welcome students to iit palm problem solving session our topic is probability and this is lecture number four in the last class we have solved several problems of probability and as i said that we shall continue with solving more problems in this session

so let us start with a simple problem suppose a and b through a pair of dice alternatively the game stops if a throws a total sum of 9 or b throws a total sum of six assuming a starts first what is the probability that b finishes the game

so let e be the event or u1 be the event that the sum of faces is equal to nine this can happen with the following cases three six four five five and 6 3 therefore probability of e 1 is equal to 4 upon 36 is equal to 1 upon 9 let e 2 be the event that the sum is six therefore possibilities are one five two four three three 4 2 and 5 1 and therefore probability of e 2 is equal to five upon thirty six we know that a starts the game therefore a finishes the game if a throws nine first time and that probability is one by nine a throws other than nine then b throws other than six and then a throws nine and this probability is 8 by 9 into 31 upon 36 into 1 by 9 they may finish the game in the fifth turn and that probability you can easily calculate is going to be eight by nine into 31 upon 36 again 8 by 9 into 31 upon 36 and finally 1 by 9 is equal to 1 by 9 into 8 into 31 upon 9 into 36 whole square as you can understand that a finishes the game only at odd turn therefore total probability that a finishes the game is 1 by 9 plus 1 by 9 into 8 by 9 into 31 upon 36 plus 1 by 9 into 8 by 9 into 31 upon 36 whole square as you can understand that this is going to be a gp series is equal to 1 by 9 into 1 plus 8 by 9 into 31 upon 36 plus 8 by 9 into 31 upon 36 whole square plus this in finite series and we know that this sum of this series is going to be 1 upon 1 minus 8 by 9 into thirty one upon thirty six is equal to one by nine into one upon one minus 248 upon 300 20 4 is equal to 1 by 9 into 1 upon 324 minus 248 upon 324 is equal to 1 by 9 into 324 upon 76 is equal to 36 upon 76 is equal to 9 upon 19 therefore the probability that if a starts the game then a finishes the game is 9 upon 19 therefore we can easily see that if a starts the game then b finishes it has the probability 10 upon 19 in a similar way you can obtain the probability that a finishes the game or b finishes the game when b starts the first move i leave it as an exercise for you to complete that part let us now do another problem let a b c be three events such that probability of a is equal to zero point three probability of b is equal to zero 0.
4 probability of c is equal to 0.
8 probability of a intersection b is equal to 0.
19 probability a intersection c is equal to 0.
2 and probability a intersection b intersection c is equal to 0.
09 if probability a union b union c is greater than equal to 0.
75 find the minimum and maximum value for probability b intersection c solution we know that probability a union b union c is equal to probability of a plus probability of b plus probability of c minus probability of a intersection b minus probability of b intersection c minus probability of a intersection c plus probability of a intersection b intersection c it is given that this is greater than equal to point seven five and it is obvious that it is less than equal to one therefore 0.
75 less than equal to probability of a plus probability of b plus probability of c plus probability of a intersection b intersection c minus probability of a intersection b minus probability of a intersection c minus probability of b intersection c which is again less than equal to 1 or 0.
75 is less than equal to zero point three plus zero point four plus zero point eight plus zero point zero nine minus zero 0.

19 minus 0.
2 minus probability of b intersection c less than equal to 1 or 0.
75 less than equal to 1.
2 minus probability of b intersection c is less than equal to 1 or minus 1 less than equal to probability b intersection c minus 1.
2 less than equal to minus 0.
75 therefore 0.
2 less than equal to probability b intersection c less than equal to 0.
45 therefore minimum and maximum values for probability b intersection c are 0.
2 and 0.
45 that is the answer ok friends let me now solve an interesting problem let a and b be two events of a random experiment e it is given that one probability of a union b is greater than equal to 0.
5 and 2 probability of a intersection b is less than equal to 0.
375 and greater than equal to 0.
125 therefore there are many different probabilities are possible for a and b satisfying the above let s be the region on 2d plane such that s is equal to x comma y where x is the probability of a and y is the probability of b satisfying the above conditions find the area and perimeter of s
so that is the question answer it is clear that s is contained in the unit square 0 comma 1 cross 0 comma 1
so if we draw the diagram suppose this is 0 1 1
so this is the square s is contained in this we have to find the area and parameter of s now given that 0.
5 less than equal to probability of a union b less than equal to 1 because this is obvious all probabilities are less than equal to 1 also it is given that 0.
125 is less than equal to probability of a intersection b less than equal to 0.
375 therefore 0.
625 less than equal to probability of a union b plus probability of a intersection b less than equal to one point three now probability of a union b plus probability of a intersection b is equal to probability of a plus probability of b therefore 0.
625 less than equal to probability of a plus probability of b less than equal to 1.
375 or this we can write it as 5 by 8 less than equal to this is given to be x this is given to be y less than equal to this is 11 upon 8 therefore suppose this is 5 by 8 and this is 5 by 8 let us draw the line also suppose this is eleven by eight and this is eleven by 8 and let us draw the line therefore s is this region
so let us give name to different points let us call it o p q let us call this line m n and suppose this line is x y and this line is or s therefore s is the region given by p x y is r q therefore perimeter of s is the sum of the six line segments what are these this is three by eight this is three by eight this is 3 by 8 and this is also 3 by 8 these are the lengths we need to compute the length of x y and we will compute the length of p q
so note that length of p q is equal to square root of 5 by 8 square plus 5 by 8 square is equal to square root of 2 into 5 by 8 square is equal to root 2 into 5 by 8.
and length of this x y is equal to root over 1 minus 3 by 8 whole square plus 1 minus 3 by 8 whole square because this point is 3 by 8 comma 1 and this point is 1 comma 3 by 8 is equal to square root of 2 into 5 by 8 whole square is equal to root 2 into 5 by 8 therefore this is is equal to 2 root 2 into 5 by 8 plus 3 by 8 into 4 is equal to root 2 into 10 by 8 plus 12 upon 8 is equal to 1.
4 into 10 divided by 8 plus 12 by 8 is equal to 26.

1 by 8 which is approximately 3.
26 therefore this is the pedimeter length we need to find out area of s if we go back to the diagram we can compute the area of this triangle from which we minus this area this area and this area therefore area of s is half into eleven upon eight whole square minus half into five by eight whole square minus half into three by eight whole square minus half into three by eight whole square is equal to 1 upon 2 into 8 square is 64 11 square is 121 minus 5 square is 25 minus 3 square is 9 minus 9 is equal to 78 upon 1 2 8.
 is equal to 0.
609
 so this is the area of s
 so the answer is perimeter is equal to 3.
26 area is equal to 0.
609 another problem suppose this sample space of a random experiment is 1 2 3 4 5 6 where each element is equally likely as i said earlier in the last class that that means that all these six elements have equal probability of occurrence that is one by six if a and b are two independent events then compute the number of ordered pairs a comma b such that one less than equal to cardinality of b less than equal to cardinality of a
 so i hope you understand the problem there are six elementary events in the sample space they are equally likely we want two events a and b such that they are independent now we want to see how many such pairs are possible such that cardinality of b less than let us make it strictly less than cardinality of a how to solve it note a and b are independent if probability of a intersection b is equal to probability of a into probability of b illustration
 so before going into solving the problem let me illustrate say b is equal to suppose 1 comma 2 and a is equal to 1 comma 2 comma 3 therefore probability of b is equal to one by three probability of a is equal to half and probability of a intersection b is equal to probability of 1 comma 2 is equal to 1 by 3 therefore probability a intersection b is not equal to probability of a into probability of b therefore in this case a and b are not independent but consider b is equal to 1 to a is equal to one comma three comma four therefore probability of b as before is one probability of a is equal to half and probability of a intersection b is equal to probability of singleton 1 is equal to 1 by 6 therefore probability a intersection b is equal to probability of b into probability of a
 so this is a case when a and b are independent and cardinality of b is less than cardinality of a how many such pairs are possible that is the question solution suppose cardinality of a intersection b is equal to x therefore probability of a intersection b is equal to x by six now probability of a is equal to cardinality of a divided by six and probability of b is equal to cardinality of b divided by six therefore x by six is equal to cardinality of a by 6 into cardinality of b by 6 implies 6 x is equal to cardinality of a into cardinality of b therefore a and b should be such that cardinality of a into cardinality of b is a multiple of six therefore if cardinality of a is equal to six then cardinality of b can be 1 2 3 4 or 5 that is if a is the full set but we take b to b any subset then how many such possibilities are there b can be of size one that can be done in six c one ways b can be of size two that can be done in 6 c 2 ways similarly 6 c 3 plus 6 c 4 plus 6 c 5 is equal to 6 plus factorial 6 upon factorial 2 into factorial 4 plus factorial 6 into factorial 3 into factorial three plus factorial six into factorial four into factorial two plus factorial six into factorial five into factorial one is equal to six plus 15 plus 20 plus 15 plus 6 is equal to 62 ways now obviously cardinality of a cannot be five if cardinality of a is equal to 4 then cardinality of b has to be

3 because 4 into 3 is 12 and that is a multiple of 6 and cardinality of a intersection b has to be 2 because that has to be 6 x that is 12 therefore x is equal to 2 therefore possible cases are we can choose out of six four elements for a in six c four ways out of these four i can choose two in four c two ways and out of the remaining two i choose one for b that is two c one ways therefore the answer is fifteen into six into two is equal to one hundred eighty now cardinality of a is equal to three then cardinality of b has to be two and a intersection b should have only one element therefore number of ways is 6 c 3 into 3 c 1 into 3 c 1 out of 6 we choose 3 for a out of these 3 we choose 1 for b and out of the remaining 3 we choose 1 for b is equal to 20 into 3 into 3 is equal to 180 therefore total number of possibilities such that cardinality of b is greater than equal to one and which is strictly less than cardinality of a when a and b are independent is equal to 62 plus 180 plus 180 is equal to 422

so that is the answer okay friends we have solved several tricky problems on probability now it is time to move on and we shall now start another chapter that is conditional probability suppose a and b are two events which are not independent in that case the occurrence or non occurrence of b may have an effect on the occurrence or non occurrence of a consequently the probability of a given that b has occurred will be different from probability of a without the knowledge of occurrence of b

so this is the basic philosophy of conditional probability

so to illustrate consider rolling a fair die therefore probability of one is equal to probability of two is equal to probability of six therefore probability of two is equal to one by six

so let a be the event that two has occurred and b be the event that an even number has occurred therefore if b is known to have occurred then probability of a is equal to probability of 2 given that 1 of 2 4 or 6 has occurred therefore probability of 2 given even number has occurred is equal to 1 by 3 because it is one of these three each are equally likely therefore one by three this is different from what we had earlier without the knowledge that b has occurred

so this is the very basic of conditional probability how do you obtain that probability of a given b is equal to probability of a intersection b divided by probability of b

so in the above case a intersection b is equal to singleton 2 and b is equal to occurrence of 2 or 4 or 6 therefore probability of a given b is equal to probability of 2 divided by probability of 2 or 4 or 6 is equal to 1 by 6 divided by half is equal to 1 by 3 which we have obtained

so this is the basic knowledge of conditional probability let me do a problem which involves the concept of conditional probability

so the problem is as follows consider a bag containing 12 red balls and eight green balls we take out three balls in succession without replacement and suppose we want to know the conditional probability that this second ball is green given that the first ball is red

so how to solve this problems let r1 be the event that the first ball is read and g2 be the event that the second ball is green therefore we are looking at probability of g two given r one now initial configuration is 12 red balls plus 8 green balls therefore suppose the first one is red then after taking out the first ball the configuration of the bag is going to be eleven are plus eight green as you can understand we started with twelve comma eight where 12 is the number of red balls and 8 is the number of green balls therefore now if g 2 happens the configuration is going to be eleven comma seven therefore probability g two given r one is equal to choosing of one of the green balls out of the total of nineteen balls therefore it is going to be eight upon nineteen now we know that probability of a given d is equal to probability of a

intersection b upon probability of b therefore we can write as probability of a intersection b is equal to probability of a given b multiplied by probability of b that is for this problem therefore we can write it as probability of r 1 and g 2 is equal to probability of g 2 given r one multiplied by probability of r one is equal to the first one is eight upon 19 and this term pr1 is equal to probability that the first ball is red that is out of the 20 balls you are choosing any of the 12 that probability is 12 upon 20 is equal to 8 upon 19 multiplied by 3 upon 5 that is how we get the joint occurrence of r 1 and g 2 its probability now suppose the second problem is what is the probability that the third ball is green given that the first two ball drawns two balls drawn are of the same color therefore the given condition is ball one drawn and ball true drawn are of the same color that is either both of them are red or both of them are green that is probability of g 3 given r 1 r 2 plus probability of g 3 given g 1 g 2 and perhaps you can easily understand what this notation is r one is the first ball drawn is red r two is the second ball drawn is red and similarly g three g one g two are corresponding to green color therefore we have this situation twelve comma eight after r one we come to 11 comma 8 after r2 we come to 10 comma 8 and g3 then will come to 10 comma 7 on the other hand if the first one is green then we come to 12 comma 7 second one is also green then we come to 12 comma six then we come to g three and that is going to be twelve comma five

so what are the probabilities this probability is 12 upon 20 drawing a red ball again drawing a red ball that probability is eleven upon nineteen and now drawing a green ball that probability is eight upon eighteen in a similar way this probability is eight upon twenty now we have twelve and seven

so drawing a green that probability is seven upon nineteen and then drawing a green that probability is going to be 6 upon 80 therefore probability of g 3 given r 1 r 2 or g 1 g 2 note that these are disjoint events is equal to 12 upon 20 multiplied by 11 upon 19 multiplied by 8 upon 18 plus 8 upon twenty multiplied by seven upon nineteen multiplied by six upon eighteen is equal to 12 into 88 plus 8 into 42 divided by 20 into 19 into 18 is equal to 8 times this is eleven into twelve is equal to one hundred thirty two plus forty two divided by 20 into 19 into 18 we can cancel this with by 4 therefore we have 2 and 5 and this i cancel with this which is 9 therefore we have 174 upon 19 into nine into five we can further cancel it with three but ok let me do it this is 58 this gives us 3 therefore we get 58 upon 19 into 15 ok friend i stop here today in the next class i shall continue with conditional probability and will solve several problems and i shall start with the same problem and i will show some more important results okay friends thank you

so much you