b cross a then firstly you have to say b one then a one that means b one followed by a one and it is not the same as a one b one but the total number is the same
so this is another point that ah needs to be mentioned let me just write it as a remark in the multiplication principle the order in which events occur does not make any difference since multiplication is commutative and also because the cardinality of cartesian products of sets does not depend upon the order in which sets are taken in a cartesian product ah
so this multiplication principle is actually one of the you can say very very fundamental ah principle of counting in in fact in most of the text books including your cbse textbook this is written as the first principle in fact here i have added one additional thing that is the addition principle as the first principle but generally in the books i will start from the multiplication principle ah
so anyway ah i just introduced it here let us look at some of the ah examples here ah in a class of 50 students 20 are measuring in say physics 20 are measuring in chemistry and 10 are measuring in mathematics
so in how many ways we can choose a representative from each group
so we want to have a ah three representatives ah in such a way that one is from physics one is from chemistry and one is from ah mathematics
so now this is a very simple thing if we apply the multiplication principle in that case if i consider this first one that means the students measuring in physics
so the representative is from there
so he can be any of the twenty students
so the total number of ways will be twenty
so in that way if i write for physics chemistry and mathematics then if we use the notation here let me give you a systematic presentation here
so let us consider even that choosing representative from students majoring in physics similarly if i consider e two as the representative measuring in chemistry and e3 is the choosing a representative from students majoring in mathematics
so then if i consider cardinality of e one that is twenty cardinality of e two that is twenty and the cardinality of e three that is equal to ten and therefore now the cardinality of e one cross e two cross e three that is nothing but

twenty into twenty into ten that is equal to four thousand
so there are four thousand different ways of choosing ah three representatives one from each group ah let us consider a ternary sequence
so a ternary sequence consists of digits say zero one and two ok like a binary sequence consists of zero one similarly a ah ternary sequence consists of digits zero one two
so how many five digits ternary sequences can be formed
so now you see i am consist considering a five digit ternary sequence
so these are five places here in the first place i can put either of zero one or two that means the first place can be filled up in three different ways in the second place also i can put either of zero one two in the third place also i can put either of zero one two in the fourth place and in the fifth place also the same logic will be repeated
so in each position we can place either of 0 1 or 2.

so the number of ways of filling each position is three total number of positions is five
so by the multiplication principle in short i can use m p the total number of such ternary sequences is 3 into 3 into 3 into 3 that is 3 to the power 5 that is 243 ah i will continue in the following lecture on this further examples of multiplication principle and then we will talk about arrangements that is ah permutations and combinations you