welcome students today's lecture will be on cartesian product before we start the cartesian product lets try to do some examples and try to understand the importance of the notion of ordered pairs lets before we proceed lets do some examples we now we have a set a consisting of three bags let us name it as x y and z

so which means that x is a bag y is a bag and that is a bag of course each bag can contain as many balls but let us restrict that a bag can at most contain only one ball and you can put only one ball in it

so let b denote set of all balls with colors either blue or red

so the first set contain exactly three bags we have named it as x y and z we have another set consisting of balls but our interest lies only on the colors of the ball the colors of the ball are exactly blue and red now what are the various possibilities in through which you can fill those bags you can fill the bag x with a blue color ball or you can fill the bag x with a red color ball because we know that each bag can contain only one ball the second one similarly y the bag y you can fill it with a blue color ball or y with a red color ball right again you have the z color bag z named bag that bag wherein you can put the blue ball or z with a red ball what is that that we have now that what is that that we here have here is that to each color we have associated a bag and to each bag we have associated a color of the ball

so let us write all those things as pairs first one says you have you have chosen the bag x and you are trying to put the blue ball inside it the second one says you have the bag x and the red ball is there third one the bag y with the blue colored ball fourth one bag y with a red colored ball fifth one the back z with a blue colored ball and finally back head with a red colored ball totally we have six possibilities right

so we

so given any let us write it formally as given any a in a and b in b we obtained a pair

so we obtained a pair a comma b well let us go to the second example which i will call it as vehicles and their name plates right let a denote the set of some of the states set of some of the states let me call it as delhi madhya pradesh uttar pradesh andhra pradesh tamil nadu we have five states and on the other hand let me have the set b as the numbers zero one zero two up to zero nine what we have is ah we have two sets a and b the first set a consisting of four five states and the set b containing nine numbers supposing that i wanted to form a name plate or a number plate for a vehicle suppose that the person is from madhya pradesh what is that possibility

so the possibility of a person well not a person of our name plate the possibility of a name plate for a person coming from say m p is well let us have it as m p zero three this is one of the possibilities well this is not the usual one that we see in day to day life but this is one of the possibilities that we have in fact what is that that is happening given a n a and small b in capital b the pair a comma b right the pair a comma b tells you the first a tells you the state in which the vehicle belongs to right pair a comma b c is the following a denotes the state b denotes the number of the vehicle

so in the last two examples we saw that there are situations which gives raise to pairs a comma b

so let us make it formally and define things like given sets a and b and elements a comma b with a in a and b in b the pair a comma b is called an ordered pair right the pair a comma b this pair i will call it as an ordered pack well let us look at one more example let us look at all those elements x comma y in all those x comma y such that x and y are x comma y or real numbers and x squared plus y squared equal to one by now it is clear that this set

represents a circle right we are only trying to write a circle as a in the form of a set right

so elements of this set elements of this set are elements belonging to a circle or those points lying on the circle are examples for ordered pass right now having said this ordered pairs let us go ahead and then define the notion of what is known as the cartesian product of two sets

so let me just write down definition let us begin with the definition let a and b be any two sets the cartesian product of a and b denoted a cross b is defined as

so this is again a set a cross b equal to set of all ordered pair a comma b with a in a and b in capital b

so this cartesian product consists of all ordered pairs with the first element coming from the set a and the second element coming from the set b

so the examples that we had initially

so the examples that we had initially the first example is an example is an example for the cartesian product in fact we had our set a as three buck name may three bags named x y and z the set b as blue and red

so a cross b this consists of all possibilities consist of all possibilities that means what is that that which which ball which colored ball are we going to put inside the bag x or y or z that is what it says now let us look at the second example second example it says about what is known as the name plates of a vehicle

so again this is an example this example the second example is also an example for a for the cartesian product now let us look at the third example that is the set of the set circle sets a set ah well let us write it as this way the ordered pairs x comma y such that x and y are real numbers and x square plus y squared equal to one first thing that we will have to see is that this set well we know that this is a circle this is what we observed in the last prime this set is not a cartesian product of two sets this set is not a cartesian product of two sets that is the that is one of the observations

so why is it this set is not a cartesian product of two sets let us look at something

so for example 0 comma 1 this belongs to this is on the circle similarly one comma zero is also on the circle we have zero comma one on the circle and one comma zero on the circle which means what does this tell us

so suppose i write the set let me call the set as a let me call it as some s suppose s is of the form a cross b now since zero comma one belongs to a cross b that implies that 1 belongs to b similarly the pair 1 comma 0 belongs to a cross b that implies that 1 belongs to a whereas the pair one comma one does not belong to s because why is it

so one square plus one square which is in fact two is not equal to one therefore one comma one does not belong to s therefore s that is the set of all pairs which lie on the circle is not a cartesian product of two sets not all sets this example gives us the fact that not all sets can be written as a cartesian product although a set can have ordered pair that does not mean that it is a cartesian product of two sets fine lets look at some of

so lets before we go to the next example lets make some remarks the first one suppose that you have two sets let us look at a cross b consider a cross b and b cross e are these two sets one and the same on what basis and how

so a cross b consists of elements of the form a comma b this is a cross b with a in a and b in b this is a cross b while the other one b cross a consists of all opposite pairs b comma a such that b in b and a in j the question is can we not write it as a in a and b in b s there is no difference between writing it as a in a and b in b but it is only this order that makes sense it is because of

this order right these two sets are totally different the pair

so a comma b is not equal to b comma a which means a cross b and b cross a are totally different sets now one more thing the second one given two elements a comma b and c comma d from a cross b we say that a comma b is equal to c comma d if a equal to c and b equal to d

so what it says is that pair y or at every position the element should match in the first position on the left we have a and on the right the first element is c the first position similarly for the second positions we have b and here we have d these two should match if these two matches at every position then we say that these ordered pairs they are equal

so let us look at an example or a problem suppose x minus half plus y comma x plus half plus y x this is equal to two comma three right what we have is x minus half plus y and the ordered pair x minus half plus y and x plus half minus y this is same as two comma three find x and y

so on the left we have an ordered pair and on the right we have an ordered pair and what is given is that these these two ordered pairs they are equal

so how to solve this what is that that is given to us is that the ordered pairs well let me write it as x plus y minus half and x minus y plus half is two comma three

so that implies that x plus y minus half the same as two is equal to two and x minus y plus half this is equal to three

so let's solve this let us add these two things what it gives is that two x equal to five that implies x equal to five by two now let us look write down the same set of equations x plus y minus half equal to two and x minus y plus half equal to three lets subtract the first one lets subtract from the first the second

so x now that x gets cancelled and what we will have is two y minus one equal to minus one sorry minus ah two y minus one equal to minus one that shows that y is zero right

so x and y

so therefore x equal to phi by 2 and y equal to z fine now let us look at some more examples ah

so ah some more examples suppose a is a set and we are given that zero comma one minus one comma zero is in a cross a find a what is that that we know is that the two pairs ordered pair zero comma one and minus one comma zero or in a cross a how to find a first question that we would like to ask is how many elements are possible in a well 0 comma 1 belongs to a a cross a that implies that zero belongs to a and one belongs to a similarly minus one comma zero this belongs to a cross a that implies minus one belongs to a and zero belongs to e therefore based on the information that is given to us what we know is that minus one zero one this set is exactly our a

so a consists of three elements minus one zero and one therefore a cross a is going to look like minus one minus one minus one zero minus one one zero minus one zero zero zero one one minus one one zero and finally one one how many elements does this a cross a contain this a cross a contains exactly nine elements

so let us again write it as a remark that if a and b are any two sets with number of elements of a equal number of elements of a as b and with number of elements elements of b as q then the number of elements in a cross b is p q

so the total number of elements in a cross b is total number of elements of a times the total number of elements in b therefore the total number of elements of a cross b is same as the total number of elements in a times the total number of elements in b right

so let us look at the last example let us look at the previous example once

again the set a what we found is has got minus one zero and one
 so number of elements of a is three that implies that the number of elements of a cross a is going to be number of elements of a times number of elements of a which is going to be three times three which is nine but well now let us look at lets write down once again what this a cross a is minus one minus one minus one zero minus one one zero minus one zero zero zero one one minus one one zero and finally one one lets count the number of elements
 so you can notice that with each element we are pairing together with another three more elements thus we have ended up with nine elements
 so a cross a half has got nine elements now let us do one more example let a be a set whose cartesian product with itself whose cartesian product with itself that is a cross a has got 16 elements
 so we are given a set a such that when you take the cartesian product of a with itself then it is going to have 16 elements this is the first clue that we are given that means a cross a has got 16 elements that means a is going to have 4 element it is going to have 4 elements fine now suppose that suppose that we know suppose we know that one comma two and three comma four are in a cross a find a and a cross a how to find a and a cross a the first observation that we made here is that
 so what is that that is given the number of elements of a cross a is 16 that implies as we did in the previous remark that number of elements of a is going to be four now the pair one comma two this is in a cross a and
 so that implies one and two they are they both are in a similarly three comma four they are in a cross a that implies both three and four are in a therefore the set a consists of four elements one two three and four now a cross a it is going to have all 16 elements with order with pairs consisting of elements bit lying between likes lying from 1 to 4 well this is going to be one comma two one comma three one comma four well one comma one two comma one two comma two two comma three two comma four three comma one three comma two three comma three three comma four four comma one four comma two four comma three and finally four comma four
 so these are the possible 16 elements are in fact the only possible 16 elements of a cross a let us do one more example from geometry and lets try to do higher examples based on this most of us are aware of what is known as the r two the two dimensional plane what is this r two r two this consists of all ordered pairs x comma y with x and y as real numbers where x and y are real numbers you can notice that this is exactly equal to r cross r
 so this is an example for a cartesian product
 so the usual two dimensional plane is exactly the cartesian product of real numbers with itself let us also look at the same thing r two from a different point of view lets look at the diagram following diagram supposing that i am here and i have a point the only thing we are given the following information one is r which is exactly the distance from the origin to this point right this is my point p i have a point p and i am given this distance r and the angle that it makes with the x axis these are the two things that i am given let me call it as theta
 so this r two i can write it as all those ordered pairs r comma theta with r positive well r positive and 0 less than or equal to theta less than 180 degrees or in fact not 180 it should be 360 degrees right you have this many things in fact
 so i should remove not just r two the way i have returned it should be i should remove this does not contain the origin zero comma zero
 so to in order that i include this zero comma zero i should have it return it as r greater than or equal to zero here

so when the distance is what we need is that the distance is bit positive that is what we expect but if you look at this one this set is a cartesian product of two sets these are what are known as polar coordinates which most of you must have seen in a course on geometry right what you have is the set of polar coordinates but this set is almost like a cartesian product thats what we have right but anyhow if i am going to include r greater than zero i can write it this is going to be a cartwheel this looks like it is not a cartesian product sorry looks like it is not a cartesian product but one can write this as a cartesian product how to write this this set is exactly equal to all those r in r such that r greater than zero cross all those theta such that zero less than or equal to theta less than three sixty degrees this is an example for a cartesian product fine

so now having said all these things let me make one more remark on the cartesian product if a and b or any two sets with at least one of them being infinite then a cross b is also an infinite set in the earlier case both the sets are infinite and therefore the cartesian product is also infinite

so let us do one simple example what is that that we have in one more example let us choose a as just small a small b and small c and capital b as 1 2 3 etcetera etcetera now a cross b it is going to contain a comma 1 a comma 2 dot dot b comma 1 b comma 2 dot dot dot and c comma 1 c comma 2 dot dot

so although a is finite but since b is infinite this says that a cross b is infinite now let us do one more example this is again a familiar example a known one for all of us the first let us look at r three the three dimensional plane this is the usual one that we look at three dimensional plane what is this three dimensional one r three this as we all know consists of all x y z triples such that x y and z all three are real numbers the question is is this a cartesian product of two sets there are two ways of looking at this one us try to identify this set as a cartesian product of two sets first and then let us see what is the other possibility i can also write this as x comma within brackets y comma z such that x y and z all these are real numbers once i have this i can write this as r cross r two the natural question is why cannot i separate x y z as x comma y comma z yes y naught this way x comma y comma z as the first two entries and then separated by the comma for the third one why not this one

so this is in fact this is right and one can identify r three with r two cross r

so what we have done is that the canonical what we see in the nature the three dimensional plane is written as the cartesian product of two sets a natural question that arises at this stage is that is it possible to write this as r cross r cross r does it make sense yes one can write r 3 as r cross r cross r now what is the meaning of this archer of r cross r

so let us go a step further and try to define the notion

so let us begin with the definition let a one a two and a three be three sets then the cartesian product of a one a two and a three denoted a one cross a 2 cross a 3 is defined as a 1 cross a 2 cross a 3 equal to all those triplets a one a two small a one small a two small a three with given that a one in capital a one small a two in capital a two and small a three in capital a three in the first case what we had is ordered pairs or the lets see two tuple now we have a triplet or a three tuple three tuple with the first element coming from a one second element coming from a two and the third element coming from a three

so now let us go back there to our previous example of the r three the three dimensional plane

so r three

so this is exactly same as r cross r cross r

so this is an example for a cartesian product with three sets cartesian product

of three sets lets do one more problem at the stage supposing that we are given this one x plus y plus z comma x minus y minus z and x plus y minus z this is equal to one two three suppose this is what is given to us find x y and z you are given this

so how to solve this problem

so before i proceed with the solution to this problem let us make one more remark which we which is exactly the same as what we remarked for the case of two cartesian product of two sets if a1 a2 a3 or any three sets if you have any three sets then and if

so if you have any three sets and if the triplet a one a two a three and also the triplet a one dash a two dash a three dash if you have if you know that these two triplets belong to a one cross a two cross a three and also that you know that the triplet a one a two a 3 is equal to a 1 dash a 2 dash a 3 dash that implies that a 1 equal to a 1 dash a two equal to a two dash and a three equal to a three dash which means that at each position they should match at the i th code each coordinate should match at or at each position they should match only then you say that those triplets two triplets are equal if they each coordinate wise or each position wise if the numbers if the real numbers mat if the elements match then you say that such such two triplets are equal now let us go to the solution for the problem how to solve this problem based on what is given what is that that we are given is that on the left we are given a triplet x plus y plus z x minus y minus z x plus y minus z equal to the triplet one two three based on the above remark it can be said that x plus y plus z equal to one x minus y minus z equal to two and x plus y minus z equal to three let me call this number each equation one two and 3 adding 1 and 2 we get two x equal to three that implies that x equal to three by two now adding one and three if i am going to add one and three what will i have we get two x plus two y equal to four but what we know is that x is three by two

so let me just substitute x as three by two

so that implies that two into three by two plus two into y equal to four that implies two y equal to four

so two into three by two which is three four minus three which is one that implies y is one by two which is half now finally let us look at the first one x plus y plus z equal to one this is what i have now the value for x is three by two and the value for y is one by two plus z equal to one that implies three by two plus one by two which is two plus z is one that implies z equal to minus one therefore the values for x y and z are three by two one by two and minus one respectively

so in the next lecture we will do some more problems based on these and also will do some more examples for cartesian product and subsets of cartesian product thank you thank you you