welcome students today's topic will be on cartesian products and relations in the last class we started with the cartesian product of two sets in fact we started with ordered pairs and use this ordered pairs to define the notion of cartesian product of two sets and finally we ended up with the cartesian product of two sets well one can also define cartesian product of n sets but that is not the motive behind these lectures now let us proceed with some more examples we begin with an example or a problem let a equal to one two three b equal to three four and c equal to four five six let us find the following cartesian products a cross b intersection with c second a cross b intersection with a cross e third a cross b union c and fourth a cross b union a cross e let us find the following sets first one we are given that a is one two three b is given that b is 3 4 and c is 4 5 and 6.

therefore b intersection with c the common element between them is exactly 4 therefore b intersection c is only four now a cross b a cross b intersection c is all possible ordered pairs from a and b intersection c the first element from an ordered pair in a cross b or in a cross b intersection c the first element should be from a and the second element should be from b intersection c

so the first element is one four and there is no other element in b intersection c

so two comma four three comma four this is a cross b intersection c now let us try to calculate a cross b which is

so b is 3 4 therefore 1 3 and one four similarly two three two four three three and three four this is a cross b now let us try to calculate b cross c sorry a cross e and we know that c is four five six

so this is one comma four one comma five one comma six two comma four two comma five two comma six three comma four three comma five and the last element three comma six now let us try to calculate a cross b intersection with a cross c if you look back into what we wrote down for a cross b then you will notice that elements of the form three comma four one comma four two comma four are there one comma four two comma four and three comma four these are the elements that are common to both a cross b and a cross e and in fact these are the only three elements fine now notice the following fact a cross b intersection c is equal to a cross b intersection with a cross c let us do the third one we will have to calculate a cross b union c

so the b that is given to us is 3 and 4 and c that is given to us is 4 5 and 6.

therefore b union c is 3 4 5 and 6 right and a what is given to us is just one two three therefore a cross b union c equal to the following ordered pairs one comma three one comma four one comma five one comma six two comma three two comma four two comma five two comma six three comma three three comma four three comma five and finally three comma six

so well we all know that the number of elements in a cross b is going to be the number of elements in a times the number of elements in b now the number of elements in a is three and the number of elements in b union c c is four therefore the number of elements in a cross b union c is going to be three into 4 which is 12 and you can notice that 1 2 3 4 5 6 7 8 9 10 11 12 therefore we haven't missed any element of a cross b union c now let us try to calculate a cross b and b cross c and then take the reunion a cross b is going to be in fact these two things were calculated earlier when we did the first one or the second one

so a cross b and let me say that and b a cross c were calculated earlier right now let us just take that union a cross b union with a cross e a cross b consists of one three one four two three two four three three three four while a cross c the other one consisted of one four which have we have already returned the other one one five one six and then the next one is two four which we have

already returned
so we will have two five two six three four is already returned
so we will have three five and three six now again let us note the following that a cross b union c is equal to a cross b union a cross c right now let us do one more problem or an example
so let p equal to a b and c
so out of this b let us form the set p cross p cross p let us co form the cartesian product of p with itself three times which is a p cross p cross p p cross p cross p equal to a a a a a b a a c a b a a b b a b c a c a a e b a c c and then b a a b a b b a c b b a b b b b b c b c a b c b b c c c a a c a b c a c c b a c b b c b c c c a c c b and the last one c c c you can notice that this cartesian product p cross p cross p is going to have
so the total number of elements in p cross p cross p is going to be 27 now with all these things lets move on to the next topic of what is known what are known as relation now yesterday we had some sets of the following form all those ordered pairs x comma y such that x and y or real numbers both are real numbers and we had one more thing what is known as x squared plus y square is one what we found yesterday is that this set is not a cartesian product of two sets that is what we had although this is a subset of r two although this is a subset of r two the of course this is at example for a geometric object the circle this is not this set although it contains ordered pairs this is not a cartesian product of two sets
so this is not a cartesian product of two sets but this is just a product this is just a subset of the cartesian product r cross r well let us look at one more thing suppose that a has got the following things which is ramu babu ramesh kumar and siva let us have the
so a consists of five names of some people someone let b denote the following names lakshmi manju money and these three if you look at the cartesian product of these three sets these two sets you will have five into three the totally set consisting of fifteen elements but if i say that just say that there is a relation between a and b if a and b the an element of a is related to b if a an element of a for example small a is related to b if they are married
so let us write down this one let me just say r as the following things ramesh mani second one ramu manju and finally babu lakshmi right
so in fact how are how are these things done ramesh and money they are married together they both are married together and similarly ramu and manju they are married and the last one babu and lakshmi they are married
so we are interested in the pair we need some relation between two tooth names or elements between two sets
so let us make it make this formally write it as a definition let a and b be any two non empty sets right a relation r from a to b is a non empty subset of is a non empty subset of a cross b
so relation is just a non empty subset of a cross b
so natural question is how many possible how many relations are possible between a and b we know that
so let us make it as a remark the number of elements of a cross b is same as number of elements of a times the number of elements of b but when we are interested in the relations which means we are interested in in subsets of a cross b
so the natural question is how many relations are possible between a and b put in other words how many subsets of a cross b are possible a is a set is any set any non empty set then the number of possible subsets sets of a is to power the cardinality of a this much is known to us but if you look at the definition of the relation it is defined as the non empty subset and one will have to always

notice that the all possible subsets of a includes even the empty set therefore the number of possible possible subsets subsets in fact i should say non empty subsets of a cross b is 2 power the cardinality of a cross b minus one therefore the total number of possible relations between a and b is two power the cardinality of a cross b minus one

so this many relations are actually possible

so the examples in fact the example that we gave the first example which is exactly the circle and the second example the natural possible one

so two persons are related when they are husband and wife this these are examples for relations now let us go with some more examples let a equal to one two three b equal to two three four

so first thing that one will have to observe is that notice that a cross b is itself a relation between a and b

so this relation this relation is called the universal relation right in fact for any two sets a and b one can always define

so a cross b makes sense and therefore this a cross b is what is known as the universal relation for any two sets a and b a cross b is called the universal relation fine now lets go back to the last example and then lets see

so what we had is a equal to one two three and we had b as two three four okay now let us come up with a new relation or a subset of a cross b one one one two sorry one two one three two two two three three four

so this is a subset of let us try to represent this pictorially the first set have got one two and three let us write down the second one which is two three and four let us look at the first one the first element of the relation is one and two

so one is related to two

so let us draw an arrow diagram between one and two second one is one and three

so let us again draw one more line this time from one to three third one two two fourth one two three and fifth one three four right let us go back to the second example that we had in terms of names right let us look at the second example again we had a as ramu babu ramesh kumar and siva the second set b contains lakshmi manju and money and the relation that we had are is ramesh kama mani ramu kama manju and babu kama lakshmi now lets try to write this or represent this again in terms of pictorial diagram ramu babu ramesh kumar siva on the other hand we have lakshmi manju and money

so ramu is associated with manju is related to manju and ramesh is associated to money and finally babu with lakshmi these are the things if you look at the previous examples the two examples that we had

so in the second the second example only very few elements are mapped to elements of the second set few elements only three elements of the first set or set a or mapped to or related to elements of b let us do one more example a equal to one two three four and five and b minus 1 0 4 9 25 now let me define r as all those x comma y in a cross b with the given thing that y is equal to x square right

so now let us try to expand this and then see that whenever the element y the second whenever you have an ordered pair the second element should be related to the first element by the condition that y should be of the form x square which means the only possibilities are two comma four three comma nine and five comma twenty five let us again try to pictorially represent this one two three four and five minus one zero four nine and twenty five two is related to four three is related to nine and five is related to twenty five right now if you look at this one only very few few element only few elements of a are mapped to few elements of b and

so not all elements of a are mapped to all elements of b

so few elements of a are left out and similarly few elements of b are also left out let us have one more definition let a and b be two non empty sets and let r b a relation between a and b then the domain of r is the set of all first elements from the ordered pairs n r the set b is called as the codomine of r and the set the the set of all second elements from the ordered pairs in r is called the range of r now let us do one example let a equal to one two three four five and six and b equal to sorry we do not need any b here r is all those x comma y in a cross a such that y equal to x plus one fine now lets write down explicitly what this r is if you notice our set a is 1 2 3 4 5 and 6 and our relation is all those ordered pairs x and x comma y such that x is of the or y is of the form x plus one and

so r is going to be one comma two two comma three three comma four four comma five and five comma six now let's write down now let us try to draw the pick try to represent this pictorially one two three four five and six one two three four five and six now let us try to represent one is related to two two is related to three three is related to four four is related to five and five is related to 6.

so now domain of r is going to be all first elements

so which is going to be one two three four and five and co domain

so all first element but you can notice that six is not related and therefore 6 is not going to be part of the domain codomain of r it is the whole thing 1 2 3 4 5 and 6 while range of r is going to be two three four five and six right you have the following let us look at one more example a is going to be 4 9 10 25 and b this is similar to the previous example minus five minus three minus two one two three and five now let me define r as all those ordered pairs x comma y in a cross b with the condition that x is the square of y or y square is x that is what we want

so let us try to write down all ordered pairs four comma minus two four comma two nine comma minus three nine comma three twenty five comma minus five and twenty five comma five now let us try to represent them pictorially four nine ten twenty five minus five minus three minus two one two three and five four is related to minus two as well as to two nine is related to both three as well as three and twenty five is related to five as well as to minus five right this is the diagram that we have the first one represents the set a while the second one represents the set b now let us try to write down the domain co domain and the range domain look at domain of r look at all the first elements of the ordered pair that appear in r the first elements are going to be 4 9 and 25 while codomin of r is going to be it is the whole of b which is minus five minus three minus two one two three and five now the range of r is going to be minus 2 minus 3 minus 2 minus five and then you will have two three and five this is going to be the range of r now lets look at the same example

so now if you look at this example the number of elements of a is going to be is exactly four and the number of elements of b in this case is seven therefore the number of elements in the cartesian product is going to be four into seven which is twenty eight therefore the number of possible relationships or relations from a to b is 2 power 28 minus 1 well this is going to be a huge number but actually the truth is this many number of relations are actually possible of course we may not be able to write all these 2 power 28 minus 1 relations but but one should know that this many relations are actually possible let us do one more example let us consider consider the set of all natural numbers and then define r as all those n comma m in cross n well this n denotes the set of natural numbers let me write down on the top n is the set of all natural numbers such that with the condition that m is of the form n plus five right

so if you look at this one
so first thing that you will have to notice that r is an infinite set
so one can also write r as all those n comma n plus five with the condition that n is a natural number now lets go go to one more example let us restrict this subset r and then see some more all those n comma m and n cross n with the condition that m equal to n plus five and n less than or equal to four n can be at most four now let us write down explicitly
so now you should notice that this r dash is a finite set and now let us try to write down this set it is going to be one
so m is going to be n plus five one comma six and then two comma seven three comma eight four comma nine these are the only possible things that you have right
so by imposing conditions or giving by imposing more relations you can notice that the subset that you are going to get is going to be smaller and smaller the first relation that just like saying that m is n plus 5 just by imposing a relation what we had is an infinite set but just by imposing one more condition like n less than or equal to 4 we are reduced to a just a finite set in fact a set with just four elements let us look at one more example let a equal to one two three five and b is four six and nine let me define r as all those x comma y in a cross b with the condition that the difference between x and y is an odd number right now let us write down r in a more explicit form let us d write down all the members of r what we wanted is that the difference should be an odd number fine now let us look at the first one what we have is one one minus four the different four minus one the difference is just 3 which is an odd number
so we will have 1 comma 4 1 comma 6 the difference is 5
so we have this 1 comma 9 the difference is 8 therefore this cannot be an element of r let us look at the second one 2 comma 4 the difference 4 by 4 comma 4 minus 2 is 2 which is an even number therefore cannot be part of r
so 2 comma 6 the difference is 4 therefore cannot be part of r similarly now on the other hand 2 comma 9 the difference is 7 and which is an odd number therefore it is part of this r now the next one three three comma four the difference is one which is an even number
so will half this three comma six the difference is three three comma nine the difference is six which is an even number therefore cannot be part of r and then five comma four the difference is one five comma six the difference is one while phi comma nine the difference is four and cannot be part of r right
so now let once again let us try to represent this pictorially this is our set a on the other hand we have the set b now one is related to four one is related to six two is related to nine three is related to four three is related to six five is related to four and five finally five is related to six
so these are the only relations that we have between the elements of a and the elements of b now let us look at the same example but instead of same set a one two three five instead of going to a cross b the one more set lets define the relation r on a that is this relation r is going to be a subset of a cross a r as all those x comma y in a cross a with the condition that the difference between x and y is an odd number right now lets write down r explicitly once again
so you will have this one comma two one comma three the difference is two therefore cannot be and similarly one comma five two comma one the difference is one two comma three the difference is one two comma three the difference is three now three comma one the difference is two therefore cannot be part of r three comma two the difference is one three comma five the is two therefore cannot be part of r now phi comma one the difference is four five comma two the difference is three and five comma three the difference is 2 5 comma 5 the

difference is 0 fine now with all these things now let us notice one thing in this whenever you have first thing that one can observe here is whenever the pair x comma y belongs to r that immediately implies us that y comma x also belongs to r

so lets use this example and then define the following thing let a be a non empty set and let rb ah relation on a that means what that is r is a non empty subset of a cross a we say that r is symmetric if x comma y belongs to r implies y comma x belongs to r right whenever the pair x comma y is in r the other the opposite power y comma x is also in r such pair such as relation is called as what is known as a symmetric relation

so in the next class we will deal more with symmetric relations and will give some more examples of relations and will also deal with the notion of what is known as the concept of a function

so will stop here thank you all you