welcome students to the iit problem solving session on mathematics our topic is probability and this is lecture number five if you remember we are working on conditional probability and in particular we are working on the following problem that there is a bag containing twelve red balls and eight green balls three balls are drawn in succession without replacement we know that probability first ball is red this event we are calling as r one and that is going to be twelve upon twenty is equal to three upon five and probability the first ball drawn is green that probability is going to be eight upon twenty is equal to two upon five we have calculated some conditional probabilities same probability the second ball drawn is green given that first ball drawn is red and we know that this probability is going to be eight upon nineteen this we have already seen my question is what is the unconditional probability that the second ball drawn is green that is the question that is we are looking at probability of g two now the event g two is actually union of two events first ball is red and second ball is green union first ball is green and the second ball is green therefore probability of g two is equal to probability r r1 g2 union g1 g2

so let us compute this our initial configuration is 12 red balls and 8 green balls event r1 first ball drawn is red that brings us to eleven comma eight and if the first ball drone is green then we come at 12 comma 7 now if i have g 2 here that this second ball drawn is green then the configuration is going to be 11 comma 7 and from here if the second ball drawn is green then our configuration is going to be 12 comma 6 therefore probability of g 2 is equal to probability of r 1 intersected with g two plus probability g one intersected with g two we know that this we can write as probability g two given r one multiplied by probability of r one plus probability of g two given g one this multiplied by probability of g one is equal to probability of g two given r one that is this probability which is going to be eight upon nineteen multiplied by probability of r 1 which is going to be 12 upon 20 which is nothing but 3 upon 5 plus probability g two given g one that is we are here therefore that probability is going to be seven upon nineteen multiplied by probability of g one which is going to be eight upon twenty which is two upon five is equal to 24 plus 14 upon 19 into 5 is equal to 38 upon 19 into 5 is equal to 2 upon 5

so that is a bit new result we knew that the unconditional probability of the first ball drawn is green is same as two by five because that was eight upon twenty is equal to 2 by 5 and now we get the unconditional probability of drawing the second ball to be green is also same as 2 by 5.

i want you to verify that the unconditional probability of r two that is the second ball drawn is red and perhaps you can guess that answer is going to be 3 by 5 i want you to verify that now let us go one more step what is the unconditional probability that the third ball drawn is red

so let us start the third ball can be read is union of four disjoint events what are they first one is red second one is red and third one is red union first one is red second one is green and then third one is red union first one is green second one is red and third one is red union first one is green second one is green and third one is red this is because this event can happen if the previous two draws are either red red red green green red and green green therefore probability of third ball drawn is red is equal to summation of this individual probabilities

so let me write them as follows probability of r one r 2 r 3 plus probability of r 1 g 2 r 3 plus probability of g 1 r 2 r 3 plus probability of g one g two r three

so let us again go by the tree diagram as i said earlier that they are very useful when you are modeling the probability of a sequence of events

so if it is r one we go to eleven comma eight if the second one is r two then we go to ten comma eight and now we are looking for r three and the configuration is going to be 9 comma 8 if the first one is g1 then we come to 12 comma 7 if the second one is red then we come to eleven comma seven and if the third one is red then we come to ten comma seven now after the first one is red if the second one is green then we come to 11 comma seven and if the third one is red again then we come to the configuration ten comma seven similarly if the first one is green and second one is also green then we come to twelve comma six and if then third one is red then we come to eleven comma six and we need to compute the probabilities of these sequence of events and then we have to add them up

so let us calculate therefore probability of r 3 the third ball is red

so this is the product of 3 by 5 into r2 therefore 11 upon 19 multiplied by r 3 that is 10 upon 18 plus 3 by 5 into now we are drawing a green therefore that is eight upon nineteen into now we are drawing a red that is eleven upon eighteen plus 2 by 5 into this is r2 therefore 12 upon 19 into r 3 therefore 11 upon 18 plus 2 by 5 this is g 2 g 2 therefore this is going to be drawing a green therefore that is 7 upon 19 multiplied by drawing a red out of 12 and 6

so that is 12 upon 18 is equal to 1 upon 5 into 19 into 18 multiplied by 3 into 11 33 into 10 330 plus 3 into 8 24 into 11 that is 264 plus 12 into 224 into 11 that is 264 plus 7 into 12 is 84 into 2 168.

is equal to six two one zero two six upon 5 into 19 into 18 is equal to let us cancel now 18 cancels with 1 zero two six

so what we have five into eighteen ninety

so one twenty six seven into eighteen is equal to one twenty six is equal to three upon five therefore we see that the unconditional probability that the third ball drawn is red is same as the initial probability that the first ball drawn is red

so this is a very interesting observation and this is true not only for third draw if we proceed then we can see that same unconditional probability will remain for all successive drawings as well okay friends let us now start a new problem suppose you have three fair dice such that probability of one is equal to probability of two is equal to probability of three is equal to probability of six is equal to one upon six for all the three dice also you have a fake dice which has four faces as five and two faces as six you choose one of the four dice at random and throw it you get a five question is what is the probability of getting a five and second question is given that you have got a five what is the probability that you have chosen the fake die

so i hope you understood that two questions

so let us solve them first is a

so you are getting a 5 you are starting from here you can choose a fair die that probability is three by four and then you are throwing a die and getting five that probability is one by six you have chosen the fake die that probability is 1 by 4 and then you are getting a 5 that probability is because there are four faces five and two faces six

so that probability is two by three therefore probability of five is equal to probability of five given fair die into probability of a fair die or choosing a fair die plus probability of 5 given a fake die and probability of choosing the fake die is equal to one by six multiplied by three by four because there are three fair die and one fake die plus two by three multiplied by one by four is equal to three upon twenty four plus 4 upon 24 is equal to 7 upon 24

so that is the answer the second question is that what is the probability that you have chosen the fake die given that you result is five this question if we analyze we see that it is somewhat different in the earlier question we are starting from this point and as we are moving with the trial we are trying to

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figure out what is the probability of a particular event in our case the event is getting a 5 on the throw but this question is saying something different it is saying that this output is 5 and based on that you are trying to figure out what is the probability of this event that you have chosen a fake die

so this probability is called a posteriori probability that is after the result we are looking at the probability of the initial event as you know that this can be solved using bayes theorem

so what it is we know that probability of a given b is equal to probability of a intersected with b divided by probability of b now this we can write as probability of b given a multiplied by probability of a divided by probability of b

so our problem is probability of fake die given that we got five therefore we can write it as probability of 5 and fake die divided by probability of 5 is equal to probability of 5 given fake die multiplied by probability of fake die divided by probability of five is equal to probability of five given affected is two by three probability of getting affected is one by four and overall probability of five is seven upon twenty four is equal to 2 by 12 multiplied by 24 by 7 is equal to 4 by 7 that is the answer now we have to remember that we got probability of 5 by considering all the disjoint events that result in five and then we have added over that the same way we have done in the previous problem let us now consider another problem suppose you have 10 coins numbered 1 2 3 up to 10.

suppose probability of getting a head in single toss of the ith coin is i upon 10 i is equal to one two up to ten you choose a coin randomly and toss it if the result you got is a head what is the probability that you have chosen the fifth coin

so that is the question

so we can solve it in this way let e be the event that you choose the fifth coin and b be the event that you got a head we want to compute probability of e given b therefore using bayes theorem probability of e given b is equal to probability of b intersected with e divided by probability of b is equal to probability of b given e multiplied by probability of e divided by probability of b now probability of b given e that is probability of a head given that you have chosen the fifth coin is equal to five by ten is equal to half and probability of e is equal to one by ten as the coin is chosen randomly therefore probability of e given b is equal to half multiplied by 1 by 10 divided by probability of b that is probability of getting a head now probability of b is equal to probability first coin is chosen and you got a head plus probability second coin is chosen and you got a head up to probability tenth coin is chosen and you got a head is equal to probability of head given first coin into probability of first coin plus probability of head given tenth coin into probability of selection of tenth coin is equal to one by ten into one by ten plus two by ten into one by ten plus up to ten by ten into one by ten this is because the probability of head for the i th coin is i by 10 is equal to 1 by 100 into 1 plus 2 plus up to 10 is equal to 1 by 100 into 10 into 11 by 2 is equal to 11 upon 20 therefore probability of e given b is equal to from here we get 1 upon 20 divided by 11 upon 20 is equal to 1 upon 11

so that is the answer that we get by applying bayes theorem let us solve another problem suppose a student is answering and mcq question with five options of which only one is correct now let p be the probability that the student guesses the answer that is he ticks randomly and let one minus p be be the probability that he knows the answer and therefore takes correctly suppose the student ticked correctly what is the probability that he guessed the answer that is the question solution we want to find out probability of guessing given ticked rightly let e be the event that he guessed it and b be the event that he ticked it correctly therefore we want to compute probability of e given b which is same as probability of b given e into probability of e multiplied by probability of b now probability of b given e because he is randomly ticking and there are five options that the probability he sticked it correctly is 1 by 5 and probability of e that he is guessing the answer is p that is given and probability of b is equal to probability of b given he is guessing multiplied by probability of guessing plus probability of b given he knows the answer multiplied by probability he knows the answer is equal to 1 by 5 multiplied by p plus 1 multiplied by 1 minus p is equal to p plus 5 into 1 minus p upon 5 is equal to 5 minus 4 p upon 5 therefore probability of e given b is equal to p by 5 p into 1 by 5 divided by 5 minus 4 p upon 5 is equal to p upon 5 minus 4 p that is the answer let us solve another problem suppose you have three bags a b and c the contents of the bags bag a has one white ball two green balls and three red balls bag b has two white balls one greed ball and one red ball back c has four white ball plus five green balls plus three red balls you choose a bag at random and take out two balls from it suppose you get one white and one red ball what is the probability that you have chosen bag egg again you understand that we are again computing the opera or posterior probability of the bag a given that you have chosen one white and one red

so solution let e be the event of selecting bag a and b be the event of getting one white and one red ball we want to compute probability of e given b therefore using bayes theorem probability of e given b is equal to probability of b given e multiplied by probability of e divided by probability of b now probability of e is equal to one by three and probability of b given e that is selecting one red and one white ball out of the bag a is equal to 3 upon 6 c 2 is equal to 3 upon factorial 6 factorial 2 factorial 4 is equal to 3 upon 5 into 6 by 2 is equal to one upon five similarly probability of one white and one a red ball given bag b is chosen is equal to now bag b has four balls out of which we can choose one white and one red in two different ways and two balls can be chosen in four c 2 therefore the answer is 2 upon factorial 4 factorial 2 factorial 2 is equal to 2 upon six is equal to one upon three and probability of one white plus one red given bag c is equal to 12 upon 12 c 2 is equal to 12 upon 11 into 12 by 2 is equal to 2 upon 11 therefore probability of b is equal to probability of one white plus one red is equal to one by five into one by three that is for the bag a plus probability 1 by 3 into 1 by 3 that is for bag b plus 1 by 3 into 2 by 11 is equal to 1 by 3 into 1 by 5 plus 1 by three plus two by eleven is equal to one by three into one sixty five thirty three plus 55 plus 30 is equal to 1 by 3 into 118 upon 165 therefore probability of bag a given 1 red and 1 white is equal to 1 by 5 into 1 by 3 upon 1 by 3 into 1 1 8 upon 1 6 5 is equal to 1 by 5 into 165 divided by 1 1 8 is equal to 33 upon 1 1 8 that is the answer in a similar way you can calculate for other three bags and you will find the answer accordingly in fact you can see from here and you can understand that the posterior probability for bag b is going to be 55 upon 1 1 8 and for back c is going to be 30 upon 1 1 8.

let me solve one more problem unconditional probability and base theorem before we stop suppose you are waiting for a later from kolkata kathmandu and musket suppose you receive a letter and the only thing legible in the address is two successive letters a t now yeti gas is there in kolkata it is there in kathmandu it is there in muscat as well what is the probability that it is from kolkata that is the question therefore what we want to know is probability of kolkata given a t successively is equal to probability of two successive letters are 80 given kolkata into probability of kolkata divided by probability of a t as the two successive letters now probability of 80 given kolkata is equal to there are

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1 2 3 4 5 and 6 there are 6 possible pairs of successive letters therefore this probability is 1 by 6 probability of 80 given kathmandu there are eight possible pair of letters therefore that is going to be 1 by 8 and probability of a t given must cut is equal to 1 by 5 therefore probability of kolkata given a t is equal to probability of 80 given kolkata which is 1 by 6 multiplied by probability of kolkata which is 1 by 3 divided by as you can well understand 1 by 6 into 1 by 3 plus 1 by 8 into 1 by 3 plus 1 by 5 into 1 by 3 is equal to 1 by 6 upon 1 by 6 plus 1 by 8 plus 1 by 5 is equal to 1 by 6 upon 5 into 6 into 8 40 plus 30 plus 48 is equal to 40 upon 40 plus 30 plus 48 is equal to 40 upon 1 1 8 which can be further simplified is equal to 20 upon 59 that is the answer

so let me finish the session with the t key problem suppose india and australia are playing a five match test series suppose virat kohli won the toss in the first four matches what is the probability that we will win the toss in the fifth match as well

so that is the question and let us put it in an mcq mode option a is one option b is one by five option c is zero and option d is half let us assume that the toss is done using an unbiased coin

so that in each toss the probability of weaning is equal for both the captains because since virat won all the previous tosses therefore he will win the fifth toss but that is not correct this will be a wrong answer can it be one by five why because we got one all the four tosses therefore now the australia captain will have four by five probability of winning but this is also not correct therefore we strike out that option can it be zero because now australia captain will have to win the toss therefore probability v dot winning is equal to 0 that is also a wrong answer therefore we cancel that therefore the correct option is d that is answer is half because probability v that winning fifth toss given he won all the previous tosses is same as probability virat winning the toss as these are independent events therefore correct answer is half therefore the student should take d ok friends i stop here today in this class we have solved quite a few problems involving conditional probability and base theorem in the next class i shall work on binomial distribution and also show the utility of binomial theorem in computing the number of certain combinations to happen okay friends thank you

so much you

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