

welcome back students welcome to the series of lectures on matrices in the last class we saw about solving a system of linear equations what we saw in the previous class is that solving a system is equivalent to solving the corresponding row reduced system and we found that these two systems have got the same set of solutions we saw some examples in fact we saw systems which have a unique solution now let us do some examples where there are no solutions let us begin with an example two x minus four y plus z equal to three x minus three y plus z equal to five three x minus seven y plus two z equal to twelve fine

so now let us write down the matrix form of this two minus four one one minus three one three minus seven two then applied on the matrix xyz when applied on the unknowns xyz should give me three five and twelve fine the augmented matrix for this is two one three minus four minus three minus seven one one two augmented with three five twelve we have this this is the augmented matrix now we will have to look we are the non zero there are no non zero rows and

so looking for the first the looking for the first non zero term in the first row it is just two

so multiplied by half

so replace r one by half times r one

so which is going to give me one minus two half and then i will have three by two the other rows are untouched one minus three one five three minus seven two twelve and then i will have to make the other thing zero other elements of the first column into zero

so replace r two by r two minus r one and replace r three by r three minus three times r one i will have one zero zero now first column is untouched first row is untouched similarly $1 \frac{1}{2}$ three by two for the second one r two minus r one

so minus three minus minus two

so i will have minus one and similarly one minus half which is again half five minus three by two which is seven by two r three

so r three minus three times minus seven minus minus six which is just one and similarly i have r three minus three times

so two minus three by two which is half

so i have a minus one over here fine and finally ah twelve minus ah nine by two right i have twelve minus nine by two

so which is twenty four minus nine right i three r three minus three times r one

so twelve minus three by minus three times three by two which is nine by two

so twenty four minus nine which is 15 by 2 right i have this now this is the second one i have second the first non-zero element in the second one is minus one

so let me just multiply it by minus one r two is replaced by minus of r two what i will have is one zero zero minus two half three by two one minus half minus seven by two and then i have minus one half and then it is going to be fifteen by two now i will have to convert these two into zero minus two and minus one into zero

so replace r one by r one plus two times r two and replace r three by r three plus r two and one can notice in this stage that what you have is one zero right this goes to zero

so r one plus two times

so half minus one which is minus half you have here this row is untest zero one minus half but then it is just for the last one what we have is r three plus r one

so we will have zero zero zero now here the second one is untouched seven by two minus seven by two r one plus two times r two r one three by two plus two

times r^2

so two times of that is

so minus of fourteen by two

so you what you will have is minus eleven by two and similarly r^3 plus r one fifteen by two minus seven by two

so i will have eight by which is just four but what you can notice here is that the last row in the last row of the coefficient matrix you have just only zeros but on the other hand what you have is a non-zero term over here if you just convert this back into the system of linear equations what you will have is the following x minus z by two equal to minus eleven by two y minus z by two equal to minus seven by two zero times x sorry zero times z zero times x plus zero times y plus zero times z equal to four

so this system has got no solution right because this last one does not make sense what it says is that zero equal to four therefore the given system has no solution let us write down again the last obtained my last resulting matrix resultant matrix what we had is one zero zero zero one zero minus half minus half zero minus eleven by two minus seven by two eight by two right

so this is the

so the rank of this coefficient matrix by looking at this one can easily conclude that the rank of the coefficient matrix A is just two while rank of the augmented matrix is three and we know that two is strictly less than three this implies that the system has no solution what we wanted for the solution to exist is that rank of A should be equal to the rank of the augmented matrix but that is not the case here which can be easily observed now let us do one more example solve the system one plus i into z one minus z two equal to i one minus i into z one plus one plus i into z two equal to one notice that this is a system with complex coefficients well the procedure is the same

so let us apply the same algorithm to reduce the coefficient matrix into its row reduced form

so the augmented matrix is given as follows one by one plus i minus one one minus i one plus i augmented with i and one

so you have the first term as one plus i lets convert this one plus i into one r one is replaced by one by one plus i into r one which is one minus one by one plus i i by one plus i one minus i one plus i one

so now lets

so lets write down the resulting matrix one next one is minus one by one plus i lets multiply and divide by one minus i

so what we will end up with is on the numerator we will have minus one plus i upon one plus i into one minus i which will lead to two augmented with i into one minus i which is minus one plus i upon two and then one minus i one plus i one i

so next i have this one minus i let me convert this one to one minus i into zero r^2 is replaced by r^2 minus one minus i into r one

so what i will end up with the first row leads to leads to one zero and similarly the first row remains the same you will have minus one plus i upon two and augmented with minus one plus i upon two sorry it is going to be sorry one plus i one plus i upon two yeah even in the previous one it is just one plus i upon a point two now let us calculate the remaining terms it is one plus i minus one minus i into minus one plus i which is same as one plus i plus one minus i whole square but 1 minus i whole square is exactly 2 sorry it is minus 2

so you will have sorry it is ah what will have

so we will have one plus i plus one minus i whole square upon two which will lead to one plus i plus one minus two y plus one minus two y minus one whole upon two

so this one gets cancelled and you will have minus two y and
so this minus two i and two gets two and two gets cancelled therefore you will
have one plus i minus i

so i will find up finally end up with just one next one one minus one minus i
into one plus i upon two one minus i into one plus i is just two and

so this gets cancelled you will have one minus one which will lead to zero now
i have one over here let me convert this other element in the second column that
is minus one plus i upon two to zero r one is replaced by r one minus minus one
plus i upon two into r two i will have one zero zero one i am just adding
something with zero

so which will essentially lead to the same thing one plus i upon two and then a
zero right

so therefore what do we have z one is one plus i upon two and z two is zero
thus the solution is one plus i upon two and zero is the solution consider the
system two x plus five y plus two z equal to minus one x plus two i minus three
z equal to five five x plus twelve i plus z equal to 10 as usual let us write
down in the matrix form $\begin{bmatrix} 2 & 1 & 5 & 2 & 12 & 2 & -1 \\ 1 & 5 & 5 & 2 & 12 & 2 & -3 \\ 1 & 5 & 5 & 2 & 12 & 2 & -1 \end{bmatrix}$ times the unknown
unknowns x y z

so the constant matrix or the constant term matrix is 1 5 right we have
returned the given system in terms of matrices now let us write down the
augmented matrix a augmented with the constant of matrix b the coefficient
matrix is just two one five five two twelve two minus three one and we are
augmenting with the constant of matrix which is minus one five and ten we have
no non zero rows no zero rows and therefore we look for the first non zero term
in the first row which is just two

so our aim now is to convert this two into one

so we replace r one by half of r one which is going to give me one five by two
one augmented with minus half the remaining rows are untouched one two minus
three five five twelve one and ten i have this now i will have to make the other
things zero that is one and five into zero replace r two by r two minus r one
and replace r three by r three minus five times r one first row remains
untouched one five by two one and then you have minus half

so one minus one which is zero two minus five by two which is five by two minus
three minus one which will give me minus four right five minus minus half which
is five plus half five plus half eleven by two right and now here it is zero it
is twelve minus five times the r one twelve minus five times r one which is five
into five twenty five by two

so i will have minus half one minus five which will give me minus four r two
minus one sorry this is not the second element is r two minus r one two minus
phi by two oh this is minus half ah this is exactly what i wanted here it is
going to be r three ten minus ah five times minus half which is going to give me
ten plus five by two which is twenty five by two right now let me just convert
this thing into one r two is replaced by minus half times r two what i will have
is one zero zero phi by two one minus half i am just multiplying it by minus two

so i will have one which is eight here minus eleven and then the remaining
stays as it is which are unchanged i will have to convert the other two elements
into zero which is five by two and minus half

so what i will do is replace r one by r one minus five by two times r one and
similarly r three is replaced by r three minus sorry plus half times r two

so the resulting matrix is you will notice that one zero zero zero one zero and
then the last terms are it is going to be eight which is going to remain
untouched the first one which is r one which is one minus sorry r one minus phi
by two times r two five by two times r two which is eight four twenty

so one minus twenty which will give me minus nineteen here it is going to be

zero and the augmented matrix is going to be i will have minus half minus five by two times minus eleven minus fifty five

so i will have minus fifty four by two i am pleased to bother about what the original one and this is minus eleven and finally i will have twenty five by two plus half times minus 11

so which will give me 14 by 2 this is the one that i have now the rank of the coefficient matrix a just look at this and one can easily conclude that it is just two and similarly rank of the augmented matrix which in this case is you have although you have a non zero zero row you have this non zero term which will easily tell us that this is going to be three therefore rank of the augmented matrix is strictly greater than the rank of the coefficient matrix that will imply that the given system has no solution now let us do one more example let us try to do ah one more example x plus three y plus four z equal to eleven two x plus three y plus two z equal to seven four x plus nine y plus ten z equal to twenty and finally three x minus two y plus that equal to one this is the system that i have again one can notice that this is an over determined system that means the number of unknowns in this case is 3 while the number of equations in this case is four

so this is an over determined system fine now let us try to write down in the form of matrix $\begin{bmatrix} 1 & 2 & 4 & 3 & 3 & 3 & 9 \\ -2 & 4 & 2 & 10 & 1 & 11 & 7 & 20 & 1 \end{bmatrix}$ when applied on the coefficient on the unknowns xyz should give me the constant matrix constant vector $\begin{bmatrix} 11 & 7 & 20 & 1 \end{bmatrix}$ and one as usual let us try down the augmented matrix for this the augmented matrix is one two four three three three nine minus two four two ten one augmented with eleven seven twenty and one this is the system that we have this is the augmented matrix now let us try to write down the

so do the row elementary operations notice that there are no zero rows and therefore the first row look for the first row it has got a the first non-zero element is one

so we do not have anything to be done will convert the other elements into of that column into zeros replace r_2 by $r_2 - 2r_1$ r_3 is replaced by $r_3 - 4r_1$ r_4 is replaced by $r_4 - 3r_1$ let us perform all these operations

so as a result and what we expect is that all these three goes to zero fine and then three remains at as it is the first row in fact whole remains unchanged r_2 is replaced by two times r_1

so three minus six which is minus three two minus two times four

so you will again have minus six seven minus two times eleven

so seven minus twenty two which is minus fifteen and then nine minus four times three which is nine minus twelve you will have minus three ten minus sixteen i will have minus six and then twenty minus forty four yes twenty minus forty four this will give me minus twenty four the last one minus two minus nine which is minus eleven one minus twelve which is minus eleven one minus thirty three which is minus thirty two right the second row the first non zero coefficient is minus three let us make it into one r_2 is replaced by one by minus three times r_2 i will have one zero zero zero three four augmented with eleven which is one minus two i will have five the other things are untouched

so my next aim is to convert this three minus three and minus eleven into zeros so replace r_1 by $r_1 - 3r_2$ replace r_3 by $r_3 + 3r_2$ and replace r_4 by $r_4 + 11r_2$ well let us do these operations the first column is going to be one zero zero zero the second one turns out to be zero one zero zero third one let us calculate r_1 one is replaced by $r_1 - 3r_2$

so four minus six which is minus two augmented with minus fifteen which is minus four second row remains as it is two five third one r_3 minus six plus

three times r_2 which will give me zero minus twenty four plus fifteen which will give me minus nine and then I have r_4

so minus 11 plus 22 you have 11 minus 32 plus 35

so which will give me just 3 I have a zero row in between right I do not have so no the next one is that well I have a zero row

so interchange r_3 is swapped with r_4 I have one zero minus two with minus four zero one two five zero zero eleven three zero zero zero minus nine convert this eleven into one but I do not have to do that because I have a zero row over here and a non-zero term over here right therefore the system has got no solution because you have a non zero row in the coefficient matrix but while in the augmented matrix you have a non zero term and therefore this system has got no solution let us proceed further with examples now let us do some examples when you have infinite number of solutions right our first example let us look for an easy let us begin with an easy example $x + 2y + 3t = 7$ $z + 4t = 1$ can notice that this is an underdetermined system that means what let us look at the number of unknowns or the number of variables that we have as four well the number of equations that we have in this case is just through two

so two is strictly less than four therefore this is an under determined system let us try to solve this lets before that first let as usual write down the matrix form of this $\begin{bmatrix} 1 & 2 & 0 & 3 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & 0 & 0 & 0 & 0 \end{bmatrix}$ evaluated at x, y, z, t equal to seven eight fine now notice that in this case the given matrix or the given coefficient matrix is itself in the is in the row reduce to form because the first non-zero coefficient is one and the other elements in that column is zero and you have it so there are only two equations in fact two rows

so the other one is zero and similarly the first non-zero element appears in the third column and the other element is zero therefore this is in the this is a row reduced form alone now how to solve the system what are the solutions for the system right

so wherever there is one and zeros right will call these the variables corresponding to these positions right

so this I will call it as the corresponding to one and three

so the variables x and z are dependent variables while the other variables that are variables y and t are independent variables

so wherever there is an independent variable

so let us try to

so we have two independent variables

so let me say that y is λ and t is μ where λ and μ or any real numbers or any real numbers are any arbitrary real numbers right now let us write down again let us look at the write down the equation $x + 2\lambda + 3\mu = 7$ $z + 4\mu = 1$ now just substitute $y = \lambda$ and $t = \mu$ and then see what is there $x + 2\lambda + 3\mu = 7$ $z + 4\mu = 1$

sorry it should be the other way right sorry ah it should be $z + 4\mu = 8$ plus four μ is eight

so the second one

so this will imply that $z = 8 - 4\mu$ just substitute in the first one what will I have

so $2\lambda + 3\mu$

so what will I have $x = 7 - 2\lambda - 3\mu$ right the second one gives z as $8 - 4\mu$ first one gives x as $7 - 2\lambda - 3\mu$

so therefore the solution set to this equation $(7 - 2\lambda - 3\mu, 8 - 4\mu, \lambda, \mu)$ with the condition that

lambda and mu both are from real numbers

so we have an infinite number of solutions for various lambda and mu you get various solutions for each lambda and mu you get various different solution right as lambda and mu varies the solution keeps changing that is what we observed through this example let us do one more example $8x + 5y + 11z = 30$ minus x minus $4y$ plus $2z = 3$ two x minus y plus five $z = 12$ equal to lets write down the matrix form eight five eleven minus one minus four two two minus one five and this is applied on the unknown vector $x y z$ should give me thirty three and twelve

so the augmented matrix in this case is eight minus one two five minus four minus one eleven two five augmented with thirty three and twelve

so there are as usual there are no zero rows

so let us convert the first one r_1 is replaced by one by eight times r_1 one what we will have is one five by eight eleven by eight thirty by eight the other rows are unchanged minus one minus four two three two minus one five and twelve will have to convert the other rows other elements of the first column into zero r_2 is replaced by $r_2 + r_1$ and r_3 is replaced by $r_3 - 2r_1$ the first column becomes one zero zero half right minus four plus five by v eight four plus five by eight minus 32 plus 5 you will have

so first column as usual remains unchanged let us write it down minus twenty seven by eight two plus eleven by eight

so sixteen plus eleven

so twenty seven by eight and then $r_3 - 2r_1$ minus one minus two times minus four

so which is eight minus one seven five minus two times two five minus four which is one you have twelve minus two times three

so twelve minus six which is $y = \frac{1}{2}$ six okay what am i trying for this is wrong

so let us have you let us convert the first element r_1 is replaced by one by a times r_1 one what will i have one five by eight eleven by eight and then it is augmented with thirty by eight minus one minus four two three two minus one five and twelve i will have to convert these two elements into zero

so i am just replacing r_2 by $r_2 + r_1$ and r_3 is replaced by $r_3 - 2r_1$ first row remains unchanged one five by eight eleven by eight and i will have thirty by eight ah zero

so i will have minus four plus five by eight which will give me twenty seven by eight two plus eleven by eight which is again twenty seven by eight

so you have a minus $y = \frac{1}{2}$ five minus one ah minus twenty seven by eight

so here you will have twenty seven by eight right and then i will have minus one plus two times five by eight

so i will have five by four

so minus one plus five by four i will have zero sorry minus four plus five

so this is zero

so minus one minus four

so i will have one by four and then five minus two times eleven by eight which will give me five minus eleven by four which is twenty minus eleven by four

so i will have nine by four

so these terms three into three plus thirty by eight twenty four plus thirty fifty four by eight and then finally i will have twelve minus two times thirty by eight which is twelve minus thirty by four

so forty eight minus thirty eight is eighteen by four now the next element is just twenties this one i will have to make it into one replace r_2 by minus eight by twenty seven times r_2 one zero zero the other rows are untouched i will just have one by four nine by four eighteen by four fine now if i multiply this will become just one the remaining term i will have twenty

so i will have minus one i will have minus two f minus two now i will have to

convert the other elements of the second column into zero

so replace r_1 by $r_1 - \phi r_2$ and replace r_3 by $r_3 - r_2$ what will A have the first column remains unchanged the second column turns out to be $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and for the third column if A convert this

so what will A have the first one is replaced by $9 - 4 = 5$ or one by four that is small change this should not be $9 - 4$ this should be $1 - 4 = -3$ by $2 \times r_1$

so you will have $11 - 4 = 7$ by $4 - 11 = -7$ by 4

so twenty minus nine sorry nine by four this term r_3 is just minus one plus two into five by four minus one five zero zero zero one zero zero and then A will have A this term is

so $r_1 - r_2$ is eleven by eight minus ϕ by $16 - 8 = 8$ which A will have it as sixteen by five sorry sixteen by eight and then this minus one remains as it is nine by four minus one by four into minus one

so which is going to be ten by ten by four augmented with r_1

so thirty by eight minus five by eight into minus two

so A will have forty by eight and then the second one remains as it is minus two the last one it is going to be eighteen by four minus one by four into minus two

so which is going to be twenty by four fine now A will have to convert this element into one

so replace r_3 by four by ten times r_3 the other rows remains unchanged zero one minus one minus two zero zero one four by ten A will have just two convert the other elements into zero these two

so r_2 is replaced by $r_2 + r_3$ r_1 is replaced by $r_1 - 16r_3$ first and second column remains unchanged in the last one this one r_2 is replaced by $r_2 + r_3$ A will have a zero this plus this which is going to give me zero and then these two things $r_1 - r_2$ is replaced by which is sixteen by eight minus this term is again zero but then A will have which is forty by eight minus sixteen by eight into two

so this is thirty two

so which is eight by eight which is one here you have a definite solution the solution in this case is the rank is three right solution you have one zero and two is the solution the solution is unique in this case in the next lectures we will do some more examples wherein you will have examples with A for over determined systems under determined systems multiple solutions and all these things thank you all you