

welcome to lecture five on trigonometric functions in the last lecture we ended with solving some problems we will continue to do

so in this lecture and will introduce another topic called trigonometric equations in this lecture and will follow it up in the subsequent lecture also

so this is the first problem of today's lecture

so we need to find the value of square root of 3 times cosec 20 minus secant 20 degrees we know that cosec is one upon sign and sec is one upon cos

so using that we get which is equal to square root of three times cos twenty degrees minus sine twenty degrees upon sine 20 degree into cos 20 degrees the denominator we see that there is a pattern because we know this formula that sign of two a is two times sin a cos a

so we have sin a cos a

so therefore the denominator becomes

so we are using the formula sine two a is two sine a cos a and therefore the denominator is equal to square root of three cos twenty degrees minus sine 20 degrees upon half times sine of 40 degrees because this there is a factor of 2 ah not here

so and then we use this formula with a equal to 20 degrees and we get this the numerator was square root of 3 times cos 20 degrees minus sine twenty degrees

so to solve this problem what we realize here is that this can be written as two times and the fact that i am using two is because this is of the form a into cos

so if you remember this formula cos a plus b is cos a cos b minus sin a sine b

so what we see here is that if you put a equal to 20 degrees here because in this expression we have cos of 20 degrees and on this after minus we have sine of 20 degrees

so and if you see this expression also we have cos of b and sine of b

so there is some similarity or there is some this it seems like this pattern is going to fit here

so that is why we recall this equation

so if we put b equal to twenty degrees here what we are going to get is cos of a plus twenty degrees is equal to cos of a cos of 20 degrees minus sine of a times sine of 20 degrees but then to exactly match this expression here with this we should have cos a to be this and sin a to be to be equal to one which is not possible because cos a cannot be the modulus of cos a cannot be more than one and what we have here is square root of three

so what we do for that is another the other thing is that we should we should choose i mean because cos square a plus sin square a is always one we need to normalize this ah expression here with something such that we should have this in the form a cos 20 degrees minus b sine 20 degrees multiplied with some other number c

so a cos twenty degree minus b sin twenty degree multiplied by some c

so we need to choose this a b and cs in such a way that because we we want this thing inside the bracket to be exactly ah like this pattern but because cos square a plus sin square a is one here what we should have is that a square plus b square should be one

so we should choose a and b in such a way that a square plus b square is one

so how do we ah do that its quite easy ah the way to do that is we because if you if you see here this a and b and c also satisfy the relation if i open up if i take the c inside we should have a times c equal to square root of three and because you have square root of three here

so a times c should be square root of three and then c times b should be one because we have c times b and we have one here right now if we ah square this and this and add

so we do $a^2 + b^2 = c^2$ what we get is this is three and this is one

so we get four here but then this can be written as this left hand side here can be written as $c^2 = a^2 + b^2$ which is equal to four but we already know that we should choose this a and b in such a way that $a^2 + b^2 = 1$ and therefore it turns if we use that fact here in this equation we get that $c^2 = 4$ therefore we can choose c to be equal to let's say two

so what we had in the last slide was this $\sqrt{3} \cos 20^\circ - \sin 20^\circ = a \cos 20^\circ - b \sin 20^\circ$ and we saw that c is equal to two and therefore it is very easy to now see that a is equal to $\frac{\sqrt{3}}{2}$ and b is equal to $\frac{1}{2}$

so what we have now is that this expression is equal to $2 \cos 20^\circ$ and now if you see if $a^2 + b^2 = 1$ and if you add them then you get $3 + 1 = 4$.

so we denote and we also if you remember the initial expansion of $\cos(a + b)$ that we wanted to use this expansion here then by comparison what we get is $\cos a = \frac{\sqrt{3}}{2}$ and $\sin a = \frac{1}{2}$ from this from these two it follows that a is equal to 30° or $\frac{\pi}{6}$

so a is 30°

so therefore finally what we get is that we can write $\sqrt{3} \cos 20^\circ - \sin 20^\circ = 2 \cos a$

so this is $\cos a$

so since a is 30° it is $\cos 30^\circ \cos 20^\circ - \sin 30^\circ \sin 20^\circ$

so that is equal to $\cos(30^\circ + 20^\circ)$ which is actually $\cos 50^\circ$

so this whole thing is equal to $2 \cos 50^\circ$ and then we go back to our problem that we were trying to solve what we initially started with

so here what we finally get is that this is equal to $2 \cos 50^\circ$ divided by $\frac{1}{2} \sin 40^\circ$ but we know that $\sin 40^\circ = \cos 50^\circ$ and therefore these two cancel out and the answer is therefore equal to 2 divided by $\frac{1}{2}$ which is four

so this is equal to four let us take another problem now

so again this problem appears to be a little difficult because we have angles which are not the common not those angles for which we usually learn the values of sine cosine and tan by heart but we do see that 6° and 66° if you take the difference it is equal to 60° and if you look at the sum of 42° and 78° that is 120°

so we do know the sine cosine and tan values for 60° and 120°

so what we will try to do is we will first try to write it this whole since $\tan x = \frac{\sin x}{\cos x}$ we will write each of these terms as

so we will write this first term as $\frac{\sin 6^\circ}{\cos 6^\circ}$ and $\frac{\sin 42^\circ}{\cos 42^\circ}$ likewise

so what we get is $\sin 6^\circ \sin 42^\circ$

so this whole left hand side is equal to this expression which I am writing upon

so all of these are degrees

so I am not writing it but these are all degrees 78° .

so now we will simplify both the numerator and the denominator one by one we start with the numerator and what we will do is we can see that there is a since we want to combine the six and sixty six first

so what we will do is we will first compute this because we know that $66^\circ - 6^\circ = 60^\circ$

6 is 60 degrees for which the value is known to us
so and this pattern is basically the two sine a sine b formula
so if you remember the two sine a sine b formula it was two sine a sine b is equal to cos of a minus b minus cos of a plus b
so with a equal to six and b equal to sixty six what we get here is that this is equal to half times cosine of a minus b which is minus sixty but cosine of minus sixty degrees is same as cosine of sixty
so cosine of sixty comes here but cosine of sixty degree is equal to half
so we write half and then minus cos of a plus b
so that is 72 degrees
so this is one of the products in the numerator sine 42 into sine seventy eight
so we have sine forty two into sine seventy eight and we again use the two sine a sine b formula
so what we get is this is equal to half of cos of a minus b which is cosine of thirty six degrees minus cosine of a plus b but cosine of a plus b is cosine of one twenty degrees and cosine of one twenty degrees is if we remember this formula cos of ninety degrees plus x is minus sine x and therefore cosine of one twenty degrees will be minus sine 30 degrees which is equal to minus half
so we put minus half here
so it becomes plus half and therefore the numerator is sine six sine sixty six into sine forty two into sine seventy eight equals one upon four into half minus cosine of seventy two degrees into half plus cosine of thirty six and we do a similar ah thing for the denominator
so if you recall the denominator was the product of all these cosine terms and again just like as we did for the numerator we will try to combine the the cosine of 6 with cosine of 66 and we will compute separately the product of cosine of 42 and the cosine of 78 degrees and here if we see we have a product of cosine
so we will use the two cos a cos b formula
so we start with cosine of six into cosine of sixty six and we remember this formula that two cos a cos b is cosine of a plus b plus sine of a minus b
so with a equal to six and b equal to sixty six what we get here is half of cos of
so a plus b is seventy two degrees and a minus b is sixty and cosine of sixty degrees is half and the other product in the denominator was cosine of forty two into cosine of seventy eight which again using the two cos a cos b formula we get this to be equal to half into cosine of a plus b is cosine of one twenty which as we just saw now is equal to minus half plus cosine of a minus b which is thirty six
so a minus b here is equal to minus thirty six but cos of minus thirty six is the same as cos of thirty six
so finally the denominator is equal to cosine of six into cosine of forty two into cosine of sixty six into cosine of seventy eight which is equal to one by four times half plus cosine of seventy two from here times this is cosine of 36 minus half and now we just need to divide the numerator divided by the denominator
so finally what we get is that the left hand side is equal to half minus cosine of seventy two times half plus cosine of 36 divided by and the denominator which we just computed now was equal to one by four times half plus cosine of seventy two times cosine of thirty six minus half of course one by four and one by four is common in the numerator and denominator let us expand the numerator and denominator now
so what we get is 1 by 4 plus half of cosine of 36 minus half of cosine of 72 minus cosine of thirty six into cosine of seventy two upon half of cos of thirty

six minus one by four plus cosine of thirty six into cosine of seventy two minus cosine of seventy two times half now we are asked to show that this is equal to one which means that we should be able to show that the numerator and the denominator are the same and what we see is that for example this term is here and this term is also present here

so if you want to show if we want to show that the numerator and the denominator are the same it suffices to show that the remaining terms in the numerator are equal to the remaining terms in the denominator which means that we just have to show that the remaining terms in the numerator is equal to cos thirty six times seventy two minus one by four

so so this is what we have to show and this can be simplified and written as of seventy two equals

so if we take this side and becomes two and

so two times cosine thirty six cosine seventy two equals half

so this is what we have to finally show if you want to solve this problem

so that is equivalent to equivalent to showing that cosine of thirty six times cosine of seventy two is equal to one upon four now we know that cosine of thirty six is the same as sine of fifty four degrees and cosine of 72 is the same as sine of 18 degrees and if we recollect from our last lecture the value of sine of 18 degrees that we had computed was square root of 5 minus 1 divided by 4 and from here we can find the value of sine of 54 degrees because if you remember this formula $\sin 3x = 3 \sin x - \sin^3 x$

so we put x equal to eighteen

so we get $\sin 54 = 3 \sin 18 - \sin^3 18$ and then we just ah instead of sine eighteen we just put this expression and what we will end up getting is

so we can ah try that out

so this will come out to be $\frac{\sqrt{5} + 1}{4}$ that is simple manipulation if you want to do it even more simpler then you can take this sign x common

so this will be $\sin x$ times in brackets $3 - \sin^2 x$ and then

so $\sin^2 18$ can be very easily computed

so you will end up getting this and therefore now if you if

so the final answer is we need to show that this is true and but this is equal to this and this is this product is equal to therefore

so this is $\frac{\sqrt{5} - 1}{4}$ times $\sin 54$ is $\frac{\sqrt{5} + 1}{4}$ which is equal to

so this final thing i rewrite it here

so this is equal to five minus one upon sixteen which is one by four and which is what we had to show

so this finishes the proof of the solves this problem also

so what was the one trick that was useful was that sometimes you have to remember the value of some of these angles like 18 degrees

so it can save time in the exam

so we discuss one more last problem before we go on to the next topic which is trigonometric equations

so here is the last problem

so we have to show that this expression is equal to $\frac{3}{2}$ and what we realize here is that $5\pi/8$ actually can be if you look at the difference between $5\pi/8$ and $\pi/8$ it is equal to $\pi/2$ similarly the difference between $7\pi/8$ and $3\pi/8$ is also $\pi/2$

so there are many ways of solving this problem you can do it either way

so i what the pattern that i saw was that $5\pi/8$ is equal to $\pi/8$ plus $\pi/2$ and therefore $\sin 5\pi/8 = \sin(\pi/8 + \pi/2)$ which you see here you see the fourth power of it is equal to $\sin^2 \pi/2$ plus and we know

that sign of π by two plus x is \cos of x

so using that result what we get here is that this is equal to \cos of π by eight

so we get an end up getting π by eight here and therefore we must combine

so essentially what we have is \sin four of this thing is \sin four of this is \cos four of π by eight

so this term here is essentially equal to \cos four π by eight and we have the same angle here π by eight π by eight

so we will try to somehow combine this ah term here with this term here and similarly ah you will we will also see that since seven π by eight is equal to π by two plus three π by eight this term here will be equal to \cos four of three π by eight and then we will combine this with this term

so that is the idea

so we we finally have the left hand side to be equal to this plus by eight plus \sin to the power four three π by eight plus \sin to the power four sorry \cos to the power four three π by eight

so this is the left hand side

so we we try to simplify this first and then we will take this later on

so this is the \sin four π by eight plus \cos four π by eight equals

so this is of the form a to the power four plus b to the power four and we can use this thing that a to the power four plus b to the power four can be written as a square plus b square the whole square minus two a square b square

so using this identity what we get here is this is equal to whole square minus two \sin square π by eight into \cos square π by eight but we immediately realized that this is of the form \sin square x plus \cos square x and therefore this is equal to one and one square is one

so this becomes 1 minus and we see here that even this thing can be written as half into 2 \sin π by 8 times \cos π by eight the whole square but we have a pattern here two $\sin a \cos a$ and this is equal to \sin of two a

so this this whole thing is equal to \sin of two times π by eight which is π by four

so finally we have this ah this term to be equal to one minus half into \sin square π by four which is equal to now \sin of π by four is one over root two

so \sin square π by four is half

so this is equal to one minus half times half which is three by four we now try to do the same thing with the other expression here

so the other expression was \sin four three π by eight plus \cos four π by eight but there is an interesting thing we do not need to do this all the way till the end because we can actually write \sin of three π by eight as \sin of now three π by eight is four π by eight minus π by eight and four π by eight is π by two

so we can write it as π by two minus π by eight and this is we know that \sin of π by two minus x is also \cos of x

so this is equal to \cos of π by eight and therefore \sin to the power four three π by eight is equal to \cos to the power four π by eight and similarly you can also show that \cos to the power of four three π by eight is equal to \sin to the power four π by eight in a similar manner and therefore adding these two is the same as adding these two

so this whole thing is equal to \cos of π by to the power four

so this is equal to this and this is equal to this plus \sin four π by eight but this is what we had just computed now if you see here this is what we are just computed now which was equal to three by four

so this is three by four and this is also three by four therefore and therefore finally we get three by four plus three by four is three over two

so that solves the problem we are going to start a new topic now which is called trigonometric equations and trigonometric equations essentially means equations which involve trigonometric functions

so all those functions that we have studied

so far of some variable

so here is an example $\sin x + \tan x = 2$

so in this lecture and the next lecture our focus will be dealing with such equations

so here we see that we have a sine function and a tangent function and the variable here is x

so mostly we will be dealing with single variable equations and our goal will be to find the solution to such equations by solution i mean the values of x for which this expression this left hand side is equal to the right hand side which is two of course a natural question that comes to mind is does the solution always exist and the clear answer is no for example if i say that find all solutions to the equation $\sin x = 2$ now since we know that the value of $\sin x$ or the range of the sine function is between minus one and plus one for no value of x can $\sin x$ be two and therefore there is no solution to this equation the other question is is the solution unique does there always exist unique solution again the clear answer is no because all these

trigonometric functions are periodic what i mean by periodic is for example \sin

so we know that the value of \sin repeats after an interval of 2π

so $\sin x = \sin(x + 2\pi)$ similarly for the cosine function we know that $\cos x = \cos(x + 2\pi)$ and $\tan x = \tan(x + \pi)$

so tangent repeats ah with π

so because these trigonometric functions are periodic it is clear that if there is a solution then suppose if we have an equation ah in variable x for example let us say $\sin x + \tan x = 2$

so suppose this we have a solution to this problem with x equal to some value θ what that means is that $\sin \theta + \tan \theta = 2$

so θ will satisfy this but θ is not the unique solution because if i instead of x equal to θ if i put x equal to $\theta + 2\pi$ then also what we will get is $\sin(\theta + 2\pi) + \tan(\theta + 2\pi) = 2$ now $\sin(\theta + 2\pi) = \sin \theta$ because \sin repeats at an interval of 2π

so so this first term here is $\sin \theta + \tan \theta + 2\pi$ is also $\tan \theta$ but since θ satisfies this equation $\sin \theta + \tan \theta = 2$ and therefore we see that even x in this equation even x equal to $\theta + 2\pi$ also satisfies this equation and therefore if x equal to θ is a solution then x equal to $\theta + 2\pi$ is also a solution and similarly you can show that $\theta + 4\pi$ $\theta + 6\pi$ in fact $\theta + 2\pi$ times any integer ah will also be a solution to this equation and therefore there are infinitely many solutions

so the solution is not unique

so let us take a very simple trigonometric equation and try to find out ah the values of x which solve this equation

so the solution to this equation now we know that ah for $\sin x$ to be equal to half if you look at the ah let me plot it very quickly for you

so then this will be zero this is $\pi/2$ and

so on $-\pi/2$

so we have $\pi/2$ somewhere here

so the

so what i am plotting is x on the horizontal axis and $y = \sin x$ on the vertical axis

so this is three pi by two
 so this maximum value is one and the minimum value is minus one and then it repeats like this similarly on the negative side also like that now we want to solve $\sin x$ equal to half
 so let us draw a line at half
 so which is here
 so we draw a line half
 so this value this ah this the y coordinate or the this displacement is equal to half
 so this is half half and then of course graphically the solution to this equation is all those values all those ah places where this red line is going to intersect the dotted curve for sine x
 so for example here and then here
 so so this is one value of x which gives you $\sin x$ equal to half then this is another value here and then another value here similarly on the negative side
 so as discussed before we will have infinitely many solutions but there are some solutions which will lie in the interval zero to two pi
 so this is the interval zero to two pi in this case we see that there are two solutions which lie in the interval zero to two pi
 so one is at x equal to pi by six which is thirty degrees which is this one here and the other is x equal to one hundred and fifty degrees which is five pi by six which is this point here now these such solutions which lie in the interval zero to two pi are called principal solutions
 so some very simple equations to start with
 so we have already discussed for example that for $\sin x$ equal to zero the general solution is that x is equal to an integer multiple of pi
 so n belongs to
 so this is the general solution of this equation
 so x is integer multiple of pi for $\cos x$ for this equation here $\cos x$ equal to zero the general solution as we have already discussed is n plus half times pi where n again is an integer
 so now we will try to generalize this concept of general solutions for that we need some tools or some results we start with the sign function
 so earlier in one of the previous slides we had seen ah how to find the solutions to $\sin x$ equal to half
 so this is something like that
 so we have $\sin x$ equal to sine of pi by six
 so let us say y equal to pi by six and then we would like to find all general solutions to this equation
 so we are not going to discuss how to do that
 so in general for x and y real if $\sin x$ is equal to $\sin y$ then we will show that x must be equal to n pi plus minus 1 to the power n times y for some integer n
 so if this if this equation is getting satisfied then it must be true that x and y should be related like this where n is some integer
 so n has to be an integer on the other hand we also see that if we take any integer n then the sign of n pi plus minus one to the power n times y will be equal to sine of y
 so that is also true
 so we have these two statements
 so these two will help us to find the general solution
 so i will take up that $\sin x$ equal to half and will try to see how using these two we will be able to find all general solutions to that equation $\sin x$ equal to half

so for $\sin x$ equal to half we have π equal to y equal to $\pi/6$ which is thirty degrees

so we have $\sin x$ equal to $\sin y$ y is $\pi/6$ and then if we go back what we had here is that for any n

so for any integer n \sin of $n\pi + 1 - 1$ to the power of n times y is \sin of y and therefore if we put y equal to $\pi/6$ then what we see is that \sin of $n\pi + 1 - 1$ to the power of n times $\pi/6$ is \sin of $\pi/6$ which is half and therefore this is the general solution for $\sin x$ equal to half

so x as long as it takes any value of this form $\sin x$ will always be half

so all those x are the way we write it is and I have again try to show it picture graphically here

so this was ah this was half and we had drawn a line like this and then if you try to see this expression $n\pi + 1 - 1$ to the power $n\pi/6$ we just have to put all integer values of n and we will get all the generals all these solutions to $\sin x$ equal to half for example if you put n equal to zero then n equal to zero is zero times π plus $1 - 1$ to the power zero times $\pi/6$ we get $\pi/6$

so that was the first ah solution the first principle solution if you put n equal to one we get 1 times π which is π plus $1 - 1$ to the power of 1 because n equal to 1 which is $1 - 1$ times $\pi/6$

so it is $\pi - \pi/6$ which is equal to $5\pi/6$

so that is this point here

so this was $\pi/6$

so $\pi/6$ and then say this is $5\pi/6$ and then if we put

so these two ah where the ah principal solution in this expression if you put n equal to two what we get is two π plus $1 - 1$ to the power two is one

so two π plus $\pi/6$

so this two π plus $\pi/6$ is is this point here it is this point

so this point is two π plus $\pi/6$ n equal to three we have $3\pi - \pi/6$ by 6

so that point is this one this one here

so this

so this value here is $3\pi - \pi/6$ and we can go on like this for n equal to four five and then similarly on the negative side also for example if we take n equal to minus one then what we get is $-\pi - \pi/6$ which is over here

so this point is $-\pi - \pi/6$

so and then we can do it for n equal to minus two

so so this is how the the general solution is written

so we write it as

so the general solution is x equal to $n\pi + 1 - 1$ to the power of $n\pi/6$ where n belongs to the set of integers

so this is how this set of all solutions is written let us try to prove it also

so what we had said is that \sin of $n\pi + 1 - 1$ to the power of n times y for any integer n is equal to \sin of y

so we will try to prove that statement

so of course this is of the form \sin of $a + b$ and we know that \sin of $a + b$ is $\sin a \cos b + \cos a \sin b$ and therefore this thing is equal to $\sin a \cos b + \cos a \sin b$ but we know that \sin of $n\pi$ for any integer n is always zero

so this whole term goes to zero

so what remains is \cos of $n\pi$ times \sin of $1 - 1$ to the power of n times y but what is \cos of $n\pi$ if you look at the graph for \cos of x versus x we will

you will realize that whenever

so whenever n is even we have \cos of $n\pi$ is equal to one and whenever n is odd \cos of $n\pi$ is equal to minus one and therefore we can obviously from this thing we can say that \cos of $n\pi$ is equal to minus one to the power of n because if you see at 1 minus one to the power of n when n is even minus one to the power of n is one and when n is odd minus one to the power of n is minus one.

so using this relation here this is equal

so again this is equal to minus one to the power of n into sine of minus one to the power of n into y again we divide the whole set of integers into n even and an odd and try to see what this expression is

so obviously when n is even

so when n is even this whole expression is equal to

so this is one and this is also one

so it is equal to sign of y and when n is odd this is minus one and this is minus one

so it is minus of sine of minus y but sine of minus y is equal to minus $\sin y$ and therefore this whole thing is also equal to sine of y

so irrespective of whether n is even or odd this whole expression is equal to sine of y and therefore we have shown that sine of $n\pi$ plus minus one to the power of n times y is equal to sine of y for all integer n and then we will show the reverse statement also that which we had stated earlier and which is that if $\sin x$ is equal to $\sin y$ for some x and y then it must be true that x should be equal to $n\pi$ plus minus one to the power of n times y for some integer n for some integer n

so we will try to show that

so we start with $\sin x$ equal to $\sin y$

so that implies that $\sin x$ minus $\sin y$ is zero and this is basically the ah the pattern here is $\sin a$ minus $\sin b$ which from one of the previous lectures is $2 \cos \frac{a+b}{2} \sin \frac{a-b}{2}$

so using that we get this left hand side to be equal to twice of cosine of $\frac{x+y}{2}$ into sine of $\frac{x-y}{2}$ equals zero but for this to be equal to zero we should either have either \cos of $\frac{x+y}{2}$ is equal to zero or sine of $\frac{x-y}{2}$ is equal to zero

so the first one is \cos of $\frac{x+y}{2}$ equal to zero but for this to be true $\frac{x+y}{2}$ should be a odd multiple of π

so so therefore what it means is that if \cos of $\frac{x+y}{2}$ is equal to zero it means that $\frac{x+y}{2}$ is equal to $m\pi + \frac{\pi}{2}$ for some integer m for some integer m this should be true but from here what we ah what we will get is that $x+y$ equals $2m\pi + \pi$ and therefore x is equal to $2m\pi + \pi - y$ for some integer m but i can also write this as x equal to $2m\pi + \pi - y$ because for any integer for because see we this statement has to be true from some integer m

so since m is an integer $2m + 1$ will be an odd valued integer and minus one to the power of an odd integer is equal to minus one

so therefore that is why this and these two are equal

so either it should be

so for $\sin x$ to be $\sin y$ either this statement should be true which is x should be equal to $2m\pi + \pi - y$ for some integer m or the other case is that sine of $\frac{x-y}{2}$ by two should be zero which is

so \sin of $\frac{x-y}{2}$ equals zero but for this to be true since we know that sine of θ equal to zero implies that θ is of the

form m times π where m is an integer

so this must be equal to m times π for some integer m and from there we get that x must be equal to $2m\pi + y$ for some m which is integer and this can be then written as $2m\pi + (-1)^{2m}y$ because $2m$ is an even number and minus one to the power of an even number is one so these two are equal

so finally what we have is that x is equal to either of this form or x must be of this form but in both cases what we see is that either what we see that the number here and the number in the power of minus one is the same because here we have $2m + 1$ and there we have $2m + 1$ and here also we have $2m$ and the same number comes here in the power of minus one further here we have all even integers because $2m$ is an even integer and here we have odd integers

so in either case it should be true that x should be equal to $n\pi + (-1)^n y$ for some integer n

so we finish this lecture with this proof in the next lecture we will try to do the same for cosine and tan functions

so we will try to find the general solution or will show how to find the general solution of equations like $\cos x = \cos y$ and $\tan x = \tan y$
thank you