welcome students to iit problem solving session of mathematics our topic is probability and this is lecture number three in today's class we shall solve a number of problems for which the background has already been made

so two things or a few things that we have to remember are as follows probability of a where a is an event corresponding to a random experiment e whose sample space is omega is cardinality of a divided by cardinality of omega and another thing that we will be using is from event algebra that probability of a union b is equal to p a plus p b if a and b are disjoint but if a and b are not disjoint then probability of a union b is equal to probability of a plus probability of b minus probability of a intersection b and thirdly if a and b are independent events then probability of a intersection b is equal to p a times p b

so these are some basic things that we have to remember and as we shall solve problems if we find that something new is coming the concept is not given to you then i shall explain that

so let us look at the first problem we have already seen that if x y z are all greater than equal to 0 and x plus y plus z is equal to 10 then number of possible solutions is 10 plus 3 minus 1 c 3 minus 1 is equal to 12 c 2 is equal to 60 six now what is the probability that in the above scenario x is an odd number

so what we have to do we know that the total number of points in omega or the cardinality of omega is sixty six

so we have to find out number of cases when x is an odd number under this condition and from there we have to compute the probability

so solution x can be odd implies x can take values 1 3 five seven nine now consider the cases x is equal to one implies y plus z is equal to nine in how many different ways y plus z can be nine when both y and z are greater than equal to 0

so the number of cases is 0 9 1 8 2 two seven up to nine zero that is ten and if we understand therefore that y plus zero is equal to k implies there are k plus 1 solutions then we can easily find out x is equal to 3 implies y plus z is equal to 7 therefore number of solution is going to be eight similarly x is equal to five implies y plus z is equal to five therefore number of solutions is six x is equal to seven implies y plus z is equal to three therefore number of solution is four and in a similar way x is equal to nine implies y plus z is equal to one and number of solutions is two therefore total is ten plus eight eighteen plus six twenty four plus four twenty eight plus two is equal to thirty therefore probability x is odd is equal to 30 upon 66 is equal to 5 upon 11

so that is the answer let us consider another problem the problem is as follows suppose a man is standing on the x axis at the origin at each step he goes either to the right or to the left each with probability half what is the probability that after six steps he will be at a distance to from the origin

so i hope you understood the question

so here is the initial position

so he can go either to this direction or to this direction with probability half and half at each step we want that after six steps he is here or here what is the probability

so how to do that let x be the number of steps he takes to the right direction and y b the number of steps he takes to the left direction therefore x plus y is equal to 6 because we are looking at total 6 steps and modulus of x minus y is equal to two this is obvious that means either he takes two steps more to the right or two steps more to the left therefore we get two equations one pair of equation is x plus y is equal to six and x minus y is equal to two implies two x is equal to 8 implies x is equal to 4 implies y is equal to 2 and second set of

equation is that x plus y is equal to six y minus x is equal to two implies two y is equal to eight implies y is equal to 4 implies x is equal to 2.

so these probabilities 0.
5 whole to the power 6 similarly this also has probability 0.
5 whole to the power 6 therefore desired probability is equal to two times zero point five to the power six is equal to zero point five whole to the power five
so that is the answer let us now consider another problem suppose a b c are three events corresponding to a random experiment e such that one probability of a union b union c is equal to one q a b and c are equally likely three a intersection b b intersection c and a intersection c are also equally likely for probability of a intersection b is equal to probability of a divided by 2 and probability of a intersection b intersection c is equal to probability of a intersection b divided by two find the probability of a intersection b intersection c complement that is the problem now before i start solving let me explain to you that a union b union c is equal to 1 that means they cover the entire sample space but it is not that they are disjoint that is a intersection b b intersection c and a intersection c they can be with positive probability and what do you mean by equally likely it means that all the three events have the same probability similarly a b c are equally likely a intersection b b intersection c and a intersection c are also equally likely and moreover probability of a intersection b is half of probability of a we need to find out the probability of a intersection b intersection c complement
so we move as follows late probability of a b equal to x therefore probability of b is equal to probability of c is equal to x therefore probability of a intersection b is equal to probability of a by 2 is equal to x by 2 and that is same as probability of b intersection c and probability of a intersection 3 and probability of a intersection b intersection c is equal to half of that therefore x by 4.
therefore probability of a union b union c is equal to we know that it is probability of a plus probability of b plus probability of c minus probability of a intersection b minus probability of b intersection c minus probability of c intersected with a plus probability of a intersection b intersection c now it is given that this is one that this implies that one is equal to three times x these are all x plus this is x by 4 minus 3 times this this and this these are all x by 2 implies 4 is equal to 12 x plus x minus 6 x is equal to 7 x therefore x is equal to 4 by 7 therefore probability of a intersection with b intersection with c complement is equal to probability a intersected with b minus probability of a intersected with b intersected with c is equal to x by 2 minus x by 4 is equal to 2 by 7 minus 1 by 7 is equal to 1 by 7.

so that is the answer now let us solve another problem suppose a pair of dice is rolled the dice is fair that is probability of one is equal to probability of two is equal to probability of six is equal to one by six what is the probability that the sum is 8 or you get even numbers on both the dice
so that is the question answer let a be the event that this sum of the two faces is eight therefore possible combinations are 2 6 3 5 4 4 5 3 and 6 2 that is 5 therefore probability of a is equal to 5 upon 36 since there are 36 points or 36 pair of points in omega let b be the event that the face on both the dice is even therefore possibilities is it can be two two two four two six 4 2 4 4 4 6 and 6 2 6 4 6 6 that is nine therefore probability of b is equal to nine upon thirty six therefore probability of a union b or both of them are even is equal to probability of a plus probability of b minus probability of a intersection b now what is the probability of a intersection b that means both the faces are

even and the sum is 8.

so we can see that this is one way this is one way and this is one way
so there are three such cases when the sum is eight and both of them are even therefore these probabilities 3 upon 36 therefore the answer is 5 by 36 plus 9 by 36 minus 3 by 36 is equal to 11 upon 36
so that is the answer
so let me do another problem suppose a and b are playing the final of a tournament against each other the final may be best of three matches or best of five matches the probability that a wins a match against b is 0.
4 but a has the advantage that he can choose whether it will be best of three or best of five which one will a choose
so that is the question answer obviously a will choose the format where he has more chance of winning
so let us compute the probabilities that a will win the tournament for both the options
so let us first consider best of three scenario here or the player who wins two matches first is the winner therefore a can win if 1 he wins both first and second match or he wins one of first and second match and then wins the third match
so this is an either or in one of the situations he is going to win the tournament therefore probability a wins the tournament if the outcome of first two matches are win win or it is the first two are win loss and then he wins the third match or the first two are he loses then then he wins and then he wins the third match
so these are the three disjoint scenarios when a wins the tournament
so this probability is 0.
4 multiplied by 0.
4 here it is 0.
4 0.
6 and 0.
4 and here it is 0.
6 0.
4 and 0.
4 therefore this probability is 0.
4 whole square plus 0.
4 whole square multiplied by 0.
6 plus again from here 0.
4 whole square multiplied by 0.
6 is equal to 0.
4 whole square into 1 plus 0.
6 plus 0.
6 is equal to 0.
16 multiplied by two point two is equal to zero point three five two let us now consider best of five scenario the final can be over with a winning in the following ways one a wins the first three matches which is probability 0.
4 whole cube to a wins two of the first three matches and then wins the fourth match its probabilities 0.
4 whole cube multiplied by 3 into 0.
6 because in the first three there are three possible ways that a wins two matches which are w w l w l w and l w w each one of them is going to give you 0.
4 whole square multiplied by 0.
6 and this will be followed by a win by a which is 0.
4 therefore this is the probability of this happening in a similar way the other

way that a can win is that a wins two of the first four matches and then wins the fifth one therefore out of the first four he wins two this can be done in four c two ways say for example win win loss loss win loss win loss win loss loss win loss win win loss loss win loss win loss loss win win and then he wins the fifth match therefore the probability is 6 into 0.
4 whole square into 0.
6 whole square into 0.
4 is equal to 0.
4 whole cube multiplied by 6 into 0.
6 whole square therefore probability a wins the tournament is equal to 0.
4 whole cube plus 3 into 0.
4 whole cube into 0.
6 plus 6 into 0.
6 whole square into 0.
4 whole cube is equal to 0.
4 whole cube multiplied by 1 plus one point eight three into zero point six plus this is point three six
so two point one six which is close to zero point three one seven that you can compute therefore a should choose best of 3 option because this is less than 0. 352 another problem suppose five candidates a b c d and e are waiting to appear in an interview it is known that the board or the interview board will call them randomly in any order find the probability that a is called before b find the probability that a comes before b and b is called before c third is find the probability that b is called just after a
so this is one paragraph based on that we are given three different questions and we need to find solutions a is called before b
so there are many possibilities a is called at number one therefore in the remaining four positions whenever becomes b comes after a therefore number of possibilities is equal to factorial four because a comes here and in these four positions the remaining four candidates can come in any order a is called at number two therefore b has to come in one of these positions therefore out of c d e one is chosen and put here that can be done in three ways and now the other two and b can be permuted here in factorial three ways therefore number of possibilities is equal to three into factorial three third is that a may be called at number 3 therefore b can be in one of these positions out of the other three you choose two and put them there that can be done in three ways and then they can be permuted among themselves
so that can be done in two ways now b and the other candidate can be permuted here in two ways therefore total possibilities is equal to twelve a is called at number four therefore b has to be here and other three can be permuted here in factorial 3 is equal to 6 ways therefore number of possibilities is equal to factorial 4 which is 24 plus 3 into factorial 3 which is 18 plus twelve plus six is equal to sixty now total number of permutations of five numbers is factorial 5 is equal to 1 20 therefore probability a is called before b is equal to 60 upon 120 is equal to half now if you are smart enough you can get this answer even without computing anything because the whole scenario is symmetric between a and b therefore half chance or probability is half that a will be called before b and similarly probability is half that b will be called before a
so i could have got the answer right at the beginning but i computed it for you to see that you arrive at the right answer and also i demonstrate before you how to solve this type of problems now b part is a before b and b before c in how many ways it can be done
so suppose a is in position 1 now b and c have to be arranged in such a way that b is before c

so if b is in second position then there are six possible ways of arranging them if b is in third position then out of d and d you put one in the second position that can be done in two ways then you put b here and c can be in this that can be done again in two ways therefore there are four possibilities b is in fourth position therefore c has only one option of coming in the fifth position therefore out of the remaining two you can arrange them in this in two ways therefore there are two possibilities implies twelve possibilities a is in position two therefore how many possibilities are there b is in third position therefore out of d and d you can choose one in first position that can be done in two ways and c and the other one can be permuted here in two ways

so there are four ways of doing that one b is in fourth position and that also can be done in two ways therefore total number of possibilities is six a is in third position therefore d and d are in first and second position and that can be done in two ways and obviously a cannot go to fourth position because b and c will have to come after him therefore total possibilities is equal to 12 plus 6 plus 2 is equal to 20.

therefore probability a comes before b and b four c is equal to twenty upon one twenty is equal to one by six a comes just before b therefore what we can do we can keep a b as one unit and therefore we have now a b c d e and they can be arranged in factorial 4 is equal to 24 ways therefore probability is equal to 24 upon 120 is equal to 1 upon 5 hence problem suppose five persons a b c d and e are sitting in a circular table you have hats of three colors white red and green in how many ways you can give one hat to each person such that no two consecutive persons hat of same color

so this type of problems can be solved by drawing a tree

so we have a b c d and e note that a and d are side by side because it is a circular table suppose you give the white color cap to a then b should get either red or green because they cannot have same color because they are sitting next to each other now b is red therefore c can get white and green and similarly when b is green c can get white and red therefore d can get red and green or red and white or green and red and green or white now at this position therefore we have eight different possibilities now since d is red he can get white and green but it cannot get white because a has white therefore in this case only green if it is green then in this case it can be only red if it is red then again it can be only green but if it is white then it can get both green and red similarly this is red this is green this is red and this is green or red therefore total possibilities are 1 2 3 4 5 6 7 8 9 10.

this 10 has come when a got white but a can get white green and red therefore total possibilities is equal to 10 into 3 is equal to 30

so before i stop i do one more problem suppose two teams mohan bagan and east bengal are facing each other in two matches for each match the winner gets three points and loser gets one point for a drawn match both the teams get one point each if muhammad has point five probability of winning point one probability of drawing and point four probability of losing what is the probability that mohn bagan ends with more points then use bengal

so that is the question

so again we start from using a tree

so 0.

5 is the probability that mohan bhagavan gets 3 and his bengal gets zero point one is the probability that both of them get one and point four is the probability that muhammad gets zero and his bengal gets three this is about the first match in the second match point five probability that mohan bagan wins

so mohan bhagavan's point is six is bengals is zero point 1 that they draw the match

so the final tally is 4 comma 1 and 0.
4 is that mohn bagan lose therefore this is going to be 3 3 from 1 1 with 0.
5 it is going to be 4 1 with 0.
1 it is going to be 2 2 and with 0.
4 it is going to be 1 4 from 0 3 with 0.
5 it is going to be 3 3 with 0.
1 it is going to be 1 4 and with 0.
4 it is going to be 0 6 therefore probability muhun bagan point is greater than point of east bengal is equal to 0.
5 into 0.
5 because that is the probability along this branch this is 4 1 therefore it is 0.
5 into 0.
1 plus this is 4 1 therefore 0.
5 into 0.
1 is equal to 0.
25 plus 0.
05 plus 0.
05 is equal to 0.
25 plus is equal to point three five instead if we asked what is the probability that they end up with same point then what you would do you will look at the situations when they have the same points
so this is 3 3 which is 0.
2 plus this is 2 2 which is point zero one and this is three three which is point two is equal to zero point 4 1 therefore it is obvious that probability moon bagan will have less point than is bengal you can easily understand that these two together is 0.
76 therefore this is going to be 0.
24 okay students i hope you understood how to compute the probabilities using a tree diagram and also using the independence of these two events because these are two matches whose outcomes are independent of each other and that is why we have used the multiplication and since these are disjoint events therefore we have made use of addition and like that we come to the final probability okay friends i stop here today in the next class i shall work out some more interesting problems for you okay then thank you you

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