

welcome to lecture six on trigonometric functions in the last lecture we had introduced trigonometric equations and we discussed principle solutions and general solutions for some type of equations for example equations where of the form where $\sin x$ is equal to $\sin y$ in this lecture also we will discuss general solutions to similar type of equations like $\cos x$ equal to $\cos y$ and $\tan x$ equal to $\tan y$ if $\cos x$ equal to $\cos y$

so we are trying to solve trigonometric equations of the form let us say if we are given that $\cos x$ equals half and we are interested in finding all the solutions to $\cos x$ equal to half

so we realize that $\cos x$ is equal to now half is equal to \cos of sixty degrees which is π by three

so we have y equal to π by three here and then we want to find the general solution to this equation

so towards that we will discuss some results and see how to find the general solution

so we will first show that if $\cos x$ is equal to $\cos y$ then it must be true that x is either equal to $2n\pi + y$ or x is equal to $2n\pi - y$ for some integer n similarly we will also show that for all integer n you take any integer n then \cos of $2n\pi + y$ is equal to \cos of $2n\pi - y$ which is also equal to \cos of y

so these two are going to help us to find the general equation general solution to equations of the type $\cos x$ equal to something

so we will start with showing that

so we have to show that \cos of $2n\pi + y$ is equal to \cos of $2n\pi - y$ for any integer n

so of course here the pattern is \cos of $a + b$ where a is $2n\pi$ and b is y

so this is from our previous lectures we know that this is equal to $\cos a \cos b - \sin a \sin b$

so using that what we get is \cos of $2n\pi + y$ is \cos of $2n\pi$ \cos of y minus \sin of $2n\pi$ \sin of y

so \cos of $2n\pi$ is \cos of 0 which is 1 and \sin of $2n\pi$ is 0 for any integer n

so \sin of integer multiple of π is zero

so this term goes to zero

so what remains is only the first term and we also know that \cos of integer multiple of 2π is equal to one

so for any integer n \cos of $2n\pi$ is always equal to one

so therefore this really is equal to \cos of y and similarly \cos of $2n\pi - y$ equals

so here we will use the formula $\cos a \cos b - \sin a \sin b$

so this becomes equal to $\cos 2n\pi \cos y - \sin 2n\pi \sin y$ again this term is zero what we saw before and $\cos 2n\pi$ is one for any integer n

so therefore this is also equal to \cos of y

so we have shown that $\cos 2n\pi + y$ is equal to $\cos 2n\pi - y$ for any integer n we now show the reverse that suppose if we are given that $\cos x$ and $\cos y$ are equal then we will try to see the relation between x and y

so from this we get that $\cos x - \cos y = 0$ now this is of the pattern $\cos a - \cos b$ and which we from our previous lectures should be able to find it out

so the formula that we will be using here is $\cos a - \cos b = -2 \sin \frac{a+b}{2} \sin \frac{a-b}{2}$ so we get $\cos a - \cos b = 0$ which gives $\sin \frac{a+b}{2} \sin \frac{a-b}{2} = 0$

so we will use this formula here with a equal to x and b equal to y
so what we get is $\cos x - \cos y = 0$ is the same as writing that
 $2 \sin \frac{y-x}{2} \sin \frac{y+x}{2} = 0$ but this implies that either $\sin \frac{y-x}{2} = 0$ or $\sin \frac{y+x}{2} = 0$ we take that further

so we start with the first condition that we derived now
so $\sin \frac{y-x}{2} = 0$ implies that $\sin \frac{x-y}{2} = 0$ is also 0 because $\sin(-x) = -\sin x$ and we know this result that $\sin \theta = 0$ implies that θ is of the form $n\pi$ where n is some integer

so therefore this statement implies that $\frac{x-y}{2}$ must be equal to $n\pi$ for some integer n and then by simplifying this what we get is that x is equal to $2n\pi + y$ for some integer n

so this was the first condition and then the second condition was that $\sin \frac{x+y}{2} = 0$

so let us examine that also

so $\sin \frac{x+y}{2} = 0$ implies that $\frac{x+y}{2}$ is equal to an integer multiple of π for some integer n and from here we get that x is equal to $2n\pi - y$ for some these are n

so essentially we have shown that if $\cos x = \cos y$ then that must imply that either of these two conditions hold now for this first condition to hold this implies that x should be of the form $2n\pi + y$ for some integer n and this condition says that x should be of the form $2n\pi - y$ for some integer n and therefore we see that $\cos x = \cos y$ is only true if x is either $2n\pi + y$ for some integer n or it is $2n\pi - y$ for some integer n

so either of these two case

so here is a little example with $\cos x = -\frac{1}{2}$ and we know that $\cos \frac{2\pi}{3} = -\frac{1}{2}$ $\cos x = \cos y$ which we just saw now where y is equal to $\frac{2\pi}{3}$ and therefore the general solution is x equal to $2n\pi + \frac{2\pi}{3}$ or $2n\pi - \frac{2\pi}{3}$

so because if you remember we had said that for any integer n we had proved that $\cos(2n\pi + y) = \cos(2n\pi - y) = \cos y$

so using that we can say that all values of x of this form such where n is integer

so so this becomes the general solution to the equation $\cos x = -\frac{1}{2}$
for example if we put n equal to zero then we get $\frac{2\pi}{3}$ or $-\frac{2\pi}{3}$

so we get two solutions here $\frac{2\pi}{3}$ and $-\frac{2\pi}{3}$ with n equal to one we again get two solutions $\frac{2\pi}{3} + 2\pi$ and $-\frac{2\pi}{3} + 2\pi$ with n equal to minus one we get the solution $-\frac{2\pi}{3} - 2\pi$ and $\frac{2\pi}{3} - 2\pi$ and we can keep on going like this and all of these are solutions to this equation $\cos x = -\frac{1}{2}$

so just like $\sin x = \sin y$ and $\cos x = \cos y$ will also try to get the general solution to any trigonometric equation of the form $\tan x = \tan y$ of course here both x and y should not be all multiples of π because \tan of an odd multiple of $\frac{\pi}{2}$ is not finite

so we will show that if $\tan x = \tan y$ then it must be true that x is equal to an integer multiple of π plus y on the other hand for any integer n we can show that $\tan(n\pi + y) = \tan y$ and this is not very difficult to see because in the previous lecture we saw that \tan of y is a periodic function \tan of x is the same as \tan of $\pi + x$ is the same as \tan of $2\pi + x$ and

so forth nevertheless we will still prove it here

so \tan of $n\pi + y$ is equal to

so this is of the form \tan of $a + b$ and we know the formula for $\tan a + b$ it is $\tan a + \tan b$ by $1 - \tan a \tan b$

so using that here we get \tan of $n\pi + y$ equals \tan of $n\pi + \tan$ of y upon $1 - \tan$ of $n\pi$ times \tan of y here n could be any integer

so n is any integer but for any integer n

so for any integer n we know that \tan of $n\pi$ is equal to zero because \tan of $n\pi$ is \sin of $n\pi$ over \cos of $n\pi$ and \sin of $n\pi$ zero for all integer n therefore $\tan n\pi$ is zero for all integer n and therefore this goes to zero and this also goes to zero

so what remains is that this becomes equal to \tan of y which proves that \tan of $n\pi + y$ equals \tan of y for all integer n and then we will also show that the reverse statement which is that if $\tan x$ and $\tan y$ are equal then it must hold true that x is equal to $n\pi + y$ for some integer n

so let us say we have $\tan x$ equal to $\tan y$ and x and y are not odd multiples of π by 2

so we get from here we get this and then since $\tan x$ is $\sin x$ by $\cos x$ we get $\sin x$ by $\cos x$ minus $\sin y$ by $\cos y$ equals 0 from where we get $\sin x \cos y$ minus $\cos x \sin y$ over $\cos x \cos y$ equal to zero but since x and y are not or multiples of π by two $\cos x$ is not zero and \cos of y is also not zero and therefore this statement here is equivalent to $\sin x \cos y$ minus $\cos x \sin y$ equals zero but this pattern we have already seen this is of the form $\sin a \cos b$ minus $\cos a \sin b$ which is equal to \sin of a minus b therefore this is the same as \sin of x minus y and we know that \sin of x minus \sin of θ equal to 0 implies that θ must be some multiple of some integer multiple of π therefore from here we can conclude that x minus y must be some integer multiple of π and that implies that x has to be equal to $n\pi + y$ for sum integer n now in the remaining lecture we will try to solve some ah find the general solution to some trigonometric equations which you might encounter

so here ah this is a very general type of a very commonly seen trigonometric equation that you will come across

so a , b and c are real numbers and we are asked to find the general solution to this equation $a \cos \theta + b \sin \theta = c$

so the way to proceed ah with this is that we divide both the left and the right hand side by square root of $a^2 + b^2$ and what we get is ah this second equation here now if you if you see a upon square root of $a^2 + b^2$ and b upon square root of $a^2 + b^2$ let us say that what we realize is that the square of this term plus the square of this term is equal to one and we draw a unit circle here with center at O and lets say that this is the point whose x coordinate is a by square root of $a^2 + b^2$ and whose y coordinate is b upon square root of $a^2 + b^2$ and of course this point is lies on the unit circle and the angle of rotation for this ray OP is ϕ in the anticlockwise direction

so we have this ϕ here

so let us now focus on this this right angle triangle here which i am drawing now in this right angle triangle from the definition of cosine and sine what we have is that $\cos \phi$ will be equal to the x component of this point which is a upon square root of $a^2 + b^2$ and $\sin \phi$ will be the y coordinate of this point which is b upon square root of $a^2 + b^2$ now using this fact here back into the trigonometric equation what we get is $\cos \phi$

so now find ϕ this angle ϕ is completely known to us $\cos \phi$ into $\cos \theta + \sin \phi$ into $\sin \theta = c$ upon square root of $a^2 + b^2$ square but if you see this expression on the left hand side here it is of the

form $\cos a \cos b + \sin a \sin b$ and therefore it is equal to \cos of $\theta - \phi$ equals c upon square root of $a^2 + b^2$ and therefore this is the trigonometric equation for which we are asked to find the solutions to θ now what we see here is that on the left hand side we have cosine of $\theta - \phi$ and we know that the range of the cosine function is between minus one and plus one and if that has to be equal to this right hand side it must hold true that $\frac{c}{\sqrt{a^2 + b^2}}$ must be less than equal to one

so a solution will exist to this trigonometric equation if and only if this condition is satisfied

so only if this condition is satisfied are we going to have a solution to this trigonometric equation otherwise there is no solution to this trigonometric equation now let us assume that this condition is satisfied

so in that case what we just need to do is we will say that we again go back to so if this if this condition is satisfied then we essentially go back and write this $\frac{c}{\sqrt{a^2 + b^2}}$ because if it is less than equal to root or if this is less than equal to one then this must be equal to \cos of some angle y and then we will use what we just studied in some in one of the previous slides where we had said that for any integer n \cos of $2n\pi + y$ is equal to \cos of $2n\pi - y$ is the same as $\cos y$

so we have a similar equation here like $\cos x = \cos y$

so we have to treat this whole thing as x and therefore the general solution is that x which is $\theta - \phi$ is equal to $2n\pi \pm y$ for all integer n belonging to

so for all n belonging to \mathbb{Z} the set of

so \mathbb{Z} is the set of integers and therefore the final general solution for θ is equal to $\phi + 2n\pi \pm y$ where n is an integer

so this is what is the general solution to this trigonometric equation but this general solution will exist if and only if this condition is valid or it holds now let us discuss another problem

so here we are asked to find

so now if you see we have two variables now whereas what we have been studying so far was to solve equations trigonometric equations where we have only one variable

so initially it might seem very tough but then what we can do is we can do some substitution and then get the solution

so we have to ask we have to find the solution set of this system of equations

so what we see now is that from the first equation here we can write that y is equal to $2\pi/3 - x$ and we substitute this value of y in the second equation

so the second equation now is $\cos x + \cos(2\pi/3 - x) = 3/2$

so finally with the help of this substitution we finally get a trigonometric equation in one variable x and we should be able to solve that

moving further ahead we have $\cos x + \cos(2\pi/3 - x) = 3/2$

so we use the $\cos(a - b)$ formula and we write $\cos 2\pi/3 \cos x + \sin 2\pi/3 \sin x = 3/2$ but $\cos 2\pi/3$ is equal to $-1/2$

so we have $\cos x - 1/2 \cos x + \sin 2\pi/3 \sin x = 3/2$

so that is equal to $\frac{\sqrt{3}}{2} \sin x = 3/2$ and

then further simplifications some simplification gives us $\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x = 3/2$ and we know that $1/2$ is \cos of $\pi/3$ and $\sqrt{3}/2$ is \sin of $\pi/3$

so we will get the form $\cos a \cos x + \sin a \sin x$

so we write the left hand side as $\cos(\pi/3 - x)$ into $\cos x + \sqrt{3} \sin x$ sorry instead of this we write $\sin(\pi/3 - x)$ into $\sin x$ equal to

so this is our trigonometric equation

so far but this is of the form $\cos a \cos b + \sin a \sin b$ which is $\cos(a - b)$

so we finally have $\cos(x - \pi/3) = 3/2$ but we know that the cosine function is limited to the range -1 to 1 and therefore cosine of any angle can never be equal to $3/2$ and therefore there is no solution to this set of equations

so the final answer is that there is no solution let us see another question here

so we are asked to find the general solution to this trigonometric equation and if we see this is actually the left hand side is actually quadratic in $\sin \theta$

so if we take $z = \sin \theta$ then we get the quadratic equation $2z^2 - 3z - 2 = 0$ equation here will be given by $z = \frac{3 \pm \sqrt{9 + 16}}{4}$ which is $\frac{3 \pm 5}{4}$ this is 2 or $-\frac{1}{2}$

so the two roots are 2 and $-\frac{1}{2}$ but we see that if we take the root 2 $3 + 5 = 8$ $8/4 = 2$ but z is equal to \sin of some angle and therefore it cannot be 2

so that

so the solution $z = 2$ is not a feasible solution

so the only other solution which remains is $z = -\frac{1}{2}$

so essentially the solution to this trigonometric equation is the solution to $\sin \theta = -\frac{1}{2}$

so we have $\sin \theta = -\frac{1}{2}$

so we know this identity that $\sin(\pi + x) = -\sin x$ will be $-\sin x$

so it will be $-\sin x$

so this identity is known to us and if we substitute $x = \pi/6$ here what we get is $\sin(7\pi/6) = -\sin(\pi/6)$ and $\sin(\pi/6) = \frac{1}{2}$

so this is $-\frac{1}{2}$

so therefore we can write that $\sin \theta = \sin(7\pi/6)$

so we essentially have to find out the general solution to $\sin \theta = \sin(7\pi/6)$ and this we have already studied in the previous lecture if you recollect we had said that the general solution to $\sin x = \sin y$ is that $x = n\pi + (-1)^n y$ for all where n is some integers for all integers n

so this is the solution now the only thing is that here x is θ and y is $7\pi/6$ therefore the final answer is that the general solution to this equation is $\theta = n\pi + (-1)^n \cdot 7\pi/6$ for all integers n

so here we have another problem where we would like to find out the general solution to this trigonometric equation here

so we write this as $\tan^2 \theta + \sec 2\theta = 1$ because $\sec^2 x = 1/\cos^2 x$

so this is $1/\cos^2 2\theta$ but we know that $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ also we know that the numerator one here can be replaced with $\cos^2 \theta + \sin^2 \theta$

so the reason why i am doing this type of substitution is because i would like this whole left hand side to be in terms of $\tan \theta$

so that i get some kind of a quadratic equation or something like that which i

can solve and then get the general solution

so what we have further is $\cos^2 \theta + \sin^2 \theta = 1$ therefore the numerator is replaced by this divided by $\cos^2 \theta - \sin^2 \theta$ equal to one and then if you divide both the numerator and denominator with $\cos^2 \theta$ what we end up getting is $\frac{1 + \tan^2 \theta}{1 - \tan^2 \theta} = 1$

so the the reason why we had to substitute this one with $\cos^2 \theta + \sin^2 \theta$ was that i wanted this whole of left hand side to be in terms of $\tan \theta$ and then what we eventually get is $\tan^2 \theta - \tan^2 \theta + 1 + \tan^2 \theta = 1 - \tan^2 \theta$

so essentially in this equation you multiply both the left hand side and the right hand side by $1 - \tan^2 \theta$ and this is what you end up getting

so if we open up this braces what we get is on the left hand side we get $2 \tan^2 \theta + 1 - \tan^4 \theta = 1 - \tan^2 \theta$ and then if we rearrange this a little bit what we get is $\tan^4 \theta - 3 \tan^2 \theta = 0$ and therefore what we have is $\tan^2 \theta (\tan^2 \theta - 3) = 0$

so for this to happen either we should have $\tan \theta = 0$ which implies that $\theta = n\pi$

so so this is $\tan \theta = 0$ can be written as $\tan \theta = \tan 0$ and therefore that implies that θ is of the form $n\pi$ for all integer n

so this is from this equation

so either this should happen or we should have $\tan^2 \theta - 3 = 0$ which implies that $\tan \theta = \pm \sqrt{3}$ now $\tan \theta = \sqrt{3}$ but $\sqrt{3} = \tan 60^\circ = \tan \frac{\pi}{3}$ and then we have the same form which we $\tan x = \tan y$ if you remember a few slides ah back we were just discussing that and what we had said was that the general solution to this type of equation is that $x = n\pi + y$ for all n belonging to integers

so using this result which we had proved earlier we get with $x = \theta$ we get that $\theta = n\pi + \frac{\pi}{3}$ and then similarly for $\tan \theta = -\sqrt{3}$ it can be written as $\tan \theta = \tan \left(-\frac{\pi}{3}\right)$ and for this the solution set will be $\theta = n\pi - \frac{\pi}{3}$

so the the final ah solution to the trigonometric equation

so from here we had come to the we had substituted and then what we got was that $\tan^2 \theta = 0$ or $\tan^2 \theta = 3$

so this implied that this holds true and then we saw that this is true when either $\tan \theta = 0$ or $\tan \theta = \sqrt{3}$ or $\tan \theta = -\sqrt{3}$

so for this the general solution was this set for $\tan \theta = \sqrt{3}$ the general solution was $n\pi + \frac{\pi}{3}$ and for $\tan \theta = -\sqrt{3}$ it was $n\pi - \frac{\pi}{3}$ but since we have an or here we have to take the union of all these three sets

so so this is the final ah general solution to this trigonometric equation it is the union of all these three sets we have another problem here but here it is said that we would like to find the values of x only in the interval $[-\pi, \pi]$ which satisfy this equation

so we have two to the power of a series here

so the for example the third the next in the series would be $\cos^3 x$

so in general for any integer m it is true that $\cos^m x$ is the same as $\cos x$ raised to the power of m and therefore using this identity we will try to simplify the exponent on the left hand side

so therefore what we get is is equal to one plus mod of cos x plus mod of cos x whole square plus mod of cos x cube and

so on but what we realize is that this is essentially a geometric progression of the form one plus c plus c square plus c cube and

so forth and we know that this infinitely long geometric progression will converge to the value one over one minus c if and only if if and only if modulus of c is strictly less than one

so let us use this result here of course ah c is equal to modulus of cos x by comparison we are told that x should belong to the open interval minus pi to plus pi

so in this open interval there is only one place where c could be one which is when x equal to zero

so but

so so with x equal to zero this sequence will not converge anyways and therefore x equal to zero can anyways not be a solution to this equation but for x not equal to zero and x belong to minus pi to plus pi the sequence will converge to the value one upon one minus modulus of cosine of x and therefore we finally have the trigonometric equation as two to the power of one upon one minus modulus of cos of x is equal to four which implies that one minus modulus of cos of x should be equal to half that is modulus of cos of x should be equal to half

so now we have two cases here either cos x is equal to half or cos x is equal to minus half and the general solution to this equation will be the union of the general solution set for this equation and the general solution set for this equation

so for cos x equal to half general solution will be the set to n pi because half can be written as cos of sixty degrees which is pi by three

so we have cos x equal to cos y

so cos x equal to cos y with y equal to pi by three

so using the formulation earlier we are shown that the general solution is the general solution for cos x equal to cos phi is two n pi plus minus y

so we will have two n pi plus minus pi over 3 n being integer

so this is the ah general solution set for the first condition cos x equal to half union with the solution set for the the other condition cos x equal to minus half we know that cos of two pi by three is minus half and therefore we have this equation is the same as cos x equal to cos two pi by three

so the solution the general solution set for this second equation will be two n pi plus minus two pi by three for integer n

so the final answer is that the general solution to this trigonometric equation is the union of these two sets but if we remember the what was asked in the question was not the general solution but we were said that we should find all solutions for x between minus pi to plus pi

so therefore we have to only see which solutions here fall in the range minus pi to plus pi and that is very easy to see the

so i will rewrite the solutions here

so this was the final

so this was the general solution set and out of this we have to find out the solution lying in the open interval minus pi to plus pi

so here for example if we take n equal to zero here in the first one we get two solutions minus pi by three n plus pi by three and both of them are in the interval minus pi to plus pi if we take n equal to one then we are outside that interval minus pi to plus pi similarly if we take n equal to minus one again we are outside the interval minus pi two plus y

so from here there are only two solutions which lie in the interval minus pi

two plus pi and then we look at the second ah general solution set here

so sorry this is pi

so here for n equal to zero we get two pi by three and minus two pi by three and both of these are in the interval they lie in the interval minus pi to plus pi for n equal to one we get two pi plus two pi by three but that of course is going to lie outside the interval minus pi to plus pi the other solution is two pi minus two pi by three which is actually equal to four pi by three and this also is outside the interval minus pi to plus pi and therefore we will not write it here

so and similarly for n equal to two and further also the solution here will not be lying in the interval minus pi two plus pi and the same thing will hold for n equal to minus one therefore the final answer is that the solutions to the trigonometric equation

so this trigonometric equation which lie in the interval minus pi to plus pi r these four values minus pi by three pi by three two pi by three n minus two pi by three there is a very ah interesting ah problem

so it says that let m be an odd integer then if this relation holds true for all integer all odd integers

so m could be 1 3 5 7 9 likewise

so it says that if this relation holds for all odd integers for every x then we need to find the value of b 0 and b 1 such that this equation is satisfied for all odd integers m and for all x but this might appear very difficult but what we can do here is that if we put x equal to 0 in this equation let us expand that first

so what we get is sine of m x is

so the first term in the summation here is b zero

so it is b zero sine x to the power of zero is one the next next term is b one sine x then b two sin square x and all the way till b m sine x to the power of m now we substitute x equal to zero the left hand side is zero on the right hand side each of these terms will go to zero

so what remains on the right hand side is b zero and therefore it must hold that b zero is equal to zero

so we have got the value of b zero now we need to find the value of b one such that this is always satisfied for all odd m and all x now let us consider the ratio of sine of m x over sine of x and that will be equal to since

so we are going to use this expansion for the right hand side b 0 is 0

so this is not there

so we divide all of this by sine of x

so we will get b one plus b two sine x plus b three sine square x all the way till b m sine m minus 1 x and then we take the limit x goes to 0 on both the left hand side and on the right hand side

so when we take the limit

so we take this limit on both the

so limit x goes to zero on both the left hand side and the right hand side we know that the limit of the left hand side is equal to m and on the right hand side if we see all the these terms sign x sin square x and sin x to the power of m minus one in the limit that x goes to zero they go to zero

so what remains is only b one and therefore the value of b one is m and b zero is zero

so now we take up another problem

so we have to find all the solutions to this trigonometric equation here

so what we see here is that we have sine x everywhere except in the first term we have a cos square x

so if we replace this with in in terms of sin x then we are going to get a

polynomial in sine x since $\cos^2 x$ is one minus $\sin^2 x$ we get which is equivalent to writing that because we have $\sin x$ here $\sin x$ here and $\sin x$ here

so $\sin x$ is a common factor in both the left hand side and on the right hand side

so we write it as $\sin x$ times $4 \sin^2 x - 1 - 2 \sin x$ for this term minus three equals zero which is same as writing $\sin x$ into one minus two $\sin x$ minus four $\sin^2 x$ equals zero

so for this to be zero either $\sin x$ should be zero or this term in the square bracket should be zero but $\sin x$ equal to zero implies that this the general solution for $\sin x$ equal to zero is that x should be of the form $n\pi$ where n is integer for all integer and let us now see how to solve the second equation the this particular term

so either this is zero or this is zero

so for the second factor on the left hand side we have $4 \sin^2 x + 2 \sin x - 1 = 0$ but this is a quadratic equation in $\sin x$ so the solutions are that $\sin x$ is equal to $\frac{-2 \pm \sqrt{4 + 16}}{8}$

so that is

so the two there are two solutions here

so one is $\frac{-2 - \sqrt{5}}{4}$ and the other solution is $\frac{\sqrt{5} - 1}{4}$ now we know that this is nothing but sine of eighteen degrees if you recollect sine of eighteen degrees was $\frac{\sqrt{5} - 1}{4}$

so this is sine of $\frac{\pi}{10}$ and we also know that sine of fifty four degrees which is $\frac{3\pi}{10}$ sine of fifty four degrees is $\frac{\sqrt{5} + 1}{4}$ and therefore this is equal to exactly the negative of this and therefore here we can write sine of $-\frac{3\pi}{10}$

so from here

so for this equation to hold we should either have $\sin x = \sin \frac{\pi}{10}$ or $\sin x = \sin \frac{-3\pi}{10}$

so this is of the form $\sin x = \sin y$ and therefore the general solution here is $n\pi + (-1)^n \frac{\pi}{10}$ n being integer and then for the other equation $\sin x = \sin \frac{-3\pi}{10}$ this is also the same form $\sin x = \sin y$ and therefore the general solution for this will also be $n\pi + (-1)^n \frac{-3\pi}{10}$ n being integer z but if we go back to the original problem

so we had to find the solutions to this equation

so and we derived it like this

so it is the left hand side is a factor of these two factors

so either $\sin x$ is zero or this factor is zero for $\sin x$ to be zero ah this is the solution set and for this other factor to be zero we saw that either ah it should

so we saw that the other solution set is nothing but the union of these two sets and therefore the final answer is this set union $n\pi + (-1)^n \frac{\pi}{10}$ n being integers union $n\pi + (-1)^n \frac{-3\pi}{10}$ n being integers

so this is the general solution set to this trigonometric equation in the next class we will discuss some more solutions to some more trigonometric equations till then thank you