welcome to the third and the last problem solving session on quadratic equations

so today we are going to solve some more problems and with this we will end our session on quadratic equations this is our question number 16.

here we have two quadratic equations x square minus px plus r equals to 0 and x square minus qx plus r equals to 0 let alpha beta be the solutions of the first quadratic equation and late alpha by 2 and 2 beta be the solutions of the second quadratic equation we have to find out what is the value of r since we are given that alpha and beta are the solutions of x square minus p x plus r equals to 0 we can write alpha plus beta is equal to p and also alpha into beta is equal to r since alpha by 2 and 2 beta are the solutions of x square minus q x plus r equals to 0 we can write alpha by 2 plus 2 beta is equal to q and alpha by 2 into 2 beta that means alpha beta is equal to r

so we have total three relations in alpha beta p q and r using these we shall find out what is the value of r note that we can write alpha by 2 plus 2 beta equals to q as alpha plus 4 beta is equal to 2 q now if we subtract alpha plus beta equals to p from this equation then we get 3 beta is equal to 2 q minus p that means we have beta is equal to 2q minus p divided by 3

so therefore alpha is equal to p minus beta that means p minus 2 q minus p divided by 3 which is 2 into 2 p minus q divided by 3 now recall that we had alpha into beta is equal to r

so therefore r is equal to 2 by 3 into 2p minus q into 2 q minus p divided by 3 that means 2 by 9 into 2 p minus q into 2q minus p

so we see here that the fourth option is the correct answer here in this question we are given the quadratic equation x square minus 5 x plus 3 equals to 0 we are told that alpha and beta are the solutions of this quadratic equation we have to then find out a quadratic equation which is having alpha by beta and beta by alpha as its solutions to do that we shall first note down alpha plus beta is equal to 5 and alpha into beta is equal to 3 we are getting these two from the quadratic equation x square minus 5 x plus 3 is equal to 0 as alpha and beta are the solutions of this quadratic equation now to construct a quadratic equation which is having alpha by beta and beta by alpha as its solutions we shall first find out what is alpha by beta plus beta by alpha let us write alpha by beta plus beta by alpha as alpha square plus beta square divided by alpha beta now we know that alpha square plus beta square is equal to alpha plus beta whole square minus 2 alpha beta

so alpha by beta plus beta by alpha is equal to alpha plus beta whole square minus 2 alpha beta divided by alpha beta now when we substitute the values of alpha plus beta and alpha beta we obtain this is equal to 25 minus 6 divided by 3 that is 19 divided by 3 and it is easy to note that alpha by beta into beta by alpha is equal to 1

so a quadratic equation having alpha by beta and beta by alpha as its solutions is x square minus 19 by 3 into x plus 1 is equal to 0 now if we multiply this equation by 3 we obtain 3x square minus 19x plus 3 is equal to 0.

so here the first option is correct and as we can see that none of the test equations are nonzero scalar multiples of the first equation all those three options are not correct here let p and q be two real numbers such that p is not equal to 0 and p cube is not equal to plus minus q if alpha and beta are two nonzero complex numbers such that alpha plus beta is equal to minus p and alpha cube plus beta cube is equal to q then we shall find out a quadratic equation whose solutions are alpha by beta and beta by alpha recall that a quadratic equation having alpha by beta and beta by alpha as its solutions would be x square minus alpha by beta plus beta by alpha into x plus alpha by beta into

beta by alpha which is 1 is equal to 0

so to write down such a equation we have to find out what is alpha by beta plus beta by alpha note that alpha by beta plus beta by alpha is equal to alpha square plus beta square divided by alpha beta

so this can be written as alpha plus beta whole square minus 2 alpha beta divided by alpha beta

so to write down the quadratic equation explicitly we have to know what is alpha into beta to find out alpha into beta let us compute this alpha plus beta whole cube is equal to alpha cube plus beta q plus 3 alpha beta into alpha plus beta we are given the value of alpha q plus beta cube and also the value of alpha plus beta

so therefore we have here minus p cube is equal to q minus 3 alpha beta into p

so therefore we have alpha beta is equal to pq plus q divided by 3p therefore alpha by beta plus beta by alpha is equal to p square minus 2 into p cube plus q divided by 3 p this whole divided by p cube plus q divided by 3 p now if we simplify this we obtain 3pq minus 2pq minus 2 q divided by p q plus q this is nothing but p cube minus 2 q divided by p cube plus q therefore a quadratic equation having alpha by beta and beta by alpha as its solutions would be x square minus p cube minus 2 q divided by p cube plus q into x plus 1 is equal to 0 now if we multiply this equation by p cube plus q we obtain p cube plus q into x square minus p cube minus 2 cube into x plus p cube plus q is equal to zero

so the second option is correct here and as all the rest three options are containing quadratic equations which are not scalar multiples of the quadratic equation given in option two

so they are not correct in this question we are given the quadratic equation x square minus 6x minus 2 is equal to 0 and late alpha and beta be the solutions of the given quadratic equation with alpha being strictly bigger than beta if a n is equal to alpha to the power n minus beta to the power n for all natural numbers in bigger than or equal to 1 then we shall find out what is the value of 18 minus 2 a 8 whole divided by 2 a 9 since we are given that alpha and beta are the solutions of x square minus 6 x minus 2 is equal to 0 we can write alpha square minus 6 alpha minus 2 is equal to 0 and beta square minus 6 beta minus 2 is equal to 0 now let us try to find out what is 8 n minus 2 a 8 whole divided by 2 a 9

so i write it here a 10 minus 2 a 8 whole divided by 2 a 9 is equal to alpha to the power 10 minus beta to the power 10.

minus 2 alpha to the power 8 plus 2 beta to the power 8 whole divided by 2 alpha to the power 9 minus 2 beta to the power 9 and this whole expression is equal to alpha to the power 8 into alpha square minus 2 minus beta to the power 8 into beta square minus 2 and then in the denominator we have 2 into alpha to the power 9 minus beta to the power 9 now note that here we had alpha square minus 6 alpha minus 2 is equal to 0

so that means alpha square minus 2 is equal to 6 alpha and from the second equation we get beta square minus 2 is equal to 6 beta now we substitute alpha square minus 2 as 6 alpha and beta square minus 2 as 6 beta in this expression and we obtain this is 6 alpha to the power 9 minus 6 beta to the power 9 divided by 2 into alpha to the power 9 minus beta to the power 9 and this is equal to 6 divided by 2 that means 3

so therefore we have the option 3 is correct and hence rest all the options are not correct this is our question number 20.

in this question basically we have two questions question a and question b

so let p and q be two integers and alpha and beta be the solutions of the quadratic equation x square minus x minus 1 is equal to 0 with alpha being not equal to beta let a n b p alpha to the power n plus q beta to the power n for

all integers n bigger than or equal to zero before we read out the question we shall keep this fact in mind if a b are rational numbers such that a plus b square root of 5 is equal to 0 that means both of a and b are equal to 0.

our first question is which is question a that is we have to find out the value of a 12

so we write down what is a 12 this is question a a 12 is equal to p into alpha to the power 12 plus q into beta to the power 12 let us write it as p into alpha to the power 10 into alpha square plus q into beta to the power 10 into beta square we know that alpha and beta are the solutions of the quadratic equation x square minus x minus 1 equals to 0

so therefore alpha square minus alpha minus 1 is equal to 0 that means alpha square is equal to alpha plus 1 and similarly theta square minus beta minus 1 is equal to 0

so we have beta square is equal to beta plus 1 now substituting alpha square as alpha plus 1 and beta square as beta plus 1 in this expression we get a 12 is equal to p alpha to the power 10 into alpha plus 1 plus q beta to the power 10 into beta plus 1.

and this is equal to p into alpha to the power 11 plus p into alpha to the power 10 plus q into beta to the power 11 plus q into beta to the power 10 taking these two parts together we can write this is equal to a11 and taking this part and this part together we can write this is equal to 18

so a 12 is equal to a 11 plus 18

so in question a option 2 is correct and rest of the options are not correct now we come to question b

so if a4 is 28 then we have to find out what is the value of p plus 2 q we have here a 4 is equal to 28 if we proceed in the similar manner as in the solution of question a we would get a 4 is equal to a 3 plus a 2 because a 4 is equal to t alpha to the power 4 plus q beta to the power 4 and again we will write this as p alpha square and alpha square we would substitute it as alpha plus 1 and here we would write q into beta square and beta square we would substitute as beta plus 1

so you would get p alpha cube plus p alpha square plus q beta cube plus q beta square

so again from these two we'll write it as a3 and from these two we'll write it as a2 in fact any a n is equal to a n minus 1 plus a n minus 2 for all natural numbers n bigger than or equal to 2 and we can note that a0 is equal to p plus q

so we have here a 4 is equal to again a 2 plus a 1 plus a 1 plus a 0

so this is equal to again a 1 plus a 0 plus 2 a 1 plus a 0.

so finally we have 3 a 1 plus 2 a 0.

note that a 1 is equal to b alpha plus q beta

so therefore we have a 4 is equal to 3 p alpha plus 3 q beta plus 2 p plus 2 q and this is also equal to 28 as a 4 is given to be 28 from here now we will find out what is p plus 2 q let us first find out what is alpha and beta our quadratic equation was x square minus x minus 1 is equal to 0

so the solutions of this quadratic equation are 1 plus minus square root of 5 divided by 2 without loss of generality let alpha be equal to 1 plus square root of 5 divided by 2 and beta be equal to 1 minus square root of 5 divided by two we substitute the values of alpha and beta in the equation 3 p alpha plus 3 q beta plus 2 p plus 2 q is equal to 28 note that this equation is symmetric in p and q now after substituting the values of alpha and beta we get this is 3p into 1 plus square root of 5 divided by 2 plus 3q into 1 minus square root of 5 divided by 2 plus 2 p plus 2 q is equal to 28 and simplifying this we obtain 3p plus 3p into square root 5 plus three q minus three q into square root five plus

four p plus four q is equal to 56 now we shall use the fact that if a and b are rational numbers such that a plus b square root 5 is equal to 0 then a is equal to b is equal to 0 therefore 3 p minus 3 q is equal to 0 that implies p is equal to q and also we get here that 7p plus 7 q is equal to 56 from these terms this implies p plus q is equal to 8 and as p is equal to q we get p is equal to q is equal to 4 therefore we can say that p plus 2 q is equal to 4 plus 8 is equal to 12.

so here in question b the fourth option is correct in this question we have the quadratic equation x square minus x minus 1 is equal to 0 and let alpha and beta be the solutions of this quadratic equation with alpha being strictly bigger than beta let a n b alpha to the power n minus beta to the power n divided by alpha minus beta for all integers n bigger than or equal to one we take v1 to be equal to 1 and let bnb a n minus 1 plus a in plus 1 for all integers n bigger than or equal to 2 then we have to find out among all the four options given here what are correct before we proceed to find out what are all the correct options let us first compute what is alpha and what is beta our quadratic equation is x square minus x minus 1 is equal to 0
so therefore the solutions of this quadratic equation are 1 plus minus square root of 5 divided by 2 and since it is given to us that alpha is strictly bigger than beta we can write alpha is equal to 1 plus square root of 5 divided by 2 and beta is equal to 1 minus square root of 5 divided by 2 we shall first say whether option 2 is correct or not we have bn is equal to a n minus 1 plus a n plus 1 for all in bigger than or equal to 2 and we know that a n minus 1 is equal to alpha to the power n minus 1 minus beta to the power n minus 1 divided by alpha minus beta plus we know a n plus 1 is equal to alpha to the power n plus 1 minus beta to the power n plus 1 divided by alpha minus beta
so therefore b n is equal to if we take alpha to the power n minus 1 common we get alpha square plus 1 and here we take minus beta to the power n minus 1 common and then we have inside beta square plus 1 and in the denominator we have alpha minus beta now since we have alpha and beta here we can find out easily what is alpha minus beta alpha minus beta is equal to square root of 5 also we know that alpha and beta are the solutions of the quadratic equation x square minus x minus 1 is equal to 0 therefore alpha squared minus alpha minus 1 is equal to 0
so we can write alpha square plus 1 is equal to alpha plus 2 similarly we can write from beta square minus beta minus 1 is equal to 0 that beta square plus 1 is equal to beta plus 2.
now as we know what is alpha and what is beta we can explicitly find out what is alpha square plus 1
so alpha plus 2 is equal to 1 plus square root of 5 divided by 2 plus 2 and this is equal to 5 plus square root of 5 divided by 2 and if we take square root of 5 out we get square root of 5 plus 1 divided by 2
so basically alpha square plus 1 is equal to square root of 5 into alpha similarly beta plus 2 is equal to 1 minus square root of 5 divided by 2 plus 2 and this is equal to 5 minus square root of 5 divided by 2
so this is equal to square root of 5 into square root of 5 minus 1 divided by 2 that means minus square root of 5 into beta
so substituting alpha square plus 1 is equal to square root of 5 into alpha and beta square plus 1 is equal to minus square root of 5 into beta we get bn is equal to alpha power n minus 1 into square root of 5 into alpha plus beta to the power n minus 1 into square root of 5 into beta divided by as we know alpha minus beta is equal to square root of 5 we substitute it here
so this is equal to alpha to the power n plus beta to the power n

so we have got here that b_n is equal to alpha to the power n plus beta to the power n for all n bigger than or equal to 2 now here we can also note that b_1 is equal to 1 and alpha plus beta is equal to 1

so therefore we can say that option 2 is correct now we shall check option 3.

write the infinite series summation b_n divided by 10 to the power n and n runs over the set of all integers bigger than or equal to 1 now using the expression which we have already got we can write b_n is equal to alpha to the power n plus beta to the power n and this is 10 to the power n now since sum of alpha to the power n divided by 10 to the power n and sum of beta to the power n divided by 10 to the power n both of them converge we can split this

so we write it in this way note that are geometric sums

so therefore this is alpha divided by 10 divided by 1 minus alpha divided by 10 plus this is beta divided by 10 divided by 1 minus beta divided by 10

so simplifying we get alpha by 10 minus alpha plus beta by 10 minus beta

so taking lcm of the denominators we get 10 minus alpha into 10 minus beta and this is alpha into 10 minus beta plus beta into 10 minus alpha

so this is 10 into alpha plus beta minus 2 alpha beta divided by 100 minus 10 into alpha plus beta plus alpha beta now we already know what is alpha plus beta we have seen that alpha plus beta is equal to 1 and we shall note alpha into beta is equal to minus 1 we can note that easily from the given quadratic equation

so therefore we have from here summation n bigger than or equal to 1 b_n divided by 10 to the power n is equal to 10 minus 2 into minus 1

so this is plus 2 divided by 100 minus 10 minus 1

so this is nothing but 12 divided by 89.

we see here that option 3 is not correct now we shall check option 4.

we consider sum a_n divided by 10 to the power n and n is bigger than or equal to 1 now a_n is equal to alpha to the power n minus beta to the power n divided by alpha minus beta

so we substitute it here and

so this is equal to 1 by alpha minus beta into sum of alpha to the power n divided by 10 to the power n minus 1 by alpha minus beta into sum of beta to the power n divided by 10 to the power n

so this is equal to 1 by alpha minus beta into alpha by 10 minus alpha minus beta by 10 minus beta simplifying this we obtain 1 by alpha minus beta into this is 10 alpha minus alpha beta minus 10 beta plus alpha beta divided by 100 minus 10 into alpha plus beta plus alpha beta

so therefore we have here 1 by alpha minus beta into 10 into alpha minus beta divided by 100 minus 10 into alpha plus beta plus alpha beta we know alpha minus beta is square root of 5 and alpha plus beta is equal to 1 and alpha beta is equal to minus 1

so from here we get that the summation n bigger than or equal to 1 a_n divided by 10 to the power n is equal to 10 divided by 100 minus 10 minus 1

so this is equal to 10 divided by 89

so we see that option 4 is correct now we shall check option 1.

before we do that we shall first write down a recurrence relation for a_n we have b_n is equal to a_{n-1} plus a_{n+1} and a_{n-1} is nothing but alpha to the power n minus 1 minus beta to the power n minus 1 divided by alpha minus beta and this is alpha to the power n plus 1 minus beta to the power n plus 1 divided by alpha minus beta

so all together we have if we take alpha to the power n minus 1 common we get alpha square plus 1 and from here if we take minus beta to the power n minus 1 common we get beta square plus 1 and in the denominator we have alpha minus beta recall that we have already got alpha square plus 1 is equal to alpha plus 2 and

also beta square plus 1 is equal to beta plus 2 we can note it here therefore we have this is alpha to the power n minus 1 into alpha plus 2 minus beta to the power n minus 1 into beta plus 2 divided by alpha minus beta

so this is equal to alpha to the power n minus beta to the power n divided by alpha minus beta plus 2 into alpha to the power n minus 1 minus beta to the power n minus 1 divided by alpha minus beta

so finally we are getting that b_n is equal to a_{n+2} into a_{n-1} also recall that we had b_n is equal to a_{n-1} plus a_{n+1} therefore we have a_{n-1} plus a_{n+1} is equal to a_{n+2} into a_{n-1}

so we have a_{n+1} is equal to a_n plus a_{n-1} this is a recurrence relation of a in which is going to be useful for us to check whether option 1 is correct or not we start with a_{n+2} and we are going to use the recurrence relation which we have got let us write a_{n+2} is equal to a_{n+1} plus a_n now we keep the part a in intact and then for a_{n+1} we use the recurrence relation and we write this is equal to a_n plus a_{n-1} and here we have a in now we keep the part a_{n-1} plus a in intact and then for a in we use the recurrence relation and we get this is equal to a in minus 1 plus a in minus 2 and here already we have a_{n-1} plus a in then next we keep this part in tag and we use the recurrence relation for a_{n-1} and continuing this way we obtain a_{n+2} is equal to a_2 plus a_1 plus a_2 up to

so on and

so forth a in now note that a_2 is equal to alpha square minus beta square divided by alpha minus beta

so this is equal to alpha plus beta and we already know that alpha plus beta is equal to 1

so therefore we have a_{n+2} is equal to 1 plus a_1 plus a_2 up to a in that means a_1 plus a_2 up to a_n is equal to a_{n+2} minus 1

so we see that option 1 is also correct this is our question number 22 we have here two quadratic equations x square plus ax plus b is equal to 0 and x square plus bx plus a is equal to 0 for a not equals to b if these two quadratic equations are having a common solution then we shall find out what is a plus b let us assume alpha to be a common solution of the given two quadratic equations therefore we have alpha square plus a alpha plus b is equal to 0 and also alpha square plus b alpha plus a is equal to 0.

if we subtract the second equation from the first one then we obtain alpha into a minus b plus b minus a is equal to 0 from here we obtain that alpha into a minus b is equal to a minus b now since a is not equal to b a minus b is non-zero therefore we can cancel a minus b from both sides and we obtain alpha is equal to 1 and then we substitute alpha is equal to 1 in this equation and we obtain 1 plus b plus a is equal to 0 that means a plus b is equal to minus 1

so therefore option 3 is correct and rest of the options are not correct in this question we are given two quadratic equations x square plus bx minus 1 is equal to 0 and x square plus x plus b is equal to 0 we have to find out for what values of b these two quadratic equations are having a common solution let us assume that alpha is a common solution for the given two quadratic equations therefore we can write alpha square plus b alpha minus 1 is equal to 0 and alpha square plus alpha plus b is equal to 0.

and when we subtract one equation from the other we obtain alpha into b minus 1 is equal to b plus 1 we shall note here that b is not equal to 1 because if b is equal to 1 then x square plus x minus 1 is equal to 0 and x square plus x plus 1 is equal to 0 these two are the quadratic equations mentioned in our question and we shall note that these two equations are not having any common solution

so therefore p is not equal to 1 and we can write alpha is equal to b plus 1 divided by b minus 1 and now since here alpha square plus b alpha minus 1 is

equal to 0 we have alpha square is equal to 1 minus b into alpha now we substitute the value of alpha here we get alpha square is equal to 1 minus b into b plus 1 divided by b minus 1 which is equal to b minus 1 minus b square minus b divided by b minus 1 that means minus of 1 plus b squared divided by b minus 1

 so let us write it as 1 plus b square divided by 1 minus b on the other hand we have alpha square is equal to b plus 1 divided by b minus 1 whole square
 so equating these two we get b square plus 2 b plus 1 is equal to 1 minus b into 1 plus t square and if we split this we get 1 minus b plus b square minus b cube

 so finally we have b cube plus three b is equal to zero therefore b into b square plus 3 is equal to 0 from here we get b is equal to 0 or b square plus 3 is equal to 0 and b is equal to 0 is not there in the options given here
 so the other possibility is b square plus 3 is equal to 0 that means b square is equal to minus 3 this implies b is equal to plus minus square root of 3 i therefore here we can see that option 1 and option 3 are correct i end here with this we conclude our problem solving session on quadratic equations you