

welcome to lecture seven on trigonometric functions in the last lecture we had discussed trigonometric equations we gave general solutions to trigonometric equations of the form  $\sin x = \sin y$ ,  $\cos x = \cos y$  and  $\tan x = \tan y$  and we solved a few problems

so in this lecture also we are going to continue solving problems let us first start with a quick recap of what we did in the last class

so we discussed general solutions of trigonometric equations of this form  $\sin x = \sin y$  and we said that the solution to this equation is  $x = n\pi + (-1)^n y$  where  $n$  is an integer similarly for the trigonometric equation of the form  $\cos x = \cos y$  we showed that the general solution is of the form

so here when I say  $x = n\pi + (-1)^n y$  basically means that  $x$  belongs to this set so this is the general solution set for this equation and for  $\cos x = \cos y$   $x$  belongs to the set  $2n\pi + (-1)^n y$  for all integers  $n$  and for the equation  $\tan x = \tan y$  we showed that the general solution set was of the form  $n\pi + y$  for integer  $n$

so let us continue solving some more problems

so in this problem we are asked to find the general solution to the equation cosecant of  $x$  equals cotangent of  $x$  plus square root three

so one technique of solving problems where you get cosecant and cotangent is to express them in terms of sine, cosine and tangent and then use all the identities for sine, cosine and tangent in solving this equation

so we know that cosecant of  $x$  is one over sine  $x$  equals we can write cotangent of  $x$  as cosecant of  $x$  over sine  $x$  plus square root of 3 and then we bring this term on the left hand side because we see that  $1 - \cos x$  is something that we know we know that  $1 - \cos x = 2 \sin^2 \frac{x}{2}$

so we will try to use that

so then this becomes or

so that is one way or the other way is that we multiply both sides by sine  $x$  and then we get  $1 = \cos x + \sqrt{3} \sin x$

so this probably is of the form  $a \cos x + b \sin x$  and then we had discussed in one of the previous lectures how to simplify that

so this can be written as  $2 \left( \frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \right)$  which can be further written as now here we know that we can replace  $\frac{1}{2}$  with  $\cos 60^\circ$  and  $\frac{\sqrt{3}}{2}$  with  $\sin 60^\circ$  or we can even do it otherwise we can even write  $\frac{\sqrt{3}}{2}$  as  $\cos 30^\circ$  and  $\frac{1}{2}$  as  $\sin 30^\circ$  as  $\sin \left( \frac{\pi}{6} - x \right) = \cos x + \sin \left( \frac{\pi}{6} - x \right)$  and this is of the form  $\sin \left( \frac{\pi}{6} - x \right) = \sin \left( \frac{\pi}{6} - x \right)$

so this thing inside the braces is sine of  $x$  plus  $\frac{\pi}{6}$  two times sine of  $x$  plus  $\frac{\pi}{6}$

so we on the previous slide we reduced that two into  $\sin \left( x + \frac{\pi}{6} \right) = \frac{1}{2}$  but half is the same as sine of thirty degrees which is  $\frac{\pi}{6}$

so here we again have an equation of the form  $\sin x = \sin y$  and for this we know that the general solution is that  $x + \frac{\pi}{6}$  should belong to the set  $n\pi + (-1)^n y$  in this case  $y$  is  $\frac{\pi}{6}$

so  $\frac{\pi}{6}$  for all integer  $n$  and this is the same as saying that  $x$  belongs to the set  $n\pi + \frac{\pi}{6} - \frac{\pi}{6}$  and  $n\pi - \frac{\pi}{6}$  one belonging to integers

so this is the general solution set for the equation cosecant of  $x$  equals cotangent of  $x$  plus square root of three just take a slightly more difficult problem

so in this problem we are asked to find all the solutions to cosecant  $\theta$

plus secant of theta equals one

so as again we will express  $\cos x \theta$  as one over sine theta plus this will be one over cos theta equals one and then when you multiply both sides with sine theta cos theta you end up getting cos theta plus sine theta equals sine theta into cos theta but this does not appear to be when this appears to be of the form  $a \cos \theta + b \sin \theta$  but then here we have a product of sine theta cos theta ah here there are many possible ways of doing this problem one possible way is that using the fact that  $\sin^2 \theta + \cos^2 \theta = 1$  what we can do is we can define  $t$  to be sine theta plus cos theta and then if you

so this is a different new variable that is defined here and then you will see that  $t^2$  is sine square theta plus cos square theta plus two sine theta cos theta

so but  $\sin^2 \theta + \cos^2 \theta = 1$  and therefore  $t^2$  is one plus two sine theta into cos theta and from here we can see that sine theta into cos theta is actually equal to  $t^2 - 1$  over two

so now if we see go back to the ah trigonometric equation the left hand side was  $t$  is equal to the right hand side is  $t^2 - 1$  over two and  $t$  has been defined to be sine theta plus cos theta

so the equation in terms of  $t$  then becomes  $t^2 - 1 = 2t$  or  $t^2 - 2t - 1 = 0$  and therefore since this is a quadratic equation in  $t$  there are two possible roots the roots are  $1 \pm \sqrt{2}$  now we know that  $t$  is equal to sine theta plus cos theta which can actually be written as  $\frac{1}{\sqrt{2}}(\sin \theta + \cos \theta)$  which is equal to  $\frac{1}{\sqrt{2}}(\sin \theta + \cos \theta)$  now we know that  $\frac{1}{\sqrt{2}}$  is equal to  $\cos \frac{\pi}{4}$  and it is also equal to  $\sin \frac{\pi}{4}$

so you can write here  $\cos \frac{\pi}{4} \cos \theta + \sin \frac{\pi}{4} \sin \theta$  this is again of the form  $\sin(a - b)$  and therefore this expression inside the braces is sine of theta plus pi by four

so this is equal to sine of theta plus pi by four and therefore from here we know that the magnitude of  $t$  can since the value of sine is between minus one and plus one the absolute value of  $t$  has to be less than equal to square root of two if you go back to the roots of this equation if we take the root which is  $1 + \sqrt{2}$  that is not a feasible solution because  $t$  is sine theta plus cos theta which is equal to this from where we had seen that the absolute value should be less than root 2.

so we cannot take the root which is  $1 + \sqrt{2}$  because that is outside this that does not satisfy this constraint

so therefore the only other solution is that  $t$  is equal to  $1 - \sqrt{2}$  and we already have this simplification here

so ultimately we end up with  $t$  equal to  $\frac{1}{\sqrt{2}}(\sin \theta + \cos \theta)$

so we only take the other the root which is satisfying this constraint which is  $1 - \sqrt{2}$

so this whole thing can be again rewritten as  $\frac{1}{\sqrt{2}}(\sin \theta + \cos \theta) = 1 - \sqrt{2}$  which implies that  $\sin \theta + \cos \theta = \sqrt{2}(1 - \sqrt{2})$  and let this be equal to sine of some angle phi because this value in the braces is between minus one and plus one

so we can always find a value for this angle phi between zero and two pi such that sine of phi is equal to this value

so let phi be that value

so now we again have the same equation of the form  $\sin x$  is equal to  $\sin y$  for which we have the solution that  $\theta + \frac{\pi}{4}$  should belong to the set  $n\pi + (-1)^n \frac{\pi}{4}$  for all integer  $n$  and therefore the solution said the general solution set for  $\theta$  will be  $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$  for all integer  $n$

so if you go back to the solution to this problem we used a little trick here because on one side we had the sum of  $\cos$  and  $\sin$  on the other side we had a product

so that is why we had to use this trick and we use the fact that  $\sin^2 \theta + \cos^2 \theta = 1$

so here is the next problem

so it is asking us to find the general solution to this trigonometric equation and again what we can do here is that we can write this as  $\frac{1 + \cos 2\theta}{\cos^2 \theta} = \frac{1 + \cos 4\theta}{\sin^2 \theta}$  and then we multiply both the left hand side and the right hand side by

so we multiply l h s and the right hand side by  $\cos^2 \theta \sin^2 \theta$

so what we end up getting is  $(1 + \cos 2\theta) \sin^2 \theta = (1 + \cos 4\theta) \cos^2 \theta$  this whole thing will be equal to  $2 \cos^2 \theta$  and this other expression will be equal to  $2 \cos^2 \theta \sin^2 \theta$  and then taking whatever is on the right hand side bringing everything to the left hand side we get zero and we see that there are some common terms in both these terms

so you factor them out

so what we get at the end is  $\cos \theta$  is a common term here  $\cos 2\theta$  is also common

so we take both of them out and then we have equals zero and here we see a pattern that  $2 \sin \theta \cos \theta$  is in fact equal to  $\sin 2\theta$

so using that fact we have  $2 \sin 2\theta = \cos 2\theta - \cos 4\theta$  and again we see the same pattern with two theta

so this thing is now  $\sin 4\theta$

so now this equation is zero if and only if  $\cos \theta = 0$  or  $\cos 2\theta = 0$  or  $\sin 4\theta - \cos 4\theta = 0$  either of these three we need to find the solution set for each of these three different equations and take the union of all of them

so of course we know that  $\cos \theta = 0$  means that  $\theta$  belongs to the set because this is of the form  $\cos \theta = \cos \frac{\pi}{2}$  and then we can use the general solution to equations of the form  $\cos x = \cos y$

so the solution to this is  $2n\pi + (-1)^n \frac{\pi}{2}$  where  $n$  is integer then for this equation  $\cos 2\theta = 0$  the general solution is that  $\theta$  belongs to

so is going to be exactly the same except that because we have a two theta here will have it as  $2n\pi + (-1)^n \frac{\pi}{4}$  for all integer  $n$  now for this last equation which is this can again be written this actually implies and is implied by the other this equation where even if i multiply with one over root two right hand side is still zero equals zero and this can then be written as going to  $\cos$  because  $\cos \frac{\pi}{4} = \sin \frac{\pi}{4}$  is equal to one over root two which further implies an ism now this is the left hand side here is of the form  $\sin a \cos b - \cos a \sin b$  which is  $\sin(a - b)$  therefore this left hand side is equal to  $\sin(4\theta - \frac{\pi}{4}) = 0$  which implies  $4\theta - \frac{\pi}{4} = n\pi$  for all integer  $n$

so here we are using the formula that of sine x equal to sin y with y equal to zero and this then implies that theta belongs to we move the pi by four inside the  $n\pi$  over 4 plus pi over 16 n being integers

so the final solution to the problem the general solution to the problem  $1 + \sec \theta$  into  $1 + \sec 4\theta$  equal to  $\cot \theta$

so the general solution is given by the union of these sets  $2n + \pi$  by 2 all integer n union with  $2n + \pi$  by 4 again integer n union with  $n\pi$  over 4 plus pi over sixteen integer n

so this is the final solution set for this trigonometric equation

so let us consider another problem

so in this problem we are asked to find the smallest positive value of x such that this trigonometric equation is satisfied

so as again we write  $\tan x$  as  $\frac{\sin x}{\cos x}$  and therefore this becomes sine of  $x + 100$  degrees upon cosine of  $x + 100$  is equal to sine of  $x + 50$ .

so i am not writing degrees in the interest of time into sine of x into sine of  $x - 50$  divided by cosine of  $x + 50$  cosine of  $x - 50$  and then we multiply both sides

so multiply lhs and the right hand side by  $\cos(x + 100)$  times  $\cos(x + 50)$  and then after this multiplication what we end up getting is  $\sin x + 100 \cos x + 50 \cos x$  times  $\cos(x - 50)$  equals  $\sin x + 50 \sin x$  times  $\sin(x - 50)$  times  $\cos(x + 100)$  we see that on both the left hand side and on the right hand side we have products of sine and cosine but now the question is we should we combine this with this or should we combine sine of  $x + 100$  with  $\cos x$  and what we see is that if we combine sine of  $x + 100$  with  $\cos x$  then we get a term which is sine of hundred and we get a similar term if we combine  $\cos x + 50$  with  $\cos(x - 50)$  and something like that will also happen on this side if we combine  $\sin x + 50$  with  $\sin(x - 50)$  and  $\cos x + 100$  with  $\sin x$

so let us try to do that and see what happens

so we start with simplifying the left hand side first and of course we can multiply both sides by four

so we need this four because we will try to recall the expansion for two  $\sin a \sin b$  two  $\sin a \cos b$  and two  $\cos a \cos b$  and two  $\cos a \sin b$

so on the left hand side will will what we can write it as  $2 \sin(x + 100)$  times  $\cos(x + 50)$  multiplied by  $2 \cos(x - 50)$  plus  $40 \cos(x - 50)$  now we know that two  $\sin a \cos b$  is sine of  $a + b$  plus sine of  $a - b$  which is hundred degrees

so this is the first this term on the left hand side and then here the we see the pattern for two  $\cos a \cos b$  and we know that two  $\cos a \cos b$  is  $\cos(a + b)$  plus  $\cos(a - b)$

so this term becomes

so this multiplied by  $\cos(a + b)$  will be  $\cos 2x$  plus  $\cos(a - b)$  is going to be  $\cos 100$  and then of course we can write all the four terms here very nicely plus sine of  $x + 100$  times  $\cos 100$  plus sine of hundred times  $\cos 2x$  plus sine of hundred times  $\cos 100$  when we try to do a similar thing for the right hand side now

so on the right hand side we had  $4 \sin(x + 50)$  times  $\sin(x - 50)$  times  $\cos(x + 100)$  and here we will combine  $\sin(x + 50)$  with  $\sin(x - 50)$  and  $\sin x$  with  $\cos(x + 100)$

so it becomes two  $\sin(x + 50)$  into  $\sin(x - 50)$  times  $2 \sin x \cos x$  plus hundred now this is of the form two  $\sin a \sin b$  which is equal to  $\cos(a - b) - \cos(a + b)$

so  $\cos(a - b)$  will be  $\cos$  of hundred degrees minus  $\cos$  of  $a + b$  will be  $2x$

so this is  $a$  and this is  $b$

so  $a + b$  is going to be  $2x$  multiplied by  $2 \sin a \cos b$  for this other term  $2 \sin a \cos b$  is  $\sin(a + b) + \sin(a - b)$

so  $\sin(a + b)$  is going to give us  $\sin$  of  $2x + 100$  and  $\sin(a - b)$  is going to give us  $\sin$  of  $100 - x$  which is  $\sin$  of  $100 - x$  and then we again write up all the four terms that we get we end up getting  $\sin$  of  $2x + 100$  times  $\cos 100$

so this is this with this minus  $\sin 100$  into  $\cos 100$  minus  $\cos 2x \sin 2x + 100$  plus  $\cos 2x$  into  $\sin 100$

so this is the right hand side and this is equal to the left hand side which is  $\sin$  of  $2x + 100$  times  $\cos$  of  $2x + 100$  plus  $\sin$  of  $2x + 100$  times  $\cos$  of  $100 - x$  plus  $\sin$  of  $100 - x$  times  $\cos 2x$  plus  $\sin 100 \cos 100$  and we can see that a few terms are going to get cancelled here because this term will  $\sin 2x + 100$  times  $\cos 100$  is both here and here

so this gets cancelled out and then  $\cos 2x$  times  $\sin 100$  is also on both the left hand side and the right hand side

so this also gets cancelled

so what remains in the end is that  $2 \sin 2x + 100$  into  $\cos 2x$  which is essentially this term ah bringing it on this side and then plus  $2 \sin 100 \cos 100$  equals  $0$  which can be further simplified as

so from the previous slide we have  $2 \sin$  of  $2x + 100$  times  $\cos$  of  $2x + 100$  plus  $2 \sin 100 \cos 100$  equals zero now this is of the form  $2 \sin a \cos b$

so we can we know that  $2 \sin a \cos b$  is  $\sin(a + b) + \sin(a - b)$

so this becomes  $\sin$  of

so  $a + b$  will give you  $4x + 100$  and  $\sin(a - b)$  is going to give us  $\sin$  of  $100 - x$  and and and this is if you see the pattern here this is  $\sin$  of  $200 - x$  this is of the form  $2 \sin a \cos a$

so  $2 \sin a \cos a$  is  $\sin 2a$  which is  $\sin$  of  $200 - x$  and then here we will try to use the  $\sin(a + b)$  formula

so this thing will become  $2 \sin$

so  $a + b$  by  $2$  is  $150$  degrees into  $\cos$  of  $a - b$  by  $2$  will be  $50$  degrees but  $\sin$  of  $150$  is the same as  $\sin$  of  $30$  degrees

so which is half

so this is equal to half

so  $2$  multiplied by half is one therefore what we have is  $\sin$  of  $4x + 100$  is equal to  $\cos$  of  $50$  degrees which is  $\sin$  of  $40$  degrees because  $\cos$  of  $50$  is the same as  $\sin$  of  $40$ .

which can also be written as  $\sin$  of  $-40$  degrees

so again we have the form  $\sin x = \sin y$  for which the sol general solution will be given by  $4x + 100$  belongs to this set  $n\pi \pm 1$  to the power of  $n$  times  $-40$  because  $y$  is  $-40$  here we should not because we are using  $\pi$  here and we are expressing the answer in terms of radians we need to convert this back to radians

so this is not correct

so since this is degrees

so from degrees if we remember from the previous lectures we just need to multiply this with  $\pi$  over  $180$

so that will convert it back to radians but then the same thing has to be done with this  $100$  also here

so in any case this statement is correct

so for all  $n$  belong to  $\mathbb{Z}$  and then we can write that  $4x$  belongs to  $n\pi \pm 1$

minus one to the power of n times minus

so this forty we can simplify this as two pi by nine and then we have to put a minus hundred over here but minus hundred this is in degrees

so in terms of radians that will be minus 5 pi over 9 and again belonging to integers and but this is 4 x

so then we have to divide everything here with by four

so what we end up getting is that the solution set is of the form x belongs to n pi over four plus we can actually bring the minus outside but into minus pi over 18 minus 5 pi over 36 for all n belong to set of integers

so this is the general solution to this problem and you can in fact find out the first for example if you put n equal to one for example then that corresponds to the solution pi by four and then minus minus pi by eighteen minus 5 pi by 36 which is pi by 4 plus pi by 18 minus 5 pi by 36 which in degrees is this is 45 degrees this is 10 degrees and this is pi is 180

so this is 25 degrees

so this is equal to 30 degrees or pi by six

so this is with n equal to one and then you put different values of n you get all the general solution all the solutions to this equation here is another interesting ah problem where we are asked to show that for all x in the interval zero to pi by two this statement is true cos of sin x is always greater than equal to sine of cos x for all x in the interval 0 to pi by 2.

so for this we it seems that we need to use the identity that sine of pi by two minus x is equal to cos x and a similar identity for cos

so we can start with sine

so this ah term on the left hand side can be written as sine of pi by two minus sine x because cos theta is equal to sine of pi by two minus theta

so if you want to show this it is equivalent to show that this is greater than equal to sine of cos x now remember we are it is said that x has to belong to the interval zero to pi by two

so when x belongs to the interval zero to pi by two let us examine the relation between pi by two minus sin x and cos x

so let us examine these two terms for when x belongs to zero to pi by two of course when x belongs to zero to pi by two cos x will be between

so at x equal to zero it will be one and at x equal to pi by two it is zero

so it is going to be between zero and one and pi by two minus sin x at x equal to zero this will be pi by two and at x equal to pi by two it will be equal to pi by two minus one

so it is going to vary between pi by two minus one two pi by two and we see that this interval as well as this interval both of them are subset of the interval zero to pi by two for example this interval is a subset of zero to pi by two the interval ah the set of all values taken by cos x are also are between zero and one is also a subset of the interval zero to pi by two

so this tells us that both pi by two minus sin x and cos x which are the arguments here belong to the interval zero to pi by two

so essentially we have something of the form sine of a and and sine of b

so a is equal to lets say pi by two minus sin x and b is cos x and we are asked to show that sign a is greater than equal to sign b and we know that both a and b

so both a and b will belong to the interval zero to pi by two if you would have seen the graph of the sign function we know that

so i'll quickly plot ah the graph of the sine function

so so if this is zero and let us say this is pi by two

so this is x and this is y equal to sine x

so at  $x$  equal to zero  $\sin x$  is zero and then between zero and  $\pi/2$  it is monotonically increasing from zero all the way till one

so what that means is that for any two values let us say  $a$  and  $b$  if  $a$  is greater than equal to  $b$  as is the case shown in this graph then  $\sin a$  which is this value over here

so this value here is  $\sin a$  will be greater than equal to  $\sin b$

so  $\sin b$  is the

so this value is  $\sin b$

so  $a \geq b$  implies  $\sin a \geq \sin b$  is implied by the fact that  $\sin$  is increasing in this interval here if you recall  $a$  was  $\pi/2 - \sin x$  and  $b$  was  $\cos x$

so if we have to show that  $\sin a \geq \sin b$  it only suffices to show that  $a \geq b$  in this interval here if you recall  $a$  was  $\pi/2 - \sin x$  and  $b$  was  $\cos x$

so as long as if we can show this statement if this holds true in the interval  $x$  belonging to zero to  $\pi/2$

so  $x$  belongs to zero to  $\pi/2$  as long as we can show this that will solve the problem right

so let us try to again further examine this particular equation showing this is equivalent to showing that  $\cos x + \sin x \leq \sqrt{2}$  for all  $x$  in the interval zero to  $\pi/2$

so we try to show this on the next slide

so these two are again equivalent statements now  $\sin x + \cos x = \frac{1}{\sqrt{2}}(\sqrt{2}\sin x + \sqrt{2}\cos x)$  which is equal to

so again we can do we can write  $\frac{1}{\sqrt{2}}$  as  $\cos(\pi/4)$  and the  $\frac{1}{\sqrt{2}}$  here as  $\sin(\pi/4)$  therefore this whole thing simplifies to  $\sin(x + \pi/4)$  but then we know that this value

so when  $x$  belongs to  $0$  to  $\pi/2$   $x + \pi/4$  is of course well there is nothing much to see now because the value of  $\sin(x + \pi/4)$  for any  $x$  it has to be less than equal to 1 and therefore  $\sin x + \cos x$  from this equality here has to be less than equal to  $\sqrt{2}$  since  $\sqrt{2}$  is less than strictly less than  $\pi/2$  it is easy to now see that because  $\sqrt{2}$  is one point four one something and  $\pi/2$  is one point five seven something from these two statements it follows that  $\sin x + \cos x \leq \sqrt{2}$  for all  $x$  belonging to the interval  $0$  to  $\pi/2$  ok let us take another problem in this problem we have to find the smallest positive number  $p$  for which this trigonometric equation is satisfied and in fact the number  $p$  should be such that this equation has a solution  $x$  in the interval zero to two  $\pi$

so here again we use the identity that  $\cos \theta = \sin(\pi/2 - \theta)$

so we write this left hand side as  $\sin(\pi/2 - p) \sin x = p \cos x$  and this is again of the form  $\sin x = \sin y$

so for this to be true it must be it must hold that  $\pi/2 - p - \sin x = n\pi + \sin x$  or some integer  $n$

so let us now try different possibilities for this integer  $n$  and see what we get

so if we try  $n$  equal to 0 for example then the equation that we get is  $\pi/2 - p - \sin x = \sin x$  which can be written as  $p \sin x + \cos x = \pi/2$  and then the  $\sin x + \cos x$  can be simplified as  $\frac{1}{\sqrt{2}}(\sqrt{2}\sin x + \sqrt{2}\cos x)$  which is equal to  $\sin(x + \pi/4) = \pi/2$  now we are in the question it is asked to find the smallest positive number  $p$  but for  $p$  to be positive we have to choose  $x$  such that  $\sin(x + \pi/4) = \pi/2$

also positive because  $\pi$  by two is positive and root two is positive plus since we want to find the smallest  $p$  we should try to choose  $x$  as that sine of  $x$  plus  $\pi$  by four is the largest possible positive value which we know is one

so when this is equal to one we get the smallest possible value of  $p$  which will be equal to  $\pi$  by two over root two but this is only for  $n$  equal to zero we can try for with  $n$  equal to one if we put  $n$  equal to one then we end up getting two minus  $p$  sine  $x$  equals  $\pi$  minus  $p$  cos  $x$  and what we if we rearrange this then what we end up getting is that  $p$  times cos  $x$  minus sin  $x$  is equal to  $\pi$  over two and even here we can again write it as  $p$  into square root of two times and then it will be cos  $x$  into one by root two minus one by root two sin  $x$  which can be written as cos of  $x$  plus  $y$  over four is  $\pi$  over two but even here the the largest possible of cos of  $x$  plus  $\pi$  by four is still one and therefore the smallest positive value of  $p$  will still be the same value and we can try like this for negative  $n$  also and we will end up in general what we can see is that this equation for a general  $n$  will only get satisfied if

so this can be written as minus 1 to the power of  $n$  plus 1 if i bring this or sorry rather it will be easier to take sine  $x$  on this side

so it becomes  $p$  into minus one to the power of  $n$  cos  $x$  plus sine  $x$  equals minus of  $\pi$  over two into two  $n$  that is because of this term minus one but one thing that is to be realized here is that if we then further write it as we take the root two outside and then we bring the root two inside here and if we see here if we take the absolute values on both the left hand side and the right hand side what we will see is that

so  $p$  anyways we wanted to be positive

so mod  $p$  into square root of two into now the absolute value of this thing inside the braces this particular term the largest possible value is still one because this particular term no matter what the value of  $n$  is this will always be less than equal to one

so the largest possible value is one and that should be equal to  $\pi$  by two into absolute value of two  $n$  minus one

so from here it is now easy to see that for  $n$  equal to zero we get this thing equal to  $\pi$  by two  $n$  equal to one also we get this thing equal to  $\pi$  by two but if we try  $n$  equal to minus one then we have three  $\pi$  by two here similarly if we try  $n$  equal to minus two or  $n$  equal to two or larger values of  $n$  then will will not get  $\pi$  by two will be will get bigger numbers but since we have to find the smallest positive value of  $p$  we have to therefore choose either  $n$  equal to zero or  $n$  equal to one for which the smallest positive value comes out to be  $p$  equal to  $\pi$  by two root two and that finishes this solution to this problem

so with this we end this lecture on solving problems for trigonometric equations in the next lecture we are going to start a new topic which is to know inverses of trigonometric functions till then thank you you