

okay friends we start some important reals on linear inequality which involves absolute value of real number that is we can say modulus function

so before we start we just discuss some concept about or the meaning of mod x
 $\text{mod } x$ is an absolute value function

so it represent plus x if x greater than equal to 0 and minus x if x less than 0 obviously where x belongs to \mathbb{R}

so important concept that we use in this important real that is modulus of x can be defined in this way

so here we have some result that is some real tone in equation involving modulus of the variable if a belongs to zero infinity it means a is positive real number then we have two situation that is $\text{mod } x$ less than a

so when $\text{mod } x$ less than a then minus a less than x less than a that is x belongs to minus a since x lies interval minus a a and second case is when modulus of x less than equal to a implies minus a less than equal to x less than equal to a that is x belongs to close interval minus a a a second is a belongs to zero infinity it means again a b is positive real number then modulus of x greater than a implies x less than minus a or x greater than a that is x belongs to interval minus infinity minus a union a infinity obviously open interval and second situation modulus of x greater than equal to a implies x less than equal to minus a or x greater than equal to a that is x belongs to minus infinity minus a minus a included union a a is included interval a infinity now third react let r be positive real number and a be a fixed real number then modulus of x minus a less than r implies a minus r less than x less than a plus r x belongs to open interval a minus r a plus r similarly $\text{mod } x$ minus a less than equal to r implies a minus r less than equal to x less than equal to a plus r that is x belongs to close interval a minus r a plus r third is modulus of x minus a greater than r implies x less than minus r or x greater than a plus r and modulus of x minus a greater than r greater than equal to r implies x less than equal to a minus r or x greater than equal to a plus r and fourth important reality let a and b both are positive real number then a less than modulus of x less than b implies x belongs to minus b minus a union a b open interval a b a less than equal to x modulus of x less than equal to b implies x belongs to close interval minus b minus a union close interval a b and a less than modulus of x minus c less than b implies x belongs to open interval minus b plus c minus a plus c union of a interval a plus c b plus c and a less than equal to x modulus of x minus c less than equal to b implies x belongs to minus b plus c minus a plus c close interval union close interval a plus c b plus c

so these are some important reals based on absolute value function now on the basis of this real we discuss some problem that is solve three x minus two less than equal to one by two

so we know that $\text{mod } x$ less than equal to a implies minus a less than equal to x less than equal to a

so $\text{mod } 3x$ minus two less than equal to one by two

so by using this result this implies minus one by two less than equal to three x minus two less than equal to one by two this implies minus one by two plus two less than equal to three x less than equal to one by two plus two

so this implies three by two less than equal to three x less than equal to five by two this implies three by two into one by three less than equal to three x by three

so this implies 1 by 2 less than equal to x less than equal to 5 by 6

so x belongs to closed interval one by two five by six

so solution of this in equation $\text{mod } 3x$ minus two less than equal to one by two is x belongs to one by two five by six obviously replacement set for all the in equation is set of real number now another example solve $\text{mod } x$ minus two

greater than equal to five x belongs to r solution mod x minus two greater than equal to five we know that mod x greater than equal to a implies minus a less than equal to x less than equal to minus a or x greater than equal to a
 so mod x minus two greater than equal to five implies x minus 2 less than equal to minus 5 or x minus 2 greater than equal to five
 so we have two linear inequality this implies x minus 2 less than equal to minus 5 or x minus 2 greater than equal to 5 this implies x minus 2 plus 5 say plus 2 plus 2 less than equal to minus 5 plus 2 or x minus 2 plus 2 greater than equal to 5 plus 2 this implies x less than equal to minus 3 or x greater than equal to 7.

so we can write it as x belongs to minus infinity minus 3 or x belongs to 7 infinity that is x belongs to minus infinity minus 3 it is included union 7 included infinity

so this is the solution set for the in equation mod x minus two greater than equal to five another example solve the system of in equation mod x minus one less than equal to five and mod x greater than equal to two

so here we have two in equation that is system of in equation is given so we solve mod x minus one five and mod x greater than equal to two this is given say this is in equation one and this is in equation two

so from one mod x minus one less than equal to five so this implies we use the inequality again mod x less than equal to a implies minus a less than equal to x less than equal to a
 so this is minus five less than equal to x minus and less than equal to 5
 so this implies minus 5 plus 1 less than equal to x less than equal to 5 plus 1.

this implies minus 4 less than equal to x less than equal to 6.

again from two mod x is greater than equal to two so this implies again use mod x greater than equal to a implies x less than equal to minus a or x greater than equal to a

so x greater than equal to implies x less than equal to minus 2 or x greater than equal to 2 this implies x belongs to minus infinity here we can also write x belongs to close interval minus four six minus infinity minus two union to infinity this is closed included this is also included to infinity

so from one when mod x minus one less than equal to five and mod x greater than equal to two

so mod x minus one less than equal to five implies x belongs to minus four six and mod x from 2 mod x greater than equal to 2 implies x belongs to minus infinity minus 2 semi close interval union to infinity

so when you plot these two solution on number line say minus infinity infinity and minus infinity say this is \emptyset

so the solution of first set say this is minus 2 and this is 2 this is 4 this is 6 and this is minus 4

so first equation give the solution from minus four to six means from minus four to six this is minus 4 this is 6 and second equation gives solution minus 2 to minus infinity means we can proceed from here to here and two to infinity

so this is from two to six
 so finally when combine this

so two solution will get solution like minus four to minus two means minus four to six means minus four to minus two

so this is one solution and this is two to six this is another solution

so combining solution one and two we get solution set equal to minus 4 minus 2 close interval union closed interval two to six

so in this way we can find solution of system of two linear equation in one

variable involving absolute value function now another example that is one less than equal to mod x minus two less than equal to three we know that a less than equal to mod x minus c less than equal to b implies x belongs to close interval minus b plus c minus a plus c union closed interval a plus c b plus c

so by using this inequality

so given in equation one less than equal to mod x minus two less than equal to three implies x belongs to

so here this is a this is b and this is c

so using this result x belongs to close interval minus b means minus three and c is equal to 2 minus 3 plus 2 and a is equal to 1

so minus 1 plus 2 union a plus c means one plus two and b plus c means three plus two this implies x belongs to close interval minus 1 1 union closed interval three five

so if you represent this one number line infinity minus infinity is \emptyset say this is 1 and this is minus 1

so this is 2 3 4 5

so minus 1 to 1 close interval it means from here to here means minus one to one this part and three two five

so solution of the given equation

so solution set for the given in equation one less than equal to mod x minus two less than equal to 3 is x belongs to minus 1 1 union three five another example that is mod x plus five mod x minus one by mod x minus two is greater than equal to zero

so given mod x minus one by mod x minus two greater than equal to zero let mod x equal to z

so this implies z minus 1 by z minus 2 is greater than equal to 0.

this implies z less than equal to one or z greater than two in fact z greater than equal to two but x is not equal to two z minus one z minus two is greater than equal to zero whenever we have to solve this type of inequality we use this concept say x minus a by x minus b greater than equal to 0 and obviously a less than b this implies x less than equal to a or x greater than equal to b

so this is one and second one is if x minus a by x minus b less than equal to zero obviously a less than b this implies a less than equal to x less than equal to b

so we can use these two types of inequality also when in equation is given in this way when equation in equation is given like this then we can use this inequality also

so what we get we get z less than equal to one or z greater than two

so this implies mod x less than equal to 1 or mod x greater than two

so this implies minus one less than equal to x less than equal to 1 or x less than less than minus 2 or x greater than two

so this implies x belongs to minus 1 1 or x belongs to minus infinity minus 2 union to infinity

so again we use the concept of number line this is infinity this is minus infinity this is \emptyset this is 1 and this is minus 1 this is 2 this is minus two

so x in first case x belongs to minus one to one x belongs to minus one two one and x belongs to minus two minus infinity minus two means this two is not included

so and x belongs to 2 infinity again this 2 is not included

so another such a situation like this

so finally when you compare these two are combine these two solution on number line will get solution like this it means the solution of this inequality will be set of whole number excluding these two portion

so on the basis of solution represented on number line the solution set solution set is equal to minus one union open interval minus infinity minus 2 union open interval to infinity

so required solution for this in equation is minus 1 close interval union minus infinity minus 2 union to infinity now another example solve minus one by mod x minus two greater than equal to one x belongs to a r n x is not equal to plus minus two given minus one by mod x minus two greater than equal to one let mod x equal to z

so this implies minus 1 by z minus 2 is greater than equal to 1.

so this implies minus 1 is greater than equal to z minus 2

so this implies minus 1 minus 1 less than equal to minus 1 z minus 2

so this is one less than equal to two minus z

so 2 minus z is greater than equal to 1

so z minus 2 is less than equal to minus 1 this implies z minus 2 plus 2 less than minus 1 plus 2 this implies z less than equal to 1

so mod x less than equal to one this implies minus one minus one less than equal to x less than equal to one

so solution set for the given inequality x belongs to minus one one now solve line equation mod two by mod x minus four greater than equal to one x not equal to four given in equation two by mod x minus four whole mod greater than one

so this implies we can write it as two by mod x minus four greater than one because mod two equal to two

so this can be written as two is greater than mod x minus four

so this implies mod x minus 4 is less than 2 now we have property mod x minus a less than r implies a minus r less than x less than a plus r

so by using this property we can write it as this is a and this is r

so 4 minus 4 minus 2 less than x less than 4 plus 2 this implies 2 less than x less than 6.

but in problem given that x is not equal to four

so in this interval implies we can write it as x belongs to 2 6 but x is not equal to four

so this implies x belongs to two four union four six

so this is the

so required solutions of solution set we can write solution set equal to two four union four six solve mod x minus one plus mod x minus two greater than equal to four solution

so given in equation mod x minus one plus mod x minus two is greater than equal to four put x minus one equal to zero and x minus two equal to zero

so this will give x equal to one and two this is a this value one and two because this is critical point

so suppose if you take number line infinity and minus infinity say this is 0 and this is 1 and this is 2.

it means at this point 1 and 2 this function will change its behavior

so we have this point one and two divide this number line into three parts

so we will discuss in three intervals that is minus infinity one then one two and two infinity

so case one when x belongs to minus infinity one

so when x belongs to minus infinity one then mod x minus one equal to minus x minus 1 and mod x minus 2 is equal to also minus x minus 2 as we know mod x equal to x if x greater than 0 and minus x if x less than zero

so mod x minus one plus mod x minus 2 greater than equal to 4 implies minus x minus 1 minus x minus two is greater than equal to four this implies minus two x is greater plus three is greater than equal to four this implies minus two x is greater

than equal to four minus three this implies minus two x is greater than equal to one

so this implies minus or two x less than equal to minus one

so x less than equal to minus one by two but x belongs to minus infinity one minus infinity one

so from these two information means x belongs to implies x belongs to minus infinity minus one by two this is one by two included and x belongs to minus infinity one

so if you discuss this one number line

so this is zero this is one and this is minus one by two

so when you compare these two real will get x belongs to minus infinity minus one by two

so this portion will be the solution for when x belongs to minus infinity one then its solution will be minus infinity minus one by two included

so this is one now case two

so when x belongs to one two then mod x minus one is positive x minus one x minus one

so mod x minus two this is minus x minus two

so mod x minus one plus mod x minus 2 is greater than equal to 4 this implies x minus 1 minus x minus two is greater than equal to four this implies one is greater than equal to four which is absolute result which is absurd result

so no solution exist for the given in equation in one two now case three when x belongs to two infinity

so mod x minus 1 is equal to x minus 1 and mod x minus 2 is equal to both positive

so mod x minus 1 plus mod x minus 2 greater than equal to 4 implies x minus one plus x minus two greater than equal to four

so this implies two x minus three greater than equal to four

so this implies two x greater than equal to seven this implies x greater than equal to seven by two this implies x belongs to seven by two infinity now we have three case

so in case one x belongs to minus infinity minus one by two included case three case two no solution and case three x belongs to seven by two infinity

so combining these three results combining case one two and three we get x belongs to minus infinity minus 1 by 2 union seven by by two infinity

so solution set minus infinity minus one by two and if you represent this one number line say 0 and this is say 7 by 2 and say this is minus 1 by 2

so solution will be like this

so solution of this in equation can be given a number line like this ok thank you we will discuss some more concept in next session you