

welcome to lecture four on trigonometric functions in lecture 3 we discussed and derived expressions for  $\sin x + y$ ,  $\sin x - y$ ,  $\sin 2x$ ,  $\sin 3x$ ,  $\cos 2x$ ,  $\cos 3x$  and we also formally defined the tangent function in today's lecture we will derive more expressions for the sum the tangent of the sum of two angles and the difference of two angles the tangent of twice of an angle and thrice of an angle and will introduce some more trigonometry functions like cosecant of  $x$  and second of  $x$  followed by some problems

so let us try to derive an expression for  $\tan$  of  $x + y$  in terms of  $\tan x$  and  $\tan y$  of course as we know that  $\tan$  of  $x$  is not defined when  $x$  is an odd multiple of  $\pi$  by two

so the following expression is going to be valid only when  $x + y$  is not an odd multiple of  $\pi$  by two now since  $\tan$  of  $x$  is  $\frac{\sin x}{\cos x}$

so  $\tan$  of  $x + y$  is equal to  $\frac{\sin(x + y)}{\cos(x + y)}$  in the previous lectures we derived expressions for  $\sin$  of  $x + y$  and  $\cos$  of  $x + y$

so using that we can write  $\sin$  of  $x + y$  as  $\sin x \cos y + \cos x \sin y$  upon  $\cos x \cos y - \sin x \sin y$  now dividing both the numerator and denominator by  $\cos x \cos y$  we get

so when we divide this term by  $\cos x \cos y$  we get  $\tan x$  because the  $\cos y$  gets cancelled with  $\cos y$  and

so this is what we get  $\frac{\sin x \cos y}{\cos x \cos y} + \frac{\cos x \sin y}{\cos x \cos y}$  and this is

so this term is  $\tan x$  this term is  $\tan y$  this is one and this is  $\tan x$  times  $\tan y$

so finally we get  $\tan$  of  $x + y$  equals  $\frac{\tan x + \tan y}{1 - \tan x \tan y}$  this expression is valid only when all of  $x$ ,  $y$  and  $x + y$  all these three are not odd multiples of  $\pi$  by two because  $\tan$  of  $x$  is not defined when  $x$  is an odd multiple of  $\pi$  by two it becomes unbounded

so this particular expression gives you a relation between the tangent of sum of two angles expressed as in terms of the tangent the separate tangents of the two angles from here it is very easy to now derive the expression for  $\tan$  of  $x - y$  because that can be written as  $\tan$  of  $x + (-y)$  and then we use this equation again

so essentially in this equation we replace  $y$  with  $-y$

so we get  $\tan x + \tan$  of  $-y$  upon  $1 - \tan$  of  $x \tan$  of  $-y$  we have seen from the previous lecture that  $\tan$  of  $x$  is an odd function of  $x$

so  $\tan$  of  $-y$  is  $-\tan y$  using that we get this to be equal to  $\frac{\tan x - \tan y}{1 + \tan x \tan y}$  let us now compute the expression for  $\tan 2x$  from the previous slide we saw that  $\tan$  of  $x + y$  is equal to  $\frac{\tan x + \tan y}{1 - \tan x \tan y}$

so using this expression and substituting  $y$  with  $x$  what we get is  $\tan$  of  $x + x$  which is  $\tan$  of  $2x$  equals  $\frac{\tan x + \tan x}{1 - \tan^2 x}$

so that is  $\frac{2 \tan x}{1 - \tan^2 x}$  but again this is well defined only when  $2x$  is not an odd multiple of  $\pi$  by two and also  $x$  because if  $x$  is an odd multiple of  $\pi$  by two then this  $\tan x$  here and this  $\tan x$  here is also not defined in a similar manner we can derive the expression for  $\tan$  of  $3x$  again using the expression for  $\tan$  of  $x + y$  we get  $\tan$  we have seen from the earlier slide that  $\tan$  of  $x + y$  is  $\frac{\tan x + \tan y}{1 - \tan x \tan y}$

so substituting  $y$  to be equal to  $2x$  we get  $\tan$  of  $x + 2x$  equals  $\frac{\tan x + \tan 2x}{1 - \tan x \tan 2x}$  and then we use the expression of  $\tan 2x$  from the previous slide which i just reproduce it again for your convenience

so  $\tan 2x$  is  $\frac{2 \tan x}{1 - \tan^2 x}$

so using that expression here what we get is  $\tan x + \frac{2 \tan x}{1 - \tan^2 x}$  and then multiplying both the numerator and denominator with  $1 - \tan^2 x$  we will get  $\tan^3 x + 2 \tan x = \tan x (1 - \tan^2 x + 2)$

so that is the numerator  $1 - \tan^2 x + 2 \tan^2 x$  which is equal to  $1 + \tan^2 x$  and then opening up this ah these braces here we finally get  $\tan^3 x + 2 \tan x = \frac{3 \tan x - \tan^3 x}{1 - \tan^2 x}$  again this is well defined only when  $3x$  is not an odd multiple of  $\frac{\pi}{2}$  we had introduced the cotangent function  $\cot x$  and we had defined that  $\cot x = \frac{1}{\tan x}$

so in this slide we will be trying to derive an expression of  $\cot(x + y)$  in terms of  $\cot x$  and  $\cot y$

so from here it follows that cotangent of  $x + y$  is  $\frac{1}{\tan(x + y)}$  and we know the expression for  $\tan(x + y)$  which is  $\frac{\tan x + \tan y}{1 - \tan x \tan y}$  using this expression here in the first equation here we get  $\cot(x + y) = \frac{1 - \tan x \tan y}{\tan x + \tan y}$  now dividing both the numerator and denominator with  $\tan x \tan y$  we get  $\frac{1}{\tan x \tan y} - 1$

so which is this term here  $\frac{1}{\tan x \tan y} - 1$  divided by  $\tan x + \tan y$  will give you  $\frac{1}{\tan x} \frac{1}{\tan y} - 1$

so here this will get cancelled and here  $\tan y$  is going to get cancelled the this particular term here is nothing but  $\frac{1}{\tan x} - \frac{1}{\tan y}$  one upon  $\tan x$  is  $\cot x$  one upon  $\tan y$  is  $\cot y$

so this term here is  $\cot x \cot y - 1$  and one upon  $\tan y$  here is  $\cot y$  one upon  $\tan x$  is  $\cot x$

so finally we end up with the expression  $\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$  now since cotangent of  $x$  is  $\frac{1}{\tan x}$  which is actually  $\frac{\cos x}{\sin x}$  cotangent of  $x$  becomes infinity plus minus infinity when  $\sin x$  goes to zero which happens when  $x$  is a multiple of  $\pi$

so this expression for cotangent of  $x + y$  is well defined only when  $x + y$  is not a multiple of  $\pi$  and of course from ah this expression of cotangent of  $x + y$  we can get the expression for cotangent of  $x - y$  we just need to replace  $y$  with  $-y$  in this equation here now since cotangent of  $x$  is  $\frac{1}{\tan x}$  and  $\tan x$  is an odd function

so it follows that cotangent of  $x - y$  will also be an odd function and therefore  $\cot(x - y) = \frac{\cot x \cot(-y) - 1}{\cot x + \cot(-y)}$

so cotangent of  $-y$  because cotangent of  $x$  is an odd function for tangent of  $-y$  will be  $-\cot y$

so this will become  $\frac{\cot x \cot y + 1}{\cot x - \cot y}$  and this expression is again well defined only when  $x - y$  is not a multiple of  $\pi$

so just like we derived the expressions for  $\tan(2x)$  and  $\tan(3x)$  we can derive the expressions for  $\cot(2x)$  in terms of  $\cot x$  we saw that  $\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$  and therefore substituting this  $y$  to be equal to  $x$

so with  $y$  equal to  $x$  what we end up getting is that  $\cot(2x) = \frac{\cot^2 x - 1}{2 \cot x}$

so we got an expression for  $\cot(2x)$  in terms of  $\cot x$  and similarly we can also derive a function for  $\cot(3x)$  in terms of  $\cot x$  we substitute  $y$  with  $2x$  in this equation we get  $\cot(x + 2x) = \frac{\cot x \cot(2x) - 1}{\cot x + \cot(2x)}$  which is  $3x = \frac{\cot x \cot(2x) - 1}{\cot x + \cot(2x)}$  and then we use the expression for  $\cot(2x)$  from the previous slide

so we get cot of  $x$  into cot of  $2x$  was  $\cot^2 x - 1$  upon  $2 \cot x$   
so multiplying the numerator and denominator with  $2 \cot x$  we finally get and  
this again is defined only when  $3x$  is not a multiple of  $\pi$

so in one of the previous slides we had defined the cotangent function to be  
one upon the tan function

so just like that we have to do we define another function called the cosecant  
function

so the name of the function is cosecant but we usually call it cosec in short  
and it is defined as cosec of  $x$  equals  $1$  upon sine  $x$

so from this definition it obviously follows that the domain of the cosecant  
function will be the same as the domain of the sine function which is the set of  
all real numbers further we know that for any  $x$  real  $|\sin x|$  is less than  
equal to one but cosecant  $x$  is  $1$  upon  $\sin x$  and therefore from this relation  
and this fact it follows that  $|\operatorname{cosec} x|$  for any  $x$  real is always greater  
than equal to one from which we can say that the range of the cosecant function  
is therefore the union of the in two intervals the first interval is from minus  
infinity to minus one union with

so so the value of minus one will be there in the set union with the other  
interval is from one to infinity

so this is the range set of the cosecant function just like the way we define  
the cosecant function another very popular trigonometric function is the secant  
function the name is secant in short we write it as sec sec of  $x$  is defined as  $1$   
upon  $\cos x$  and therefore the domain of the secant function will be the same as  
the domain of the cos function which is the set of all real numbers and again  
just like and as in the case of the cosecant function since for any  $x$  real  $|\cos x|$   
is less than equal to one

so using this fact and this definition it follows that for any  $x$  real  $|\sec x|$   
has to be greater than equal to  $1$  and therefore we can write that the  
range of the secant function is again the same as the range of the cosecant  
function which is the union of the interval minus infinity to minus one union  
one to infinity now that we have ah learnt a lot of identities and relations  
between different trigonometric functions and also expressions for the sum and  
differences the tangents and the sine and cosine of the sum and differences of  
angles let us try to solve some problems

so in this problem it is asked to show that the expression on this left hand  
side is equal to cot of  $2x$

so the main idea of solving problems is to find patterns and to try to apply  
the expressions and identities that we have learnt to patterns that you find out  
in the expressions in the question for example here we see that on the left hand  
side we have sum of two cosines and if you remember we had this identity that  
 $\cos a + \cos b$  equals  $2 \cos \frac{a+b}{2} \cos \frac{a-b}{2}$

so this was ah taught in one of the previous lectures and

so this is the slide from the previous lecture where we have shown this  
identity of  $\cos a + \cos b$  to be equal to  $2 \cos \frac{a+b}{2} \cos \frac{a-b}{2}$  and since we are getting this pattern here  $\cos a$

so this is  $a = 7x$   $b = 3x$

so by putting  $a$  equal to  $7x$  and  $b$  equal to  $3x$  what we get is  $\cos 7x + \cos 3x$  is two times

so  $a + b$  is  $10x$  and that divided by two will give you  $\cos 5x$   
and  $a - b$  is  $4x$

so upon two will be  $2 \cos 5x \cos 2x$

so  $\cos 2x$  and then in the denominator we see a pattern of the form  $\sin a - \sin b$   
and this was also discussed in one of the previous slides in

lecture three which i reproduce it in front of you now

so  $\sin a - \sin b$  this expression was derived

so we will try to use it here

so let me rewrite it for you  $\sin a - \sin b = 2 \cos \frac{a+b}{2} \sin \frac{a-b}{2}$

so again putting  $a = 7x$  and  $b = 3x$  we get  $\sin 7x - \sin 3x = 2 \cos \frac{7x+3x}{2} \sin \frac{7x-3x}{2}$  will be  $5x$  and  $\frac{a-b}{2}$  will be  $2x$

so we

so finally we get these expressions for the numerator and the

so this is for the numerator and this is for the denominator and then when we divide these two it is equivalent to dividing this by this

so what we get is that the left hand side here will be equal to the numerator is equal to  $2 \cos 5x \sin 2x$  and this denominator is equal to this expression here  $2 \cos 5x \sin 2x$

so two and two gets cancelled  $\cos 5x$  and  $\cos 5x$  gets cancelled what remains is  $\cos 2x$  upon  $\sin 2x$  which is equal to  $\cot 2x$  which is the right hand side here and that completes the proof of the fact that this is equal to this

so what we learnt here was to find out patterns in the question or in the expressions that are there and try to see if we can apply whatever expressions we have previously learnt to the question here for example in this case we identified the patterns  $\cos a + \cos b$  and  $\sin a - \sin b$  and we express them as product of  $\cos$  and  $\sin$  which led to the cancellation and then final answer another similar question here we have the sum of three cosines

so either we can start with adding this and this first and then we add this to the sum of these two or we can add  $\cos 3x$  and  $\cos 5x$  first and then later on add  $\cos 4x$

so which way do we go with do we add  $\cos 3x$  with  $\cos 4x$  using the  $\cos a + \cos b$  formula and then add  $\cos 5x$  the problem is that if we add  $\cos 3x$  and  $\cos 4x$  first if you remember the formula for  $\cos a + \cos b$  it is  $2 \cos \frac{a+b}{2} \cos \frac{a-b}{2}$  the terms that we will get is  $\cos 7x$  by two

so so if you take  $a = 3x$  and  $b = 4x$

so we are adding this and this we get  $2 \cos 7x$  by two and  $\cos x$  by two the problem is that then these two terms have nothing in common with  $\cos 5x$

so fact

so it will be very difficult to factor out something common because ultimately if you see what we need here on the right hand side is  $4x$

so therefore choosing trying to add this and this first is not the right strategy it will lead to waste of time in the exam

so the other option could be to add  $3x$  and  $5x$  and that is better because when you add  $3x$

so when you write  $\cos 3x + \cos 5x$  what you get is  $2 \cos 4x \cos x$

so this is  $\cos 3x + \cos 5x$  now you also need to add  $\cos 4x$  now but the good thing now is that this this this already has this  $\cos 4x$  factor

so it becomes easy to further combine this term and this  $\cos 4x$  another thing is that the right hand side also has  $4x$

so that is why we should take this root instead of adding  $3x$  and  $4x$  first we should add  $3x$  and  $5x$  first

so then the numerator can be written as

so the numerator of the left hand side can be written as  $2 \cos 4x$  times

cos x which is the sum of cos three x and cos five x and then plus cos four x  
so cos four x can be factored out and it can be written as cos four x into one plus two cos x

so for the same reason on the denominator we will try to add sin three x with sine five x first

so if you remember the formula for sine a plus sign b and this was derived in the previous lecture to be equal to two sine a plus b over two times cos of a minus b over 2

so therefore sine of 3 x plus sine of five x equals two times sine of four x because a is three x and

so a plus b is eight x upon two it becomes four x into cos of a minus b by two so cos of

so a minus b is minus two x

so cos of minus two x upon two is cos of minus x but cos of minus x is the same as cos of x

so this is what we get and then the final expression for the denominator on the left hand side will be we just add this expression to sign four x

so what we get is sin four x again is common

so we can write it as sine four x times one plus two cos x

so this is the denominator

so this is the numerator and this is the denominator and then finally when we divide the numerator by the denominator what we see is that this term 1 plus 2 cos x is there in both the numerator and the denominator

so when we divide it gets cancelled and then we get cos 4 x divided by sin 4 x which is equal to the right hand side which is cot 4x

so through this example we saw that it is very important that we decide which factors should be added first otherwise it might result in loss of time another interesting problem is the following

so it is asking us to compute the value of sine of 18 degrees now what we realize here is that since sine of x is equal to cos of pi by 2 minus x this is another identity that we had discussed in the previous lecture we see that sine of 36 degrees is equal to cosine of 54 degrees the reason why we have chosen 36 and 54 is because first of all they add up to 90 degrees the other reason being that they are both multiples of 18 degrees

so the idea is to use the formula for sine 2 theta and cos 3 theta with theta equal to 18 degrees because with theta equal to 18 degrees 2 theta is 36 degrees and 3 theta is 54 degrees now we know that sine 2 theta is 2 sine theta cos theta and we also know that cos of three theta is four cos cube theta minus three cos theta both these expressions we derived in the previous lecture

so from here what we get is that with theta equal to eighteen since this and this is equal we can write that 2 sine theta cos theta minus 4 cos cube theta minus 3 cos theta equals zero because this is sine two theta and this is cos three theta and for theta equal to eighteen degrees they are equal now we see that cos theta is a common factor in all these terms

so we can write this as cos theta into 2 sine theta minus 4 cos square theta plus three equals zero but

so the possible solution here is that either this term is zero or this term is 0 but for theta equal to 18 degrees we know that cos of 18 degrees is not equal to 0 therefore the only way this equation can be satisfied is if this term is equal to zero that is for theta equal to eighteen degrees this equation is satisfied four cos square theta minus two sin theta minus three equals zero but we know that cos square theta is equal to one minus sin square theta

so using that we get 4 minus 4 sine square theta minus 2 sine theta minus 3 equals 0 and well taking it to the other side that can be written as 4 sine

square theta plus two sine theta minus one equals zero

so theta equal to eighteen degrees satisfies this equation now this is basically as the left hand side here is a quadratic polynomial in sin theta

so let us say that z we define it to be sin theta

so what we get is four z square plus two z minus one equals zero

so there are two possible solutions to this quadratic equation and the solutions are z equals minus two plus minus of twenty by eight since sign of eighteen degrees is positive the only possible route that makes sense is with the plus sign here

so we finally get that sine of 18 degrees is equal to minus 2 plus root over 20 upon 8 which can also be written as square root of five minus one upon four some more problems moving ahead

so in this problem we are asked to again prove that this expression on the left hand side and this expression on the right hand side are both equal

so if we see the expression on the right hand side we know that cosecant of x is one upon sine x and cotangent of x is one upon tan x but we can also write it as cosine of x upon sine x

so this becomes equal to one minus cosine of x by sine of x

so what we see now is that this one minus cosine x of comes here as well as here

so if you want this numerator to be equal to but the problem is that here one minus cos x is inside the square root

so for one minus cos x to be outside the square root one way to do that is that we multiply the left hand side both the numerator and the denominator with square root of 1 minus cos x

so we multiply both the numerator and the denominator of the left hand side with square root of one minus cosine x times

so the numerator now becomes one minus cos x as what we wanted and the denominator becomes root of one upon cos square x but we know that one minus cos square x is sin square x and then the root of sin square x will be sin x

so this is equal to which is nothing but the right hand side here that finishes the proof for this question some more tricky problems

so in this question we are asked to find the value of cos of 40 degrees minus cosine of 20 degrees plus cosine of 80 degrees

so it at first this might appear to be a very tough problem because all these angles are angles for which the sine and cosine is not known to us we typically usually remember sine and cosine of 45 degrees or 30 degrees or 60 degrees or maybe we can calculate it for 15 and 75 degrees

so this might appear a little daunting but then the the trick to be seen here is that again we see that we are adding and subtracting cosines

so immediately we should try to recollect the cos a plus cos b formula which is cos a plus cos b equals two cos plus b upon two into cos minus b upon two

so this gives some hope because if we choose this a and b correctly from these three terms then maybe one of these cos angles either a plus b by two or a minus b by two could be an angle for which we know the value of cosine and that might help us to solve the problem now looking at these three angles we see that if we take a to be 40 degrees and b to be 80 degrees then we see that 40 plus 80 divided by 2 is 120 divided by 2 which is 60 degrees and it is we know that cosine of 60 degree is half

so let us try this route this path

so cos of 40 degree plus cos of 80 is using this formula 2 times cos of 60 degrees times cos of 80 minus 40

so that is minus that is 40 upon 2

so that will be 20 degrees

so this becomes equal to now  $\cos 60$  degrees we know that  $\cos$  of sixty degree is equal to half

so putting it here we get this to be equal to  $\cos$  of twenty degrees and then the final expression is obviously 0 because this the sum of  $\cos 40$  and  $\cos 80$  is  $\cos 20$  and we are subtracting  $\cos 20$  here

so the final answer is 0.

this is a problem from one of the je exams

so again to start with this problem looks very terrifying because you go all the way to  $8\theta$  starting from  $\theta$  but ah again what we have to always do is to see patterns in the expression

so the pattern here is that from the first expression to the second from the first term to the second term the the angle inside the tangent is doubling and again from here to here it is doubling and then again here to here

so probably there it seems like like the formula of  $\tan$  of  $2x$  might be handy so if you remember  $\tan$  of  $2x$  was equal to  $2 \tan x$  divided by  $1 - \tan^2 x$  now let us start from let us start with the left hand side the the last term in the left hand side is  $8 \cot 8\theta$  which can be actually written as

so  $\cot 8\theta$  is  $1 / \tan 8\theta$

so taking  $x$  to be equal to  $4\theta$  we have  $\tan 8\theta = 2 \tan 4\theta / (1 - \tan^2 4\theta)$

so to get this term we need to invert this expression

so what we get is  $8 \cot 8\theta + 4$  and we realize that ah this  $\cot 8\theta$  expression will have  $\tan 4\theta$  and the next expression the next term is  $\tan 4\theta$

so we will try to combine it with it

so  $8 \cot 8\theta + 4 \tan 4\theta$  will be equal to  $8 / (1 - \tan^2 4\theta) + 4 \tan 4\theta$

so this becomes  $4 / (1 - \tan^2 4\theta)$

so that is equal to  $4$  times because we have  $4$  here and here also and we simplify it

so it becomes

so this becomes  $1 / \tan 4\theta$  because  $\tan 4\theta \times \tan 4\theta$  is  $\tan^2 4\theta$  that gets cancelled with this minus  $\tan^2 4\theta$  here

so the left hand side now reduces to  $\tan \theta + 2 \tan 2\theta + 4 \tan 4\theta$  and then plus four by ten and then we repeat the same process we write  $\tan 4\theta$  to be equal to  $2 \tan 2\theta / (1 - \tan^2 2\theta)$  because now the next term after before this term is  $\tan 2\theta$

so we would like to express this  $\tan 4\theta$  in terms of  $\tan 2\theta$

so that probably will be when we combine this and this term we might be able to cancel some terms there

so that is the idea

so then when we look at this sum what we get is  $4 / (1 - \tan^2 2\theta) + 2 \tan 2\theta$  is equal to  $4 / (1 - \tan^2 2\theta) + 2 \tan 2\theta$  plus  $2 \tan 2\theta$  is two

so we get here two upon

so if you simplify this you will get two upon  $\tan 2\theta$  and then finally the left hand side

so this is the left hand side

so the finally the left hand side becomes  $\tan \theta + 2 / \tan 2\theta$  and now again using the we need to express  $\tan 2\theta$  in terms of  $\tan \theta$

so that there could be some cancellation of some terms here we know that  $\tan$  of

$2 \tan \theta$  is  $2 \tan \theta$  upon  $1 - \tan^2 \theta$   
 so using this expression here what we get is this to be equal to  $\tan \theta$  plus  $2 \tan \theta$  upon  $1 - \tan^2 \theta$  by  $2 \tan \theta$   
 so this gets cancelled and  
 so this  $\tan \theta$  will get cancelled with this  $1 - \tan^2 \theta$  by  $\tan \theta$   
 so ultimately what will remain is  $1$  by  $\tan \theta$  which is actually  $\cot \theta$   
 so that is what was the right hand side  
 so that finishes the proof of this seemingly very difficult problem  
 so let us discuss another problem here  
 so again we have to show that this left hand side is equal to this right hand side and even here we see a pattern that there is an angle  $a$  and then  $2a$  and then  $4a$  and then  $8a$   
 so what we see is we see a  $\sin 8a$  and a  $\cos 4a$  and that should immediately ring an alarm bell in your minds that we know that  $\sin 2\theta$  is  $2 \sin \theta \cos \theta$   
 so if we put  $\theta$  equal to  $4a$  what will get here is  $\sin 8a$  is  $2 \sin 4a \cos 4a$   
 so we get a  $\cos 4a$  here and that should hopefully cancel this  $\cos 4a$   
 so if you look at this right hand side what we will get is  $\sin 8a$  upon  $8 \sin a$  will be simply  $2 \sin 4a \cos 4a$  upon  $8 \sin a$   
 so at least now we have got one term which is there on the lhs  
 so now we just need to show that this is exactly equal to  
 so we need to show that this is equal to  $\cos a$  into  $\cos 2a$  and that can be done similarly because we know that  $\sin 4a$  will have this  $\cos 2a$  term again using this formula but with  $\theta$  equal to  $2a$  what we get is  $\sin 4a$  equals  $2 \sin 2a \cos 2a$   
 so therefore  
 so we have to show that  $2 \sin 4a$  upon  $8 \sin a$  equals  
 so this is what we have to show and just now we derived this expression that  $\sin 4a$  equals  
 so this is what we have to show actually we derived this expression we wrote that  $\sin 4a$  is  $2 \sin 2a \cos 2a$   
 so now ah putting replacing this  $\sin 4a$  with this over here what we end up getting is  $\sin 2a$  into  $\cos 2a$  upon  $2 \sin a$   
 so if  
 so so this thing is equal to this and you can see that now we also get this  $\cos 2a$  term here and now it is very easy because we know that  $\sin 2a$   
 so if you now just look at this term that term is nothing but  $2 \sin a \cos a$  is  $\sin 2a$   
 so this gets cancelled and then this becomes equal to  $\cos a$  and  $\cos 2a$   
 so this becomes equal to  $\cos a \cos 2a$   
 so what should be realized by these examples is that we should always try to see for patterns and try to make the right judgment in terms of the route to be followed for the proof because most of these competitive exams are time bound and in the next lecture we will continue to solve some more problems ah which will ah which will basically make you comfortable with solving these type of problems thank you you