welcome to the third and final lecture on sets

so in the first two lectures we introduced the basic notions of sets and then we discussed some of the operations on sets and some properties on sets today we will do some problems on sets let me first recall if n of a is the number of elements in the set a then for given two sets a and b the number of elements in the union is given by n of a plus n of b minus n of a intersection b and also we have for three sets n of a union b union c this is equal to n of a plus n of b plus n of c minus the number of elements in the intersection of any two

so n of a intersection b minus n of b intersection c minus n of a intersection c plus the number of elements in the intersection of all three a intersection b intersection c right

so these formulas are very useful also let me write for any sets a n of a is n of a minus b plus n of a intersection b this is because a is the disjoint union of a minus b and a intersection b

so let us do some problems based on this formula

so problem one suppose we are given n of a is equal to six n of b is equal to four then what is the minimum and maximum value of n of a minus b

so we are given the number of elements in a and number of elements of in b and we want to write the number of elements in a minus b

so we know that n of a is equal to n of a minus b plus n of a intersection b

so therefore n of a minus b is equal to n of a minus n of a intersection b n of a is given to us this is equal to 6 minus n of a intersection b

so to find the minimum value of n of a minus b we have to ask what is the maximum value of n of a intersection b and the maximum of n of a minus b will be when n of intersection b is minimum

so we know that a of intersection b since a intersection b is a subset of b this implies that n of a intersection b is less than equal to n of b which is given to be equal to four right

so number of elements in the intersection of a and b cannot be more than four

so n of a minus b is greater than equal to six minus four which is equal to two

so this is the minimum value and for the maximum value we have to ask what is the minimum value that a intersection b can take

so given that a and b are any two sets it is possible that a intersection b is the empty set

so also if a intersection b is empty then n of a intersection b is equal to 0

so therefore the maximum value of n of a minus b this will be equal to 6 minus 0 which is equal to 6 because if a and b are disjoined then in that case the number of elements in a minus b is same as number of elements in a

so the next problem this will involve three sets

so suppose there are sixty people out of these 25 read newspaper h 26 read newspaper t and another 26 read newspaper i

so we have three newspaper ht and i say hindustan times times of india and indian express and we know that 25 people read the h 26 read t 26 read i and we are also given nine read both h and i eleven read both h and t and eight read both t and i also we know how many read all three finally three people read all three now what do we have to find find first one is number of people who read at least one of the three and second find the number of people who read exactly one newspaper right

so there is lot of information given let me show again

so out of 60 25 read h 26 read t 26 read i 9 read h n i 11 read h and t 8 rate i and three read all three

so the best way to do this is you try to draw venn diagram

so so we have three sets h t and i and lets write the intersection of all three there are three people who read all three newspapers then we know that eleven

people read h and t

so the intersection number of elements in the intersection of h and t that is eleven

so the number of elements in this part must be eight because eight plus three is eleven let me use another pin

so this part is eight this is three and then the number of people who read h and i that is nine

so there are six people who read only h and i but not all three and eight people read t and i

so three are who reads all three

so rest five read only t and i right and then you can fill in the other things also

so number of people who read only h we know that twenty five people read h

so here we have eight plus three eleven plus six seventeen read at least two of the newspaper including h

so the rest is 25 minus 17

so this is 8 people who read h only h and then only t is twenty six

so we have eight plus three eleven plus six six eleven plus five is sixteen

so 10 are here and then only i is 26

so 6 plus 3 9 plus 5 14

so 12 are here now you can easily write what is being asked

so the first one is number of people who read at least one of the three either you can use the formula of this means that we want what is n of h union t union i

so we are given n of h n of t n of i and n of the intersections

so you can use that formula and calculate or once you have drawn this diagram then we need to just add these numbers

so this is equal to in h we have twenty five plus ten plus five plus twelve

so that is equal to fifty two or use n of h union t union i is equal to n of h plus n of t plus n of i minus n of h intersection t minus n of t intersection i minus n h intersection i plus n of h intersection t intersection i right

so the first part you can use directly this formula and do but this easier if you draw the diagram the other one is it is asking the number of people who read exactly one newspaper

so that number again is clear from this right there are eight people these are 8 who read only at the 10 who read only t and 12 who read only i

so and there is nothing in common

so eight plus ten plus twelve that means thirty people who read exactly one from the venn diagram number of people who read exactly one this is equal to eight plus ten plus twelve which is equal to thirty right

so this way you can quickly calculate these numbers if you have objective questions

so so next problem ok let me write another problem based on this concepts

so let n of a intersection b v x n of a minus b this is six x and n of b minus a is eight plus two x and also given is that n of a is equal to n of b and fine x right

so we are given the number of elements in the intersection and number of elements in a minus b and b minus a and number of elements of in a is equal to number of elements in b

so to do this we know that n of a is equal to n of a minus b plus n of a intersection b

so this is equal to six x plus x which is seven x and n of b is equal to n of b minus a plus n of a b intersection a

so this is equal to eight plus two x and plus x

so that is eight plus three x and since n of a equal to n of b we have seven x is equal to eight plus three x and this immediately gives you x is equal to two ok here you could also use the formula for n of a union b and calculate it using n of a intersection b plus n of a minus b plus n of b minus a and also using the other formula that n of a union b is n of a plus n of b minus n of a intersection b and then equate them but here if you notice this one its quicker to solve it this way ok next problem

so suppose seventy percent indians like apple and 82 percent like mango let x percent of indians like both and find they minimum and maximum possible x

so here what is given is if i write a to be the percentage of people who write like apple then n of a is seventy n of m is the number of people who like mango this is 82 percent and n of a intersection m is x percent

so what we know is that since n of a union m this has to be less than equal to 100 right the total hundred percent people are there

so people who like either apple or mango cannot be more than hundred percent

so we have n of a plus n of m minus n of a intersection m this is less than equal to hundred which implies seventy plus eighty two minus x is less than equal to hundred and this gives x must be bigger than equal to this is 152 minus 152 right

so number of percentage of people who like both apple and mango cannot be less than 52 percent now how do we get the maximum value of x also we know that a intersection m this is a subset of a

so therefore n of a intersection m is less than equal to n of a which is equal to seventy

so x has to be less than equal to seventy percent and bigger than equal to fifty two percent ok

so next thing let us recall this power set of this is all subsets of a and we have seen that the number of elements in the power set of a is equal to two times the number of elements in a

so the next problem what is the number of elements in the power set of power set of power set of phi where phi is the empty set

so first of all you have to know what is the number of elements in power set of phi

so this is equal to what since the number of elements in the empty set is 0 the number of elements in power set of empty set this is equal to 2 to the power 0 which is equal to 1 empty set has only one subset which is empty set

so number of elements in the power set of empty set is one and then this will imply that number of element in power set of power set of phi this is equal to 2 to the power number of elements in power set of 5 which is 1

so this is 2 and this will imply that number of elements in power set of power set of power set of phi this is equal to two square which is equal to four

so this problem was just to recall the number of elements in the power set

so this was problem number five let me do one more problem where we will use this number of elements in the power set

so suppose we are given n of a is equal to some number m n of b is n and the number of elements in power set of a minus number of elements in power set of b is given to be 112 then find m and n

so if you know the difference of the number of elements in power sets of a and b we want to find the number of elements in a number of elements in b

so what is given here is we have number of elements in power set of a is 2 to the power m minus two to the power n this is equal to hundred and twelve

so you might be thinking that we have only one equation and from here we have to find both m and n but you can do it because

so this first of all implies that m is bigger than n and then you can write

this as we can take out two to the n common and then we have two to the power m minus n minus one this is equal to hundred twelve

so here note that we have taken out powers of 2 outside and then the rest is 2 to the power m minus n which is an even positive integer minus 1

so this part is an odd integer

so what you try to do is just write hundred twelve as product of

so this can be written as sixteen times seven which is equal to 2 to the 4 times 7

so this implies n must be equal to 4 and 2 to the m minus n minus 1 this is equal to 7

so that means that 2 to the m minus n is equal to 8 which is 2 cubed

so this implies n equal to 4 and 2 to the m minus n is equal to eight which is two cube

so this implies n equal to four n m minus n is three

so n is equal to four and m is equal to seven this was problem six the next problem we are given sets a and b a is equal to one two three and four b is the set containing two four and six fine the number of sets c such that a intersection b is a subset of c and c is a subset of a union b right

so we have to find all sub all sets c such that c contains a intersection b and c is a subset of a union b

so in this case a intersection b is equal to the set containing 2 and 4 and a union b this is the set containing 1 2 three four and six

so what we want is that

so we want 2 4 this is a intersection b to be a subset of c and this is a c is a subset of 1 2 3 4 and 6.

right

so one way you can think of is you write down all the set c such that it contains the elements two and four and it is a subset of one two three four and six and that way you can count the number of such sets but if these things have many elements then that will be difficult

so what we can do is here

so this implies c is equal to the set 2 4 union with another set let me call c prime where c prime is a subset of one three and six

so you write c as disjoint union of we know that two four must be there and then it can have other elements which has to be from the set one three and six right

so the number of elements in c

so therefore number of c of such c is equal to number of c prime which is a subset of one three and six and this you know this is nothing but the number of elements in the power set of 136

so one three six has three elements

so power state will contain two to the power three which is eight elements

so even if this sets had like 10 or 20 elements you can do like this and calculate the number of set c ok

so ok let me write down one more problem let x be the set containing the numbers of the form 4 to the n minus 3 n minus 1 such that n is a natural number and y is the set nine times n minus one where n is a natural number then x union y equals we are given four choices this is all of natural number b this is equal to y minus x c is this is equal to x and d is this is equal to y right

so this is a multiple choice question and we are given the set x and y written in this set builder form and we have to find what is x union y

so to do such problem you you can try to write down the first few elements of x and y

so if you see x if i put n equal to 1 then this is 4 minus 3 minus 1

so the first element here is 0 if i put n equal to 2 then 4 square sixteen minus six minus one

so that gives me nine and we put n equal to three will give four q is sixty four minus nine minus one

so that is fifty four and if you want you can calculate more

so these are the elements of x and y consists of for n equal to one this is zero then we have nine eighteen twenty seven and

so on

so y is clear it consists of all multiples of nine all non negative multiples of nine

so now if you see clearly one does not belong to x union y

so a is false also x union y we see that y is not a subset of x right because 18 belongs to y but 18 is not in x

so therefore x union y is not equal to x right this will be equal to x if y is a subset of x

so c is also false and b says that y minus x can this be equal to x

so this contains say all the elements in y which is not in x

so all the elements in y which is not in x

so this is again not equal to x union y this is because if you see zero this belongs to x union y but zero is not in y minus x because 0 belongs to both x and y

so if you are doing this multiple choice question only thing left is

so by elimination d must be true if it is given that one of the statements is true but let us try to prove this

so what i want to say is that x union y is equal to y which means that i have to show that x is a subset of y right

so claim is x is a subset of y this will imply that x union y is equal to y

so why is this

so that is we have to show that four to the n minus three n minus one this is divisible by nine for all natural number n

so we have to see that four to the n minus three n minus one is divisible by nine for every n

so so note that for n equal to one four to the n minus three n minus one this is equal to zero which is divisible by nine and to actually prove this we will need what is called principle of mathematical induction which you will also learn in details in another chapter but let let me show you

so suppose four to the k minus three k minus one is divisible by nine we will show that 4 to the k plus 1 minus 3 k plus 1 minus 1 this is also divisible by 9

so then we know that for n equal to one this is true

so this will imply that it is true for n equal to two and then if it is true for n equal to two it will be true for n equal to three hence by induction it will be true for all n

so why is this

so let us write four to the k plus one minus three k plus one minus one this can be written as we can write as 4 times 4 to the k minus 3 k minus 1

so this gives me 4 to the k plus 1 minus 12 k minus four but here i have four to the k plus one minus three k minus four

so i have to add plus nine k now what we know is that this part is divisible by nine and this is also divisible by nine

so the sum is divisible by nine the next problem let us take a to be the set n cube plus n plus one cube plus n plus two cube where n is a natural number and b is nine n n is in n then which of the following is or are true first is a is a subset of b second is b is a subset of a third is a is equal to b and d is a is a proper subset of b

so b consists of all the positive multiples of nine a consists of the sum of cubes of three successive natural numbers
 so if you see a if i put n equal to 1 then i have one plus two cube plus three q
 so that is equal to thirty six
 so and then you can find other one this will be two cube plus three cube plus four cube and
 so on
 so the smallest number in a is thirty six
 so a is not equal to b right because b contains nine which is not in a also b is not a subset of a right
 so b and c are false is a subset of b
 so the first element 36 is a multiple of nine the next one again if you calculate you can see that that is also a multiple of nine
 so it seems like all these elements are all these numbers n cube plus n plus one cube plus n plus two cube are multiples of nine but how do we prove that
 so claim is n cube plus n plus one cube plus n plus two cube is a multiple of nine for every natural number n
 so you can simply calculate what is this n cube plus n plus one cube plus n plus two cube this is equal to n cube if you expand n plus one cube this will give you n cube plus three n square plus three n plus one and then n plus two cube is n cube plus six n square plus twelve n plus eight and then this whole thing is equal to 3 times n cube plus 3 n square and 6 n square
 so that is 9 n square plus fifteen n plus nine we want to prove that this is a multiple of nine we already have this is equal to nine times n square plus one and then we have three n cube plus fifteen n
 so that is equal to three n times n square plus five right
 so to show that this is a multiple of 9 you have to just see that n times n square plus 5 is a multiple of 3s
 so so another claim is n times n square plus five is a multiple of three
 so if n is a multiple of three then of course this is a multiple of three if n is not a multiple of three then n must be of the form three k plus one or three k plus two and then you just calculate
 so if n is equal to three k then this is fine if n is three k plus one then n square plus five this will be three k plus one square plus five
 so three k plus one square plus five this is equal to nine k square plus six k plus six and this is a multiple of three also 3 k plus 2 square plus 5 this is equal to 9 k square plus 12 k plus 4 plus 5 which is again 3 times 3 k square plus 4 k plus 3 this is a multiple of 3 again
 so hence this n cube plus n plus one cube plus n plus two cube is a multiple of nine for all n in n
 so we have the set a is a proper subset of b hence a and d are correct
 so note that if a is a proper subset that also means that a is a subset of b ok
 so this completes the chapter on sets of course you need to practice more questions on these things to be comfortable with these concepts thank you you