

okay friends today i am going to discuss about linear inequality that is in class 11

so before we start linear inequality we must have some idea about what is equation

so an equation is defined as statement involving variables and sign of equality

so for example suppose we say an equation is defined as statement involving variables and sign of equality for example x equal to two three x minus one equal to four $a x$ plus $b y$ equal to c $a x$ square plus $b x$ plus c equal to zero two statements are connected by sign of equality that is why this is called equation in equation a statement involving variables and sign of inequality like greater than less than greater than equal to or less than equal to is called an inequation or an inequality the statements like 7 less than 11 5 and half greater than minus 3 are examples of numerical inequalities the statements like x less than 3 x plus 5 less than equal to 7 $2 i$ minus 3 greater than 8 $3 y$ greater than equal to 11 y minus 3 by two less than two i plus one are examples of literal inequality in general we can say a linear inequality in one variable can always be written as ax plus b less than zero ax plus b greater less than equal to zero ax plus b greater than zero $a x$ plus b greater than equal to zero where a and b are real numbers a not equal to d zero this is most important condition suppose if a is not a is equal to 0 it means $a x$ plus b less than 0 means b less than 0 it means this this doesn't satisfy that is a linear equation in one variable for example in equality like this three x minus one less than zero five by two x plus three by four less than equal to zero two x plus three greater than zero seven x minus one by two greater than equal to zero are some examples of linear inequality in one variable on the basis of sign of inequality we can say inequality are of two types strict inequality a strict inequity means the statement two statements are either connected by less than or greater than sign only like x less than three two x minus three greater than eight y minus three by two less than two i plus one are called strict inequality due to this less than and greater than sign this inequity is called strict in inequity but when two statements are connected by this type of sign like less than equal to greater than equal to are called slack inequality for example x plus five less than equal to seven three y plus five greater than equal to eleven are examples of slag inequality now how can you solve linear inequality in one variables or any inequality consider the in equation we have this in equation three minus two x by five less than x by three minus four

so here two parts that is lhs three minus two x by five and rhs x by three minus four now we just try to check for any particular value whether this lhs and rhs satisfy this sign of inequity or not for example suppose we take x equal to nine in lhs if you put $1 x$ equal to 9 then we get 3 minus 2 into 9 by 5 equal to minus 15 by 5 equal to minus 3 and $r h s$ 9 by 3 minus 4 equal to 3 minus 4 equal to minus one

so this shows that minus three less than minus one it means lhs less than rhs so this for x equal to nine this in equation three minus two x by five less than x by three minus four satisfy

so this x equal to nine will be the solution for this in equation again let us consider another value that is say x equal to 6 again we check lhs rhs in lhs if you put x equal to 6 we get 3 minus 2 into 6 by 5 equal to minus 9 by 5 rhs 6 by 3 minus 4 equal to minus 2

so minus 9 by 5 less than minus 2 is not correct is not true that is lhs is not less than rhs

so this in equation 3 minus $2 x$ by 5 less than x by 3 minus 4 not satisfied for x equal to six

so x equal to six is not a solution

so in general we can say solving an equation is a process to find all possible solution of an inequation we have to solve any equation we have two types of solution set that is first is solution set the solution set means the set of all possible solution of an inequation is known as its solution set and replacement set the set from which values of variable involved in the inequation are chosen is called the replacement set for example suppose we take an example like this say we take $3x$ minus 1 less than two and the replacement set that is x belongs to n and second say replacement set is x belongs to say z means set of integer and third x belongs to r

so first of all we just try to find the value of this three x minus one less than two

so three x minus one less than two when we solve it we will get three x less than three this implies x less than one now for one since x or replacement set is set of natural number

so for this situation the solution set equal to ϕ because we know that no natural number is less than one if you consider replacement set for second situation the replacement set is z means x belongs to z means we have to consider this all the possible solution from set of integer

so x belongs to z x less than one it means the solution set solution set equal to $\dots -2 -1 0$ and since less than 1

so up to 0 only

so all the integers or set of integer which is less than 1 is called solution set for the same in equation because we change the replacement set again third situation is x belongs to real number means replacement set is set of real number and solution is x less than one

so when you consider these two condition

so set solution set solution set equal to set of all x such that x belongs to r and x less than one or we can write it as $-\infty$ one

so this is a solution set for the same in equation only change is the replacement set

so for different replacement set same in equation have different solution set for example consider x less than four here replacement set is this and solution set is this

so for x less than 4 if displace replacement set is 1 2 3 4 5 6 7 8 9 10 then solution is because x less than four

so we have to consider only those element which satisfy this condition

so solution set is one two three only and with chain replacement set the second situation is $-\infty$ zero one two five eight and 4 this replacement set the solution set will be $-\infty$ 1 2 again set of real number x less than 4

so its solution set is $-\infty$ four and four set of integer its solution set is

so all the integer which satisfy x less than four means up to three $-\infty$ to plus three and for set of natural number we have solved solution set is one two three and again for this solid replacement set five six seven eight nine ten

so since x less than four

so no element is less than four

so solution set is five ok

so here we have to note that if replacement set is not given then we have to consider real number only how to solve linear in equation in one variable

so to solve linear in equation one variable we have to consider or we have to bear in mind the things that when you multiply or divide in equation by negative number then its the inequality sign of inequality will be reversed otherwise it

will work just like equation

so only changes when you multiply or divide by negative number the order of inequality will reverse for example if $x < 2$ then $-x > -2$ when you multiply it by minus one then its sign of inequality will change less than two greater than another example if you consider $3x - 1 > 5$ then and when you multiply it by minus 4 both side then its again order of inequality will change like $-12x + 4 < -20$ into $12x > 24$ and another example $-6x < 12$ then $x > -2$ when you divide it by minus six

so in this way only changes when you multiply or divide by inequality by negative sign its order of inequality will change procedure to solve a linear inequality in one variable two three points are important that we have to consider first of all whatever inequity is given simplify it and collect like terms in same side then remove fraction if any fraction involved then remove fraction or a decimal by multiplying both side by an appropriate number that is factor or lcm of denominator or a power obtained in case of decimal

so this is important now isolate all variables terms one on one side and all constant constant on other side means we have two side

so collect all the like terms all the variables in left hand side and all the constant on right hand side then make the coefficient of variable one now the variable we divide left hand side in such way the coefficient of variable will be one only and then choose a solution from replacement set

so in this way we can solve linear equation in one variable now we just take example given x belongs to $\{-3, -4, -5, -6, -9\}$ less than minus two x find the possible values of x also represent its solution set on the number line

so let us start

so given in equation $9 < 1 - 2x$ this implies $-1 + 9 < -2x$ we know that when you add or subtract same number both side the sign of inequality will not change in only in case of multiplication and division

so we will get $8 < -2x$ this implies $-8 > 2x$ we multiply both side by minus sign

so the sign of inequity will change

so we will get $-8 > 2x$ now change the side of variable that is $2x < -8$ this implies now divide both sides by two since we divide both side by plus two

so its sign will not change

so this implies $x < -4$

so we will get $x < -4$ since replacement set set is given as x belongs to $\{-3, -4, -5, -6, -9\}$ and $x < -4$

so solution set mention replacement set only two numbers or two elements satisfy this condition that is less than minus four is minus five and minus six so the solution set for the given in equation $9 < 1 - 2x$ for the given replacement set $\{-3, -4, -5, -6, -9\}$ is $\{-5, -6\}$ now another part is you have to also represent its solution set on number line let us take a number line say this is 0 and this is 1 and this is 2 this is minus 1 this is minus 2 minus 3 minus 4 minus 5 minus 6

so solution set is minus five minus six

so these two points represent the solution set of it means the solution set

so for $9 < 1 - 2x$ the solution set will be minus five and minus six for this replacement set now another example solve inequality $3 -$

$2x$ greater than equal to x minus 32 given that first x belongs to n x belongs to w and x belongs to z means these are three placement sets
so given in equation

so given equation three minus two x greater than equal to x minus thirty two
this implies minus 3 plus 3 minus 2 x greater than equal to minus 3 plus x minus 32

so this implies minus 2 x greater than equal to x minus thirty five
so this implies minus three x greater than equal to minus 35

so this implies minus 1 minus 3 x less than equal to minus one minus thirty five multiplying both sides multiplying both side by minus one

so sign of inequality will change this is 3 x less than equal to 35

so this implies x less than equal to thirty five by three

so this is the condition on x that x is always less than equal to thirty five by three we have three replacements that is given first is x belongs to natural number second is x belongs to whole number and third is x belongs to integer

so now discuss case one when x belongs to set of natural number since x less than equal to thirty five by three

so we can say it as like eleven two by three it means the solution set is equal to we have to consider all those number which are less than equal to 11 for natural number

so 1 2 3 dot dot dot up to 11.

so this is a solution set when x belongs to set of natural numbers now when case two when x belongs to set of whole number since x less than equal to 35 by 3 means 11 2 by 3

so solution set is starting from 0 because in set of whole number all the natural number including 0

so 0 1 2 dot dot dot up to 11

so only change is in first case and second guess that zero included now case three when x belongs to z since x less than equal to 35 by 3 equal to eleven two by three

so solution set equal to dot dot minus 1 0 1 2 dot dot up to 11

so this is the solution set when x belongs to set of integer we discuss another example that is x belongs to w means set of whole number and find the solution set of three by five x minus two x minus one by three greater than one given in equation three by five x minus two x minus one by three greater than one

so now take lcm of this five and three lcm of five and three equal to fifteen

so we can write it as fifteen three into three x minus five into two x minus one greater than one or we can multiply both side by fifteen

so this is nine x minus ten x plus five by fifteen greater than one

so this implies minus x plus five greater than fifteen

so this implies minus x plus five minus 5 greater than 15 minus 5 this implies minus x greater than 10 this implies minus 1 minus x less than minus 1 10 means multiplying both sides multiplying both side by minus 1 which will change the sign of inequality

so you will get x less than minus ten now replacement set is given replacement set is given as if x belongs to w it means we have to find solution within set of whole number since x belongs to w and x less than minus ten

so solution set is equal to

so solution set is equal to phi because in whole number no negative number exist now another example solve the inequality for real x given in equation

three two minus x three into two minus x greater than equal to two into one minus x this implies simplify it six minus three x greater than two minus two x

so minus six plus six minus three x greater than equal to minus six plus two

minus two x

so minus three x greater than equal to minus 4 minus 2 x this implies minus x greater than equal to minus 4 this implies minus 1 into minus x less than equal to minus 1 into minus 4 again multiplying both side multiplying both side by minus one which will change the sign of inequality

so this is x less than equal to 4 since x belongs to r replacement is set is given as set of real number and x less than equal to four this implies solution set equal to set of all x such that x belongs to r and x less than equal to four we can write it as minus infinity four

so this is the required solution set for the given in equation again another example solve two x plus one by three greater than equal to three x minus two by five x belongs to r graph the solution set on number line given in equation two x plus one by three greater than equal to three x minus two by five lcm of three and five equal to fifteen

so multiply both sides multiply both side by fifteen

so when you multiply both side by fifteen this implies fifteen within bracket two x plus one by three greater than equal to fifteen within bracket three x minus two by five

so five into two x plus one greater than equal to three into three x minus two simplify it ten x plus five greater than equal to nine x minus six nine x minus six

so we will get x greater than equal to minus eleven since x belongs to r and x greater than equal to minus eleven

so solution set equal to set of all x such that x belongs to r x greater than equal to minus 11 or we can write it as minus 11 infinity now another part of this problem is graph of solution on on number line means on real line because x belongs to r

so say this is real line minus infinity infinity and this is \emptyset

so solution set is solution set is minus 11 x greater than equal to minus eleven

so say this is minus eleven

so since greater than equal to

so just dark this circle means this minus 11 is also included and from this point to continuous towards positive direction of infinity or x axis

so in this way we can represent the graph of any solution on number line now let us take one example of this problem for application purpose

so the longest side of triangle longest side of triangle is three times shorter and the third side is two centimeter shorter than longest side if perimeter of the triangle is at least 61 centimeter find the minimum length of shortest side

so longest side of triangle say this is a triangle solution this is a triangle say a b c now let the shortest side is x this is shortest side is x the longest side of triangle is three times the shortest side say this ac is shortest side this is three x and the third side

so this say this side bc is third side now this is longest side i say this is longest side longer side and they say shortest side and this is third side

so name it a b is shortest side a c is longest side and b c third side

so according to question we assume or let us assume that the shortest side is say x and the longest side of the triangle say this is this s is longest side is three times the shortest side

so this s is three s three x and the third side is two centimeters shorter than the longest side means this is three x minus two

so in this way we define all the three side in terms of x now what is given if the perimeter of the triangle is at least 61 centimeter find the minimum length of shortest side ok

so according to question the perimeter of triangle is greater than equal to sixty one centimeter at least

so greater than one greater than equal to sixty one centimeter and perimeter means sum of three side

so x plus three x plus three x minus two is greater than equal to sixty one this implies seven x minus two is greater than equal to sixty one

so this implies seven x is greater than equal to sixty three

so this implies x greater than equal to sixty three by seven

so this implies x greater than equal to nine

so minimum length of shortest side

so minimum length of shortest side of triangle equal to 9 centimeter now let us take another example that is a man wants to cut three length from a single piece of board of length 91 centimeter the second length is to be three centimeter longer than the shortest and the third length is to be twice as long as the shortest what are the possible length of the shortest board if the third piece is to be at least five centimeter longer than the second

so let the length half length of saw test shortest piece equal to x centimeter

so the length of second piece equal to x plus three now the third length is to be twice as long as the shortest and the third length length of third piece is two x we have three piece shortest piece is x centimeter second piece is x plus three and third piece is two x now the length of board is ninety one centimeter

so we cut all these three piece from a length of board whose length is

so according to question x plus x plus 3 plus 2 x less than equal to ninety one this implies four x plus 3 less than equal to 91 this implies $4x$ plus 3 minus 3 less than equal to 91 minus 3 this implies $4x$ less than equal to 88 this implies $4x$ by 4 less than equal to 88 by 4 this implies x less than equal to 22 say this is 1 also given if the third piece is to be at least five centimeter longer than the second third piece third piece is two x third piece is two x which is which is longer than five centimeter from second piece

so again two x is greater than equal to x plus three plus five this implies two x greater than equal to x plus eight this implies two x minus x greater than equal to x plus eight minus x this implies x greater than equal to eight this is second

so in first case we get x less than equal to twenty two and in second case we get x greater than equal to eight

so from one and two 1 and 2 8 less than equal to x less than equal to 22

so in this way we can get the value of shortest piece means x belongs to x is larger than eight centimeter and less than twenty two centimeter now we have problem x example eight

so find all pair of consecutive even positive integer both of which are larger than five such that their sum is less than twenty three solution find all pair of consecutive even positive integer both of which are larger than five

so let the two consecutive two consecutive positive integers are x x plus two since both are greater than five

so x greater than five say this is first also according to question sum of these two integer x x plus two is less than twenty three in problem it is given that such that their sum is less than twenty three

so the sum is less than twenty three

so this implies two x plus two less than twenty three this implies two x plus two minus two less than twenty three minus two this implies two x less than twenty one this implies x less than twenty one by two

so we have x greater than five and x less than twenty one by two

so this implies five less than x less than twenty one 2 or we can say 5 less than x less than 10.

5 the even numbers

so we can take even number in between five and ten is six eight ten

so possible pairs are

so possible pairs are six eight eight ten and ten twelve

so ok we will discuss another concept in next session thank you you

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