

in the last lecture we discussed conjugate of complex number and also modulus of a complex number let me just recall any general complex number  $a + i b$  the conjugation that is  $\bar{z}$  we defined it as  $a - i b$  and modulus of  $z$  we defined it as square root of  $a^2 + b^2$  now let us see some properties of modulus

so throughout this discussion for this we will assume that  $z$  is of the form  $x + i y$  and let us just prove this real part of  $z$  is always less than or equal to modulus of  $z$  as well as it is greater than equal to minus of mod  $z$  it is nice inequality that real part of  $z$  or modulus of real part of  $z$  is less than or equal to mod  $z$

so the proof is simple what we have real part of  $z$  which is  $x$  and when we consider its square real part of  $z$  consider its square that is  $x^2$  that is certainly less than or equal to  $x^2 + y^2$  that is we are adding some non negative number

so now immediately you see that whenever you have  $a$  is less than or equal to square root of  $a^2 + b^2$

so this implies that real part of  $z$  is less than or equal to square root of  $x^2 + y^2$  this is nothing but mod  $z$  and what we see here it is in fact the way we have derived it is just the even in the absolute sense this will be less than or equal to mod  $z$

so that concludes the first proposition similarly we can prove that imaginary part of  $z$  is greater than or equal to minus mod  $z$  less than or equal to mod  $z$  proposition two is certainly it is straight forward one that is mod  $z$  always a non negative number for all  $z$  in complex numbers and what we need to observe is whenever modulus of  $z$  is zero if one only if  $z$  is zero ok this is basically like again which is easy to observe because once you consider mod  $z$  equal to zero it means it is its square is zero

so this implies that  $x^2 + y^2$  this is zero and what we are saying is that sum of two non-negative numbers are zero that implies basically each non negative element must be zero that is  $x^2$  must be zero as well as  $y^2$  must be zero that concludes that  $x$  equal to zero  $y$  equal to zero

so this is nothing but same as saying that  $z$  is a zero element and  $\bar{z}$  is a zero element which is basically like its follows and what are the other simple observations when you consider modulus of  $z$  which is also same as minus  $z$  modulus against the same as mod  $z$  which is a trivial one maybe i will add to this which is basically its conjugation as well same modulus

so what we are basically saying is when you take a  $z$  then minus  $z$  you have so let us say this is basically  $x + i y$  then minus  $z$  you have minus  $x$  minus  $i y$

so this distance is the same as this distance similarly you take its reflection about the  $x$  axis that is  $\bar{z}$  mod  $z$  is also same now the fourth one when you multiply  $z$  with  $\bar{z}$  we get mod  $z$  square

so by definition it is clear

so i am not basically discussing this let us go to the fifth one modulus of  $z_1 z_2$  it will be same as mod  $z_1$  multiplied with mod  $z_2$  proof is consider mod  $z_1 z_2$  to the whole square by the above proposition we are getting  $|z_1 z_2|^2$  multiplied with  $\overline{|z_1 z_2|^2}$  and again we know the relation about the conjugation

so the conjugation for  $z_1 z_2$  is  $\bar{z}_1$  multiplied with the  $\bar{z}_2$  and using the commutative law we can finally let me just write down

so what we have here is product with  $\bar{z}_1 \bar{z}_2$  and we get basically the associative law you could use and then use the further commutative law

so you can get the mod  $z_1$  square mod  $z_2$  square ok

so this gives the conclusion that modulus of  $z_1 z_2$  it gives mod  $z_1$

multiplied with mod  $z$  two you first take the product take its modulus which is same as first take the modulus for the each complex number then multiply it okay and the sixth proposition which is a famous or nice inequality which is repeatedly will be used which is called triangle inequality

so the inequality states that if we consider modulus of  $z$  one plus  $z$  two this is less than or equal to mod  $z$  one plus mod  $z$  two for every  $z$  one  $z$  two in the complex numbers let us try to understand this inequality what it says

so we have two points let us say this is  $z$  one and let us say this is  $z$  two and then we can associate naturally a vector for this

so we can visualize  $z$  one as a vector and  $z$  two as another vector then the sum gives the what we basically get is which is the diagonal for this parallelogram that is  $z$  one plus  $z$  two

so as i mentioned in the previous lecture modulus it means that it is a distance between the complex number and the origin

so modulus of  $z$  one plus  $z$  two indicates the distance between this number to the origin ok

so the distance is less than or equal to distance to go to  $z$  2 and then further you go using the vector  $z$  1 to reach this point

so it means like taking a distance go first  $z$  2 and then further use  $z$  1 to reach here that will be always bigger than the distance

so it in other words this is the shortest distance what we are observing ok

so let us try to prove this result

so consider the term left hand side and with the square and this is written by mod  $z$  if i recall the property mod  $z$  square is written by  $z$  into  $z$  bar now just by direct say say applying the property what we are getting it is  $z$  one bar plus  $z$  two bar and further you apply the distributive law then we get that  $z$  one multiplied by  $z$  one bar plus  $z$  one  $z$  two bar plus  $z$  two multiplied with the  $z$  one bar plus  $z$  two multiplied with the  $z$  two bar and we know that the quantity this one is modulus of  $z$  one square plus here what we observe is  $z$  one  $z$  two bar treat it as a one complex number then the next number we are just seeing it as its exactly the conjugation of previous one

so if you just observe

so if i apply this bar then it is at one bar multiplied with  $z$  to double bar that gives again  $z$  to itself

so we see that this is mod  $z$  2 square and we know that  $z$  plus  $z$  bar gives a real part of  $z$  multiplied by two

so we get that here it is two times of real part of  $z$  one  $z$  two bar plus this now try to recall again what is the inequality we are trying to prove ok we are trying to get the modulus of  $z$  one plus  $z$  two less than or equal to mod  $z$  one plus mod  $z$  two now we are seeing that almost we are reached here if we are able to get something like the right hand side as mod  $z$  one plus mod  $z$  two to the whole square then we are almost done but only we need to see that how do we bring the term mod  $z$  one and mod  $z$  two here ok

so if i recall the proposition which i wrote that is the first one it states that the real part of  $z$  real part of  $z$  always bigger than equal to minus mod  $z$  as well as less than or equal to mod  $z$

so so what we

so we are going to use this particular relation here

so what we know is the real part is always less than or equal to

so one need to be very careful about when we are using this relation as i warned you that for complex number such a relation is not applicable if you see the quantity above one it is completely real numbers ok now when we are comparing here it is 2 times of modulus of  $z$  1  $z$  2 bar again it is a non-negative number and mod  $z$  two square and again what you see is that the property  $z$  one

$\text{mod } z$  one square plus two times of  $\text{mod } z$  one multiplied by modulus of  $z$  to bar that is same as modulus  $z$  two plus  $\text{mod } z$  to the whole square and we know that this is nothing but this is  $\text{mod } z$  one plus  $\text{mod } z$  to the whole square ok again what we have what we derived is a square less than or equal to  $b$  square that implies  $a$  is less than or equal to  $b$  okay

so that proves our triangle inequality

so i would like to point out that question one can ask when this equality holds for this relation that is when modulus of  $z$  one plus  $z$  two equal to modulus of  $z$  one may be the conclusion i will write it here okay or the next page

so we just derived  $\text{mod } z$  one  $z$  two less than or equal to  $\text{mod } z$  one plus  $\text{mod } z$  two now question is when do i see the equality

so when this will happen now go back when we derive this inequality this is the place where we use the inequality relation

so the equality appears if and only if real part of  $z$  one  $z$  to bar equal to modulus of  $z$  one  $z$  two bar

so the equality appears

so the equality appears in the let me call it as equation one one provided real part of  $z$  one  $z$  two bar this is equal to  $\text{mod } z$  one bar this is equivalent

equivalent to saying that  $z$  one is a  $t$  times of  $z$  two where  $t$  is a non negative real number let me again repeat we proved this triangle inequality question is when the equality appears we just observe that the equality of appears if and only if  $z$  one and  $z$  two they are just like linearly dependent type that is like constant times of  $z$  two you are you are having like  $z$  one is just a constant times of  $z$  two in that case we get the equality let us see further say other consequences from this triangle inequality

so we just observed that modulus of

so let me just repeat this we have this relation and from this we can derive that we can derive that  $\text{mod } z$  one minus  $\text{mod } z$  two is less than or equal to thus how do we derive this just say let us call it as this is one

so consider  $\text{mod } z$  one we just add and subtract  $z$  two then apply one then by one we get  $\text{mod } z$  one is less than or equal to  $\text{mod } z$  one plus  $z$  two and further you get modulus of minus  $z$  two but we know that modulus of minus  $z$  two is again same as  $\text{mod } z$  two

so from this we get that  $\text{mod } z$  one minus  $\text{mod } z$  two less than or equal to modulus of  $z$  one plus  $z$  two ok similarly we can interchange the role of  $z$  one and  $z$  two

so in this case we get

so if i interchange the role because  $z$  one and  $z$  two is arbitrary

so you can just interchange the role of interchange  $z$  one and  $z$  two then we observe that this is  $z$  two minus  $\text{mod } z$  one that is less than or equal to modulus of  $z$  one plus  $z$  two and as a conclusion what we observe is this implies modulus of  $\text{mod } z$  one minus  $\text{mod } z$  two that is less than or equal to modulus of  $z$  one plus  $z$  two ok now one can ask question is this plus is really necessary ok can i have here minus true ok you can have minus sign again

so it may be here plus or minus still this is greater than or equal to modulus of  $\text{mod } z$  one minus  $\text{mod } z$  two similarly this is less than or equal to modulus of  $z$  one plus  $\text{mod } z$  two

so here i see something more general from the triangle

so we have triangle inequality and now what we see is much more general inequality for this complex numbers

so proposition 7 modulus of  $z$  inverse  $s$   $\text{mod } z$  inverse where  $z$  is a non zero complex number again it is like similar to the conjugation property we can derive this result that is start with  $z$  inverse that is  $z$  inverse is given by  $z$  into  $z$  inverse you get identity then its modulus again modulus of one is one and

we already discussed modulus of  $z_1$  into  $z_2$  gives  $\text{mod } z_1$  multiplied by  $\text{mod } z_2$

so what you get is  $\text{mod } z$  multiplied by modulus of one by  $z$  this is equal to one so what we observe is inverse

so  $\text{mod } z$  inverse is exactly  $z$  inverse  $\text{mod}$

so this is the conclusion

so what we see is that this implies that  $\text{mod } z$  inverse is nothing but modulus of  $z$  inverse that is one by  $z$

so that is our proposition and proposition eight which is modulus of  $z_1$  by  $z_2$  which gives  $\text{mod } z_1$  by  $\text{mod } z_2$  you assume that  $z_2$  is non zero the result is again say just  $c$  as a product of two complex number that is  $z_1$  multiplied by  $z_2$  to inverse then we see that modulus is applied for each factor and the previous proposition we just proved that modulus of  $z_2$  inverse is same as  $\text{mod } z_2$  the whole inverse and this is nothing but  $\text{mod } z_1$  by  $\text{mod } z_2$  ok now another interesting result which is called parallelogram law it states that modulus of  $z_1 + z_2$  whole square plus modulus of  $z_1 - z_2$  whole square which is same as two times of  $\text{mod } z_1$  square plus  $\text{mod } z_2$  square okay why they call it as parallelogram law let us try to observe this

so let us call say point as  $z_1$  and let us say that this is  $z_2$  and we know that the  $z_1 + z_2$  which is exactly gives the vector which is diagonal of this parallelogram that is  $z_1 + z_2$  and similarly the other diagonal gives the  $z_1 - z_2$  vector now if you see the identity it says that square of the magnitude of this diagonals considered its sum that is equal to twice of magnitude square sum for the sides of the parallelogram parallelogram law

so this is very interesting property

so and proof is simple you need to just expand for the left hand side then we can easily derive the you can arrive to the right hand side lhs which is modulus of  $z_1 + z_2$  square plus  $z_1 - z_2$  to the whole square

so by definition this is  $z_1 + z_2$  multiplied with  $z_1 + z_2$  bar plus  $z_1 - z_2$  multiplied with  $z_1 - z_2$  bar

so the quantity what we are calculating is  $z_1 + z_2$  the whole square plus  $z_1 - z_2$  to the whole square thus we written it as  $z_1 + z_2$  multiplied with  $z_1$  bar  $z_2$  bar plus  $z_1 - z_2$  multiplied with  $z_1$  bar minus  $z_2$  bar by simplifying it we see that this is  $z_1$  into  $z_1$  bar that is  $\text{mod } z_1$  square and the other term which is  $\text{mod } z_2$  square and the remaining term what we have here it is  $z_1 z_2$  bar plus  $z_1$  bar  $z_2$  on this term we again get the term which is  $\text{mod } z_1$  square and  $\text{mod } z_2$  square remaining factors are comes with the opposite sign to the these factors that is minus  $z_1 z_2$  bar minus  $z_1$  bar  $z_2$  say these terms are cancel each other then what we get is twice of  $\text{mod } z_1$  square plus  $\text{mod } z_2$  square and let us discuss a problem which involves the modulus sign prove that if modulus of  $z_1$  equal to one as well as modulus of  $z_2$  is also equal to one and further their product is not equal to minus one then we can show that  $z_1 + z_2$  divided by one plus  $z_1 z_2$  is a real number okay

so let us try to observe this problem what is given is  $z_1 z_2$  they are they are basically on the unit distance from the origin and their product is not equal to minus one then the quantity what we defined is a real number its really nice to see that the expression is looks complicated but what we get at the end it is a real number let see how to show this as i mentioned earlier to say a complex number is a real number just

so that  $z$  and  $z$  bar it is equal which is same as saying that imaginary part is zero

so let us define like consider  $a$  to be the number  $z$  one plus  $z$  two by one plus  $z$  one product with  $z$  two

so our claim is  $a$  equal to  $a$  bar that shows that the imaginary part is zero

so now consider  $a$  bar now the property whatever we have studied before we are going to apply it

so the conjugation for the old factor

so when we are having conjugation for this which is  $z$  one bar  $z$  two bar plus  $z$  one bar  $z$  two bar ok now we need to relate  $z$  one bar to  $z$  somehow ok we haven't used the condition modulus of  $z$  one equal to one similarly modulus of  $z$  two equal to one

so we need to use this condition

so now let us say see that since  $\text{mod } z$  equal to one

so this is given to us which is same as its square is one let us call it as a first number  $z$  one square

so this is same as this implies  $z$  into  $z$  bar is one and ok we are able to get what is  $z$  bar is  $z$  bar is  $z$  inverse

so by this relation we just obtain that  $z$  bar is given by  $1$  by  $z$

so we are we have fixed with the notation that we have consider the  $z$  one as our complex number

so the  $z$  one bar is thus similarly  $z$  two bar is  $1$  by  $z$  two ok now we need to just use this relation in the  $a$  bar

so  $a$  bar is given by we have  $z$  one bar that is replaced by  $1$  by  $z$  one and similarly  $z$  two bar is replaced by  $1$  by  $z$  two and one plus one by  $z$  one multiplied with  $1$  by  $z$  two by simplifying this expression what we end up as  $z$  one plus  $z$  two divided by one plus  $z$  one multiplied with  $z$  two which is nothing but  $a$  ok this implies  $a$  is a real number ok

so let us just write your remark here or a comment what we observed is  $\text{mod } z$  equal to one then what we see is that  $z$  into  $z$  bar is one and  $z$  bar can be written as  $1$  by  $z$  ok and just notice that again that modulus of  $z$  bar is same as  $1$  by  $\text{mod } z$  that is again one

so let us may be try to understand again about this say all those  $z$  complex number whose modulus is one let us try to understand this

so consider the unit circle as set  $U$  which is defined as set of all complex numbers such that whose modulus is one now try to describe what are all the elements which is in the set

so when we write  $\text{mod } z$  equal to one this is precisely if we consider  $z$  as  $x$  plus  $i$   $y$  then  $\text{mod } z$  which is square root of  $x$  square plus  $y$  square if we consider it square then it is  $x$  square plus  $y$  square and it is given that  $\text{mod } z$  equal to one then it is one now you ask what are all the pair of elements  $x$   $y$  satisfies this equation and which we are very familiar that it describes the unit circle with center as origin

so let us draw the picture

so we have a unit circle

so any point on this take the pair  $x$   $y$  on the circle which satisfies this equation and it means that the set  $U$  exactly describes this unit circle now we know the points on this for example one which lies on this and  $i$  which lies on this minus  $1$  and minus  $i$  they lie on this unit circle the previous problem what we discussed where  $\text{mod } z$  equal to one then we concluded that  $z$  bar is nothing but  $1$  by  $z$  which is very easily seeable just this equation means that  $z$  into  $z$  bar equal to one  $z$  into  $z$  bar is nothing but  $\text{mod } z$  square equal to one that is holes because  $\text{mod } z$  is one

so let us try to visualize on this graph when we take  $i$  its conjugation is minus  $i$  and this equation tells that  $z$  inverse is exactly its conjugation which means that  $i$  inverse with respect to the complex product is nothing but its

conjugation that is minus  $i$  if you take any element on this line on this circle consider its mirror image that is exactly the conjugation consider any point on the circle  $z$  consider its mirror image which is that bar that is exactly the inverse of  $z$  which is also lies on the circle

so if you take one its mirror image itself

so inverse is one itself and if you take minus one its mirror image again itself

so the inverse is just minus one with respect to complex product

so what are the observations we made here whenever we take two complex numbers on the unit circle their product again in the unit circle this is just simple property that modulus of  $z_1 z_2$  is  $\text{mod } z_1 \text{ mod } z_2$  and each modulus is one

so their product is one and second important property which we observed whenever  $z$  in  $U$   $z$  inverse is also in  $U$  and not only that this  $z_1 z_2$  inverse is just the its conjugation of  $z$

so with this observation let us prove a nice identity consider four complex numbers  $z_1 z_2 z_3 z_4$  or complex numbers then it satisfies the following equation  $z_1 - z_2$  product with  $z_3 - z_4$  plus  $z_1 - z_4$  product with  $z_2 - z_3$  this is equal to  $z_1 - z_3$  product with  $z_2 - z_4$  observe this identity it can be any four complex number then the following equation satisfied proof is simple just expand the left hand side and the right hand side then you will see that both expressions are equal let us consider the left hand side left hand side is the expression which is  $z_1 - z_2$   $z_3 - z_4$  plus  $z_1 - z_4$  multiplied with  $z_2 - z_3$  now just expand it  $z_1 z_3 - z_1 z_4 - z_2 z_3 + z_2 z_4$  plus  $z_1 z_2 - z_1 z_3 - z_4 z_2 + z_4 z_3$  and further plus  $z_1 z_2 - z_1 z_3 - z_4 z_2 + z_4 z_3$  ok now we see that there are some common terms with the they cancel each other that is  $z_2 z_4$  fine maybe just let us go for the right hand side which is  $z_1 - z_3$  let us expand this  $z_1 - z_3$   $z_2 - z_4$  then we identify these two elements and similarly  $z_3$  and this we identified and  $z_1 - z_4$  we identified that one  $z_2$  has been identified and these two gets cancelled

so what we verified as the left hand side equal to the right hand side

so it is very interesting that it is a non trivial identity you consider any four complex number it satisfies this particular identity now we can see a simple problem which demonstrate that how nice this identity is problem suppose say we are given four points in a plane

so show that if  $a b c d$  or points in a plane then the following inequality satisfies that is  $a d$  multiplied with  $b c$  less than or equal to  $b d$  multiplied with  $c a$  plus  $c d$  multiplied with  $a b$  ok

so let us try to say understand this particular inequality

so let us try to draw a diagram though the points can it be distributed in any manner ok

so for the simplicity i am trying to assume something like they are not really lying in a line in fact let us say it basically gives some sort of shape

so let us call it as this is  $a b c d$  then this inequality says that you consider the length  $a d$  multiplied by the length  $b c$  this will be always less than or equal to  $b d$  the diagonal with the  $ac$  you multiply plus  $c d$  multiplied with the  $a b$  ok

so proof is which is simple just by using the previous identity let me just write down the identity

so what we have

so what we can do is once these points are kept in the plane then we can identify each vertices or the end points to a complex number

so let us say that this  $a$  is associated with the  $z_1$  and this is basically associated to  $z_2$  and  $c$  is associated to point  $z_3$  and let us say that this is associated with the  $z_4$  then the previous identity says that  $z_1 - z_4 - z_2 - z_3$  that is equal to  $z_1 - z_3 - z_2 - z_4$  multiplied with  $z_3 - z_4$

so it is just rewritten in a way that I get the term the  $a$  to  $d$  that is  $z_1$  to  $z_4$  and  $b$  to  $c$  which is  $z_2$  to  $z_3$  when I take the absolute value for this identity then the modulus of  $z_1 - z_4$  that gives the magnitude of this vector that gives the length that is the  $a$   $d$

so now take the modulus sign to this identity then the modulus of  $z_1 - z_4$  product with modulus of  $z_2 - z_3$  and now you have modulus of the entire term and apply the triangle inequality we get that  $z_1 - z_3 - z_2 - z_4$  plus  $z_1$  modulus of  $z_1 - z_2$  modulus of  $z_3 - z_4$  now we see that this describes the length of  $a$   $d$  multiplied with  $z_2 - z_3$  that is length of  $b$   $c$  which is less than or equal to  $z_1 - z_3$  which is the length of  $ac$  multiplied by  $z_2 - z_4$  which is  $b$   $d$  plus  $z_1 - z_2$  which is  $a$   $b$  multiplied with  $c$   $d$  ok

so we are able to derive the desired inequality using this identity now let us do a one more problem

so before going to the problem let me just mention that  $z$  is said to be unimodular  $z$  is said to be unimodular if  $\text{mod } z$  is one just a terminology we say that a complex number is unimodular if modulus of  $z$  is one which means the  $z$  is lying on the unit circle problem

so let  $z_1$   $z_2$  to be complex numbers and suppose the number  $z_1 - z_2$  divided by  $2 - z_1 - z_2$  bar is unimodular and  $z_2$  is not unimodular then we need to choose the one of the following choice the point  $z_1$  lies on a it is lying on a straight line parallel to  $x$  axis option b straight line parallel to  $y$  axis option c whether it lies on a circle of radius two

so the meaning of this one modulus of  $z$  is whether  $z_1$  is two  $d$  whether the circle of radius root two ok

so we are given that  $z_1 - 2z_2$  divided by  $2 - z_1 - z_2$  bar is unimodular meaning is that the modulus of this number is one and modulus of  $z_2$  is not unimodular then we need to find out whether  $z_1$  lies in a straight line or a circle with certain option that whether the radius is two or root two if at all lies in a circle let us try to derive what happens with the conditions to  $z_1$

so let us try to see that the given point is modulus of  $z_1 - 2z_2$  divided by  $2 - z_1 - z_2$  bar modulus is one and modulus of  $z_2$  is not equal to one these are the given assumptions and with the first assumption we can see that modulus of  $z_1 - 2z_2$  square which is same as modulus of  $2 - z_1 - z_2$  bar square and we know that  $\text{mod } z$  square is same as  $z$  into  $z$  bar we just write down that this implies  $z_1 - 2z_2$  multiplied by  $z_1 - 2z_2$  bar this is same as  $2 - z_1 - z_2$  bar multiplied by  $2 - z_1 - z_2$  bar the whole bar

so this implies that  $z_1 - z_1 - 2z_2$  and multiplied with  $z_1$  bar two times  $z_2$  bar this is same as  $2 - z_1 - z_2$  bar two minus  $z_1$  bar and  $z_2$  and now just do the simple calculation we see that this is  $\text{mod } z_1$  square and the other factor is four times of  $\text{mod } z_2$  square and the remaining factor is minus  $z_1$  multiplied with us two times of  $z_1$  bar and minus two times of  $z_1$  bar multiplied with  $z_2$  the right hand side this is equal to four plus its mod square that is  $\text{mod } z_2$  bar square does not matter because the modulus of  $z$  bar is again modulus of  $z$  the remaining factors are minus two times of  $z_1$  bar  $z_2$  minus two times of  $z_1$   $z_2$  bar we see that these two factors are commonly appears they cancel

so we get the following equation that  $|z|^2 = 1$  square plus  $4|z|^2$  times  $|z|^2$  minus  $4|z|^2$  minus  $|z|^2$  the whole square this is equal to zero now we see that from this we can just say split this term as  $|z|^2$  multiplied by  $|z|^2$  square  $|z|^2$  square then the common factor if we try to bring out  $|z|^2$  square  $|z|^2$  square one minus  $|z|^2$  square and the remaining term which is if we take out minus 4 common then  $1 - |z|^2$  square this is equal to zero now we can write it as a product term ok which is easily see that it comes that  $1 - |z|^2$  square multiplied with  $|z|^2$  square minus four equal to zero and given assumption is  $|z|^2$  is not equal to one this immediately tells that this is a non zero term this is a non zero term

so immediately concludes that  $|z|^2 = 1$  and  $|z| = 1$  is two this immediately says that option c is correct that is  $|z| = 1$  lies on the circle of radius two ok

so if i summarize we discuss the more properties about modulus of complex numbers and using the properties of conjugation of complex number together with the modulus of complex number we derived certain problems and in the next lecture we discuss about argon plane and polar representation for complex numbers thank you you