

welcome to the second problem solving session on quadratic equations in the first problem solving session we have solved total six problems

so today we will start with the seventh problem this is our seventh question here we have the equation  $(x - \text{integral part of } x)^2 + 2(x - \text{integral part of } x) + a = 0$  for a real number  $a$  we are given the information that this equation is not having any integer solution we then have to find out the possible range of  $a$  first we can note that  $a$  is not equal to  $0$  because if  $a$  is equal to  $0$  then clearly every integer is a solution of this equation because for  $x$  in  $\mathbb{Z}$  we know  $x - \text{integral part of } x$  which is the fractional part of  $x$  is equal to  $0$

so putting  $a$  is equal to  $0$  we can see that every integer is a solution of this equation therefore we have here  $a$  is not equal to  $0$ .

now we substitute  $x - \text{integral part of } x$  is equal to fractional part of  $x$  in the given equation and then we obtain  $(\text{fractional part of } x)^2 + 2(\text{fractional part of } x) + a = 0$

so we are getting a quadratic equation in fractional part of  $x$  now since we always want to keep the coefficient of the 2 degree term of any real quadratic equation to be always positive we write it as  $3(\text{fractional part of } x)^2 - 2(\text{fractional part of } x) + a = 0$  now solving this equation we will get possible choices for fractional part of  $x$  and they are  $\frac{2 \pm \sqrt{4 - 12a}}{6}$  after simplifying we get this is equal to  $\frac{1 \pm \sqrt{1 - 3a}}{3}$  now note that  $1 - 3a$  is strictly bigger than  $0$  as we have  $a$  is not equal to  $0$

so therefore we have  $\frac{1 - \sqrt{1 - 3a}}{3}$  is strictly less than  $0$  but we know that fractional part of  $x$  is always bigger than or equal to  $0$

so therefore this cannot be a possible choice for fractional part of  $x$  we can see that  $\frac{1 + \sqrt{1 - 3a}}{3}$  is strictly bigger than  $0$

so therefore if fractional part of  $x$  is equal to this then we can write  $\frac{1 + \sqrt{1 - 3a}}{3}$  is strictly less than  $1$  and we know this is also strictly bigger than  $0$

so from here we get that  $1 - \sqrt{1 - 3a}$  is strictly less than  $3$  and this is strictly less than  $2$  now squaring the inequality we obtained  $1 - 3a$  is strictly less than  $1 - 3a + 3a$  this is strictly less than  $4$  therefore we obtain  $0$  is strictly less than  $3a$  and  $3a$  is strictly less than  $1$

so this is a possible range of  $a$  and from here we get that  $a$  belongs to the open interval  $(-\frac{1}{3}, 0) \cup (0, \frac{1}{3})$

so option 3 is correct also note that here the open interval  $(-\frac{1}{3}, 0) \cup (0, \frac{1}{3})$  is a subset of the open interval  $(-\frac{2}{3}, \frac{1}{3})$  and as we are to find out all the possible sets where the values of  $a$  lie in therefore we see that the option 1 is also correct and since the option 2 and option 4 are disjoint from option 3 that is the set  $(-\frac{1}{3}, 0) \cup (0, \frac{1}{3})$  we can directly conclude that option 2 and option 4 are not correct for non-negative real numbers  $x$  let us consider two functions  $g(x)$  which is given by  $\cos(x)$  and the function  $f(x)$  which is given by  $\sqrt{x}$  we are given here one quadratic equation  $18x^2 - 9\pi x + \pi^2 = 0$  and two solutions of this quadratic equation are  $\alpha$  and  $\beta$

so that  $\alpha$  is strictly less than  $\beta$  we have to find out the area bounded by the curve  $y = g(x)$  and the lines  $x = \alpha$ ,  $x = \beta$  and  $y = 0$  to do this our first job would be to find

out what is alpha and what is beta let us do that the solutions of the quadratic equation are  $9\pi \pm \sqrt{81\pi^2 - 72\pi}$  divided by 36 after simplifying we can write  $9\pi \pm 3\pi$  divided by 36 now as we know alpha is strictly less than beta we can write alpha is equal to  $9\pi - 3\pi$  divided by 36 which means  $\pi/6$  and beta is  $9\pi + 3\pi$  divided by 36 which means  $\pi/3$  now note that the function  $g \circ f$  of  $x$  is nothing but the function cosine of  $x$  now to find out the area bounded by all those four curves we shall try to draw a picture let this be our  $x$  axis and this be our  $y$  axis let us write this is  $x$  and this is  $y$  and then we draw the line  $x$  is equal to alpha and let this be the line  $x$  is equal to beta next we draw the graph of the function cosine of  $x$

so this point is  $\pi/2$  this point is  $\pi/3$  and this point is  $\pi/6$  this is  $y$  is equal to cosine of  $x$  this one is  $x$  is equal to alpha and this one is  $x$  is equal to beta we have to find out the area of the shaded region as it is very clear from the picture we have to find out the integral from  $\pi/6$  to  $\pi/3$  of cosine  $x$   $dx$  after integrating we get sine  $x$  from  $\pi/6$  to  $\pi/3$  and solving this we obtain sine of  $\pi/3$  minus sine of  $\pi/6$  which is square root of 3 by 2 minus half if we write it in a composite form this is square root of 3 minus 1 divided by 2.

therefore we see that here the fourth option is the correct answer in this question we are given a region  $r$  which contains all pairs of real numbers  $x$  comma  $y$  whose  $y$  coordinate lies between  $x^3$  and  $x$  and whose  $x$  coordinate lies between 0 and 1.

we are also given a real number alpha in the closed interval  $[0, 1]$

so that the line  $x$  equals to alpha divides the region  $r$  into two equal parts then we are given here four options we have to find out what are all the conditions satisfied by alpha to solve this we shall try to first draw the region  $r$  let this be our  $x$  axis and this be our  $y$  axis next we draw the graph of  $y$  is equal to  $x^3$  then we draw the graph of  $y$  is equal to  $x$  i can erase this okay

so this is the point one this is  $y$  is equal to  $x^3$  and this is  $y$  is equal to  $x$  and this is 0

so the region i am shading is the region  $r$  now alpha is a real number between 0 and 1 and the line  $x$  equals to alpha divides the region  $r$  into two equal parts

so let the line  $x$  equals to alpha be this

so this point is alpha and this is the line  $x$  equals to alpha we call this region as region a and this region as region b

so we know that the area of the region a is equal to the area of the region b let us first compute the area of the region a it is very clear from the picture that this is integral from 0 to alpha  $x$   $dx$  minus integral from 0 to alpha  $x^3$   $dx$  after integrating we obtain  $x^2$  divided by 2 minus  $x^4$  divided by 4 and we have to evaluate this from 0 to alpha and finally we get alpha square divided by 2 minus alpha to the power 4 divided by 4.

next we will compute the area of the region b and there the limiting value will be from alpha to 1.

so we do that now this is alpha  $\int_1^\alpha x$   $dx$  minus alpha  $\int_1^\alpha x^3$   $dx$

so we have here  $x^2$  divided by 2 minus  $x^4$  divided by 4 and we evaluate it from alpha to 1

so we have  $1/4$  minus alpha square divided by 2 minus alpha to the power 4 divided by 4 now as we know the area of the region a and the area of region b are equal then we can equate them and we obtain i can write it here

so the area of region a was alpha square divided by 2 minus alpha to the power 4 divided by 4 and here we have obtained  $1/4$  minus alpha square divided by 2

plus alpha to the power 4 divided by 4 simplifying this we obtain alpha to the power 4 divided by 2 minus alpha square plus 1 by 4 is equal to 0 now if we multiply this equation by 4 then we obtain 2 alpha to the power 4 minus 4 alpha square plus 1 is equal to 0

so we are getting a quadratic equation in alpha square now if we look at the options here we can see that third option is correct that alpha satisfies the third condition now we shall check also rest of the options we have already obtained a quadratic equation in alpha square i write the equation here again 2 alpha to the power 4 minus 4 alpha square plus 1 is equal to 0.

by solving this we get possible choices for alpha square and they are 4 plus minus square root of 16 minus 8 divided by 4 which is 1 plus minus square root of 2 divided by 2 now we can note that 1 plus square root of 2 divided by 2 is strictly bigger than 1 therefore this cannot be a possible choice for alpha squared as we have alpha is less than equal to 1

so we have alpha square is equal to 1 minus square root of 2 divided by 2 now if alpha is less than or equal to half then alpha square is less than or equal to 1 by 4 therefore we get 1 minus square root of 2 divided by 2 is less than or equal to 1 by 4 and from here we obtain that 3 by 4 is less than or equal to square root of 2 divided by 2 now squaring this we obtain 9 by 16 is less than or equal to 2 by 4 that means half but we know this is not possible therefore alpha cannot be less than or equal to half hence the option one is not correct and

so therefore option two has to be correct

so now only we have the option 4 to check we can see that here the option 4 is a quadratic equation in alpha square if we solve this for alpha square we get alpha square is having the possible choices minus 4 plus minus square root of 16 plus 4 divided by 2 that means minus 2 plus minus square root of 5 and as we can clearly see that none of these values agree with the choice of alpha square which we have already got

so therefore the fourth option is not correct this is our tenth question for a positive integer n let us consider the quadratic equation  $x^2 + x + 1 + x + 1 + x + 2 + \dots + x + n - 1 + x + n$  is equal to  $10n$  we have to find out what is the value of n for which this quadratic equation is having two consecutive integer solutions now note that in the left hand side of this quadratic equation there are total in many summands

so we can see that the coefficient of x square is n now we have to find out what is the coefficient of x for that let us first split the first command and we get x square plus x if we split the second sum on we get x square plus x plus 2x plus 2 which is basically x square plus 3x plus 2 and then we split the last summand and then we get x square plus n minus 1 into x plus n x plus n into n minus 1

so here we get x square plus 2 n minus 1 into x plus n into n minus 1

so therefore the coefficient of x is equal to 1 plus 3 plus up to 2n minus 1 now if we sum it for summing it let us add 2 plus 4 up to 2n and then we subtract what we have already added 2 plus 4 plus

so on

so forth up to 2n

so this is nothing but 2 n into 2 n plus 1 divided by 2 minus if we take 2 out here then this is n into n plus 1 divided by 2

so finally we are getting here n into 2 n plus 1 minus n into n plus 1

so this is 2 n square plus n minus n square minus n

so we are getting this is n square

so the coefficient of x in this quadratic equation is n square now we find out the constant term from the first summand that is from x into x plus 1 the

contribution to the constant term is 0.

and from the second summand that is  $x + 1$  into  $x + 2$  the contribution to the constant term is 2 for better understanding we write the third term which is  $x + 2$  into  $x + 3$  this term contributes 6 to the constant term now the last summand is  $x + n - 1$  into  $x + n$  from here we can see that this term contributes  $n - 1$  into  $n$  to the constant term and also in the right hand side we have  $10n$

so all together we have we write it here  $0 + 2 + 6 + \dots + n$  into  $n - 1$  and we had  $10n$  in the right hand side

so this is  $10n$  now note that this part is already in the form sum of  $k$  into  $k - 1$   $k$  is from 1 to up to  $n$  and this is  $10n$  if we split this one we obtain  $k^2$   $k$  is from 1 2 up to  $n$  and this is  $k(k - 1)$  up to  $n$  and this is  $10n$

so therefore this is  $n$  into  $n + 1$  into  $2n + 1$  divided by 6 this is  $n$  into  $n + 1$  divided by 2 and this is  $10n$  now after simplifying we get let us first take  $n$  into  $n + 1$  divided by 2 out this is  $2n + 1$  divided by 3 minus 1 and this is  $10n$

so therefore we have here  $n$  into  $n + 1$  into  $n - 1$  divided by 3 minus  $10n$

so therefore this is  $n$  into  $n^2 - 1$  divided by 3 minus  $10n$  in now if we write down the quadratic equation in a simplified form we obtain  $n^2x + n^2x$  and the constant term is  $n$  into  $n^2 - 1$  divided by 3 minus  $10n$  is equal to 0 as  $n$  is a positive integer we have  $n$  is not equal to 0

so we can cancel  $n$  from this equation and we have  $x^2 + nx + n^2 - 1$  divided by 3 minus  $10$  is equal to 0

so our quadratic equation is  $x^2 + nx + n^2 - 31$  divided by 3 is equal to 0 we have that this equation is having two consecutive integer solutions let's say  $m$  and  $m + 1$

so therefore we have  $m + m + 1$  is equal to  $-n$

so we have  $2m + 1$  is equal to  $-n + 1$  and

so we have  $m$  is equal to  $-n + 1$  divided by 2 and also we have  $m(m + 1)$  is equal to  $n^2 - 31$  divided by 3 now we substitute the value of  $m$  which we have got in this equation and then we obtain  $-n + 1$  divided by 2 into  $1 - n + 1$  divided by 2 is equal to  $n^2 - 31$  divided by 3 after simplifying we get  $-n + 1$  into  $1 - n$  divided by 4 is equal to  $n^2 - 31$  divided by 3 and this one is nothing but  $n^2 - 1$  divided by 4 and the right hand side is  $n^2 - 31$  divided by 3

so therefore we have  $3n^2 - 3$  is equal to  $4n^2 - 124$  and solving this we get  $n^2$  is equal to 121 and as  $n$  is a positive integer we can conclude from here that  $n$  is equal to 11

so therefore the third option is correct here here in this question we consider a real quadratic equation  $px^2 = 0$  we are told that this equation is having only purely imaginary solutions that means the solutions are of the form  $i\alpha$  where  $\alpha$  belongs to the set of real numbers then we consider the equation  $p$  of  $px^2 = 0$  we have to find out what are all the correct information about the solutions of  $p$  of  $px^2 = 0$ .

let us recall that a real quadratic equation is of the form  $ax^2 + bx + c = 0$  where  $a$  is strictly positive and the solutions are of the form  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$  now since it is given to us that the solutions are purely imaginary then we can conclude that  $b^2 - 4ac$  is strictly less than 0 because since we know that the solutions are complex from there we first conclude that  $b^2 - 4ac$  is strictly less than 0 and then next we conclude that  $b$  is equal to 0 as the solutions are purely imaginary because if  $b$  is nonzero then from here we can see

that  $b$  would contribute to the real part of the solutions

so therefore we have  $b$  is equal to zero

so we have here  $ac$  is strictly bigger than  $0$  that means we have  $a$  and  $c$  both of them have same signs now we write what is our first equation  $px$  is equal to  $0$

so this is  $a x^2 + c$  is equal to  $0$  from here we can write this in the form  $x^2 + \frac{c}{a}$  is equal to  $0$  let us call  $\frac{c}{a}$  as some constant  $c'$  and since  $a$  and  $c$  both are having same signs we can conclude that  $c'$  is strictly bigger than zero now let us write what is  $p$  of  $px$  is equal to  $0$

so this is nothing but  $x^2 + c'$  whole square plus  $c'$  is equal to  $0$  now let us split this part after splitting we get  $x^4 + 2x^2 c' + c'^2$  is equal to zero note that this is a quadratic equation in  $x^2$

so let us solve this for  $x^2$  after solving the equation we obtain  $\frac{-2c' \pm \sqrt{4c'^2 - 4c'^2}}{2}$  are the possible choices for  $\beta^2$  where  $\beta$  is a solution of  $px$  is equal to  $0$  and now simplifying this we obtain  $\frac{-2c' \pm 0}{2}$

so we see that solutions of  $px$  is equal to  $0$  are neither real nor purely imaginary because if  $\beta$  is of the form say  $i\alpha$  or  $\beta$  is of the form  $\alpha$  where  $\alpha$  is real then we get  $\beta^2$  is equal to  $-\alpha^2$  or  $\beta^2$  is equal to  $\alpha^2$  but we have already got here that  $\beta^2$  is not real

so therefore the fourth option which says neither real nor purely imaginary solutions is correct and immediately looking at all other options we can say that rest three options are not correct this is our question number 12.

here we have four distinct numbers  $a, b, c$  and  $d$  we have two quadratic equations  $x^2 - 10cx - 11d$  is equal to  $0$  and  $x^2 - 10ax - 11b$  is equal to  $0$ .

we are told that  $a, b$  are the solutions of the first quadratic equation and  $c, d$  are the solutions of the second quadratic equation we have to find out what is the sum of  $a, b, c$  and  $d$  let us do that since we know that  $a, b$  are the solutions of  $x^2 - 10cx - 11d$  is equal to  $0$  we can write  $a + b$  is equal to  $10c$  and since  $c, d$  are the solutions of  $x^2 - 10ax - 11b$  is equal to  $0$  we can write  $c + d$  is equal to  $10a$

so therefore summing these two we obtain  $a + b + c + d$  is equal to  $10a + 10c$

so from here we see that to know the sum of these four numbers  $a, b, c$  and  $d$  it is enough to know the sum of  $a$  and  $c$  now since  $a$  is a solution of the first quadratic equation we can write  $a^2 - 10ca - 11d$  is equal to  $0$  and since  $c$  is the solution of the second quadratic equation we can write  $c^2 - 10ac - 11b$  is equal to zero now using these two we will try to get a relation in  $a$  and  $c$  if we subtract the second equation from the first one we obtain  $a^2 - c^2 - 11d + 11b$  is equal to  $0$  that means we are getting  $a^2 - c^2 + 11(b - d)$  is equal to  $0$  we will now find out what is  $b - d$  recall that we had  $a + b$  is equal to  $10c$  and  $c + d$  is equal to  $10a$

so from here we also get that  $a + b - c - d$  is equal to  $10c - 10a$  that means we have  $b - d$  is equal to  $10c - 11a$

so let us write it in this form  $b - d$  is equal to  $11a - 10c$  now we substitute this here and therefore we obtain  $a^2 - c^2 + 11(11a - 10c)$  is equal to  $0$  now we can cancel  $a^2 - c^2$  from both sides because we have  $a, b, c$  and  $d$  are all four distinct numbers

so  $a$  is not equal to  $c$  and therefore  $a - c$  is not equal to  $0$

so we get from here that  $a + c$  is equal to 121 and hence we have  $a + b + c + d$  is equal to  $10 + a + c$  is equal to  $10 + 121$

so therefore we have here the fourth option is the correct one this is our question number 13.

let  $S$  be the set of all nonzero real numbers  $\alpha$  such that the quadratic equation  $\alpha x^2 - x + \alpha = 0$  has two distinct real solutions  $x_1$  and  $x_2$  with the property that  $|x_1 - x_2|$  is strictly less than 1 we have to identify the possible subsets of the set  $S$  from the list given below for that we first recall that for a real quadratic equation  $ax^2 + bx + c = 0$  we know it has distinct real solutions if and only if  $b^2 - 4ac$  is strictly bigger than 0

so using this we get here one condition that  $1 - 4\alpha^2$  has to be strictly bigger than 0 that means  $4\alpha^2$  has to be strictly less than 1 that is  $\alpha^2$  has to be strictly less than  $\frac{1}{4}$  now as  $\alpha$  is non-zero we are given in the question we can conclude that  $\alpha$  has to be in the open interval  $(-\frac{1}{2}, \frac{1}{2})$  excluding the number zero now we use the second condition which is given to us that the distance between the solutions  $x_1$  and  $x_2$  is strictly less than one that means  $(x_1 - x_2)^2$  is strictly less than 1 in fact this is an if and only if condition now we can write  $(x_1 - x_2)^2$  as  $(x_1 + x_2)^2 - 4x_1x_2$  recall that our equation is  $\alpha x^2 - x + \alpha = 0$  therefore  $x_1 + x_2 = \frac{1}{\alpha}$  and  $x_1x_2 = \frac{\alpha}{\alpha} = 1$  substituting these two values in the inequality here we get  $1 - 4\alpha^2 < 1$  that means  $1 - 4\alpha^2 < 1$  that is  $\alpha^2 > \frac{1}{5}$  that is  $\alpha > \frac{1}{\sqrt{5}}$  or  $\alpha < -\frac{1}{\sqrt{5}}$  now we can write down the set  $S$  explicitly

so the set  $S$  is equal to  $(-\frac{1}{\sqrt{5}}, -\frac{1}{2}) \cup (\frac{1}{2}, \frac{1}{\sqrt{5}})$  hence clearly the set given in option 1 is a subset of the set  $S$  also the set given in option 4 is a subset of the set  $S$  but the sets given in option 2 and 3 are not subsets of  $S$  therefore here the first and the fourth options are correct let us now look at this question here we have  $p$  to be a nonzero number and then we have the quadratic equation  $px^2 + qx + r = 0$  with the property that  $p, q$  and  $r$  are in an arithmetic progression we are given that  $\alpha$  and  $\beta$  are the solutions of this given quadratic equation with the property that  $1 + \alpha + \beta = 4$  we have to find out the value of  $|\alpha - \beta|$

so since  $\alpha$  and  $\beta$  are the solutions of the given quadratic equation we can immediately write  $\alpha + \beta = -\frac{q}{p}$  and also  $\alpha\beta = \frac{r}{p}$  since  $1 + \alpha + \beta = 4$  from here we can conclude that  $\alpha + \beta = 3$  into  $\alpha\beta = \frac{r}{p}$  now we know that  $\alpha\beta = \frac{r}{p}$  therefore  $\alpha + \beta = 3$  into  $\frac{r}{p}$  now equating these two we obtain that  $-\frac{q}{p} = 3$  into  $\frac{r}{p}$  and as  $p$  is non-zero we can write  $q = -3r$

so we have got a relation in  $q$  and  $r$

so in the question we are told that  $p, q$  and  $r$  are in an arithmetic progression therefore we can write  $q = \frac{p + r}{2}$  that means  $-3r = \frac{p + r}{2}$  as we have got already that  $q = -3r$  this implies  $p = -9r$  and hence we have  $\frac{r}{p} = -\frac{1}{9}$  and hence therefore we have  $\alpha + \beta = 3$  into  $-\frac{1}{9}$  we have here  $\alpha + \beta = -\frac{3}{9} = -\frac{1}{3}$  we are going to

use this relation to find out what is modulus of alpha minus beta now we know that alpha minus beta whole square is equal to alpha plus beta whole square minus 4 into alpha beta and we know the value of alpha plus beta here if we substitute we get 16 by 81 and here if we substitute the value of alpha beta we get this whole thing 4 into alpha beta is equal to 4 into r divided by p which means minus 1 divided by 9 and hence we are getting this is 16 divided by 81 plus 4 divided by 9 and this is nothing but 52 divided by 81

so therefore we get alpha minus beta is equal to plus minus 2 square root of 13 divided by 9 that means modulus of alpha minus beta is equal to 2 square root of 13 divided by 9 and

so we see that the second option is correct let us look at this question now we have three real numbers a b and c

so that a is non-zero we have three quadratic equations  $a^2 x^2 + b x + c = 0$  and  $a^2 x^2 - b x - c = 0$  and  $a^2 x^2 + 2 b x + 2 c = 0$ .

we are told that alpha is a solution of the first quadratic equation and beta is a solution of the second quadratic equation with the property that  $0 < \alpha < \beta$  our job is to find out what are the properties satisfied by a solution of the third quadratic equation since alpha is a solution of the first quadratic equation we have  $a^2 \alpha^2 + b \alpha + c = 0$  and since beta is a solution of the second quadratic equation we have  $a^2 \beta^2 - b \beta - c = 0$  let us call  $f(x) = a^2 x^2 + 2 b x + 2 c$

so we have to find out the properties satisfied by a solution of  $f(x) = 0$ .

we shall first compute what is  $f(\alpha)$  and what is  $f(\beta)$   $f(\alpha) = a^2 \alpha^2 + 2 b \alpha + 2 c$  we can write it as  $a^2 \alpha^2 + b \alpha + c + b \alpha + c$  now note that we have  $a^2 \alpha^2 + b \alpha + c = 0$  as alpha is a solution of the first quadratic equation

so therefore  $f(\alpha) = b \alpha + c$  and since  $a^2 \alpha^2 + b \alpha + c = 0$

so from there we can write  $b \alpha + c = -a^2 \alpha^2$  which we substitute here  $b \alpha + c = -a^2 \alpha^2$  since  $a^2 \alpha^2$  is positive we get that  $f(\alpha)$  is strictly less than zero

so from here we can conclude that alpha is not a solution of the third quadratic equation

so therefore if we call a solution of the third quadratic equation by gamma then gamma is not equal to alpha that is we see that the third option gamma is equal to alpha is not correct now we compute what is  $f(\beta)$   $f(\beta) = a^2 \beta^2 + 2 b \beta + 2 c$  we can note that here  $a^2 \beta^2 = b \beta + c$  therefore substituting  $b \beta + c$  in place of  $a^2 \beta^2$  we get  $f(\beta) = 3 b \beta + 3 c$  and this is equal to  $3(a^2 \beta^2)$

so therefore if beta is strictly bigger than 0 note that  $f(x)$  is a continuous function and at alpha  $f$  is strictly less than 0 and at beta  $f$  is strictly bigger than 0

so therefore there must exist a gamma in between alpha and beta

so that  $f(\gamma) = 0$  as we have  $0 < \alpha < \beta$  strictly less than beta

so we can write there exists gamma which is between alpha and beta

so that if  $f(\gamma) = 0$  we see here that the fourth option is

correct now to check whether the first option is correct or not let us compute  $f$  of  $\alpha + \beta$  divided by 2.

$f$  of  $\alpha + \beta$  divided by 2 is equal to  $a$  square into  $\alpha + \beta$  divided by 2 whole square plus  $2b$  into  $\alpha + \beta$  divided by 2 plus  $2c$  and this is equal to  $a$  square into  $\alpha + \beta$  divided by 2 whole square plus  $b$   $\alpha + c$  plus  $b$   $\beta + c$  now since we have  $\beta$  is strictly bigger than  $\alpha$  we can write the first term is strictly bigger than  $a$  square into  $2$   $\alpha$  divided by 2 whole square and note that the second term is equal to minus  $a$  square  $\alpha$  square and the third term is equal to plus  $a$  square  $\beta$  square so all together we are getting this is equal to  $a$  square into  $\beta$  square which is strictly bigger than  $0$

so we have  $f$  of  $\alpha + \beta$  divided by 2 is strictly bigger than  $0$  and this proves that  $\alpha + \beta$  divided by 2 cannot be a solution of  $f$  equals to  $0$

so the first option is not correct now we need to check only the second option for that part let us try to draw a picture for the purpose of this picture let us draw only  $x$  axis let this be the graph of  $y$  equals to  $f$  of  $x$  recall that we have already got  $f$  of  $\alpha$  is strictly less than zero and  $f$  of  $\beta$  is strictly bigger than  $0$  also we have got  $f$  of  $\alpha + \beta$  whole divided by 2 is strictly bigger than  $0$  therefore  $\alpha$  is somewhere here in this region and  $\beta$  is somewhere in this region and  $\alpha + \beta$  by 2 is somewhere in this region for simplicity let us take this point to be  $\alpha$  this point to be  $\beta$

so  $\alpha + \beta$  whole divided by 2 will be somewhere here now note that  $\alpha + \beta$  by 2 is strictly bigger than  $\alpha + \beta$  whole divided by 2 as we have  $\alpha$  is strictly bigger than  $0$

so therefore from the picture it is very much evident that  $f$  of  $\alpha + \beta$  by 2 is strictly bigger than  $0$

so  $\alpha + \beta$  by 2 cannot be a solution of  $fx$  equals to  $0$

so the second option is also not correct we end this session here we have one more session on quadratic equations

so in the next session we are going to solve some more problems you