welcome to the second lecture on sets

so in the last class we talked about what are sets and then we studied about some operations on sets and we introduced many notations

so sets are very useful concepts which is used in almost every branch of mathematics

so when you study about functions relations everywhere you will see that sets comes

so it is very important to understand the concepts of sets well

so let me start

so first let me recall that we talked about two different ways of representing a set one was roster form and the second one is set builder form in the roster form let us recall that we list all the elements all elements are listed note that in general for in finite sets like the set of real numbers it is not possible to use the roster form

so we need set builder form though for some infinite sets like if we have a set of natural numbers or even numbers we can still use roster form as well but set builder form in general is more useful

so i will introduce some notations

so we will define what are called intervals in the set of real numbers

so let a and b are real numbers with a less than b then we write the interval a b this is equal to set of all x such that x is a real number and a is less than x is less than b this is called open interval

so this is set of all real numbers which are strictly between a and b and we use this closed bracket a b this will mean all x says that x is a real number and a is less than equal to x is less than equal to b this is closed interval and we also use half open or half closed intervals

so a b this will mean x is that x is a real number and x is strictly bigger than a and less than equal to b similarly this will denote all x says that x is in r and a is less than equal to x is less than equal to is less than b and we use some infinite intervals also

so if i write a comma infinity this will mean all x is that x is a real number and x is bigger than a closed a infinity will mean all x such that x is a real number and x is bigger than equal to a minus infinity to b this means x says that x is less than b minus infinity to b closed this will mean all x such that x is a real number and x is less than or equal to b and we use minus infinity infinity to denote the set of all real numbers ok

so infinity here is just a symbol and this gives you the intervals these are all subsets of all these intervals are subsets of r

so these interval will be very useful when you learn calculus or in other subjects

so next we will also introduce

so i we define what we mean by subsets of a set

so we also sometimes see this note this term called proper subset

so we say a is a proper subset of x if a is a subset of x but a is not equal to x right and the notation we use is

so for subset we use a subset of x and for proper subset we write it like this

so this will mean that a is a proper subset of x this means that every element of a is in x and there is at least one element in x which is not in a

so for example one two this is a proper subset of the set one two three another term use is superset

so we say b is a superset of a if a is a subset of b and the notation here is we use this to denote that b is a superset of a and then if b is a superset of a but b is not equal to a we can say proper superset of a b is a proper superset of a if we write like this this means this and b is not equal to a ok in the

last class we learnt about some operations on sets like union intersection complement and set difference

so today we will list down some properties of these operations union and intersection

so first one if i have two sets then a union b is same as b union a this is called the commutative law second is if i have three subsets then a union b union c this is same as a union with b union c this is called the associative law

so you must have heard these terms for addition of real numbers that addition is commutative addition is associative

so similarly taking union this is commutative associative third is that we have this is empty set

so if you take a union with empty set that is equal to a

so this is like for real number a plus 0 is equal to a this is true and a union a is equal to a this sometime we call identity law and this is called idempotent also i like one more

so if a is a subset of u then a union with u is equal to u and we have similar properties for intersection again intersection is commutative a intersection b is same as b intersection with a a intersected with b intersected with c is equal to a intersected with b intersected c a intersected with empty set gives empty set a intersected a is a and if a is a subset of b then a intersected with b is same as a

so these properties are very easy to check the next one important thing is what is called the distributive law

so if we have three sets a b and c then a intersected with b union c is equal to a intersected with b union a intersected with c right

so this says that the intersection distributes over the union

so this again analogous thing is that if you have product and sum then a times b plus c is a times b plus a times c and also a union with b intersected c this is a union b intersected with a union c

so this let me explain the first one by using venn diagram

so if we have a b and c are three sets then let me write a intersected with b is denoted by this red and a intersected with c is this part and if you take the union of these two

so red is a intersected with b and blue part is a intersected with c and the union of these is exactly a intersected with b union c similarly you can do for the second one another thing some properties of complement

so if i have u is universal set then a compliment is nothing but u minus a right

so first property is that if we take compliment of the complement then we get equal to a this is because a compliment complement is equal to u minus a complement and this is equal to u minus u minus a which is equal to a itself and what is the complement of the empty set since empty set has no element the complement will contain all the elements of u and the complement of u is just the empty set another thing is that if a is a subset of b and both are subsets of the universal set u then the complement of b is a subset of complement of a right because anything in b complement means that all those elements which are in u but not in b

so if an element is not in b it cannot be in a

so x belonging to b complement this implies x is not in b which implies x is not in a since a is a subset of b if a x was in a then it has to be in b also this implies that x is in a compliment

so next we will relate unions and intersection with complement

so these are two very important properties and these are called de morgan's law

so the first one is if i take complement of the union that is equal to the intersection of the compliments a union b complement is same as a compliment intersected with b complement and second one if i take complement of the intersection then i get union of the compliments again this one you can see by drawing diagram

so if you draw venn diagram if you see a union b complement this is everything which is not in a and it is not in b

so this part is a union b complement what about a complement and b complement a complement is all the elements which are not in a but it includes these points which are in b but not in a similarly b complement is all the elements which are not in b but it can contain the elements which are in a but not in b now if you see the intersection of the elements which are common to these two red ones is exactly equal to the blue one

so this is our a complement this is our b complement and then this is equal to a compliment intersected with b complement the next thing that we will do is suppose we have two sets a and b and we know the number of elements in a and b then can we say anything about the number of elements in the union

so so first let me write let a and b are two finite sets such that a intersection b is empty that is a and b are two disjoint sets then in that case what is the number of elements in the union

so n of a union b this is equal to n of a plus n of b right because the number of elements in the union anything in the union means that it is either in a or in b and the number of elements in a is n of a this is number of elements in b is n of b this is number of elements in a and because there is no element which is common to both a and b this is clear that the number of elements in the union is sum of the number of elements in a and b

so in this case we have two sets a and b which are disjoint

so counting the number of elements which are in either a or b is same as just adding the number of elements in a and number of elements in b next we will see that what can we say in general

so in general n of a union b this is equal to n of a plus n of b minus n of a intersection b this is a very important formula for the number of elements of the union which you will find very useful when learning probability

so let us prove why this is true

so proof we can write the union a union b as the disjoint union of a minus b union with a intersection b union with b minus a right

so a union b can be broken into these three parts and these are disjoin where a minus b a intersection b and b minus a are pairwise disjoint now we know that for disjoint sets the number of elements in the union is sum of the number of elements in each of them

so therefore n of a union b is equal to n of a minus b plus n of a intersection b plus n of b minus a now we want to express this in terms of the number of elements in a and b

so note that the number of elements in a is nothing but the number of elements in a minus b and plus number of elements in a intersection b

so this is equal to n of a minus b plus n of a intersection b plus n of b minus a plus n of a intersection b and minus n of a intersection b

so but a is equal to a minus b union a intersection b this is disjoint union

so n of a is equal to n of a minus b plus n of a intersection b similarly n of b is n of b minus a plus n of a intersection b

so if you see the previous page the first two terms n of a minus b plus n of a intersection b this is n of a n of b minus a plus n of a intersection b this is n of b and then we have minus n of a intersection b therefore n of a union b is equal to n of a plus n of b minus n of a intersection b

so this is very important formula

so here we have to subtract the number of elements in the intersection because when we are counting the number of elements of a and number of elements of b the elements in the intersection are counted twice

so we need to subtract

so that is one way that you can remember this formula now what about n of a union b union c suppose i have three sets then can we write similar formula for the number of elements in the union of a b and c

so n of a union b union c this by the previous formula we can write as n of a plus n of b union c minus n of a intersected with b union c and n of b union c we know again that this we can write as n of b plus n of c minus n of b intersection c minus n of a intersection b union c now by de morgan's law sorry by the distributive property a intersection b union c this is equal to a intersection b union a intersected with c

so n of a intersected with b union c this is equal to n of a intersected with b plus n of a intersected with c minus n of the intersection of the two sets which is a intersected b intersected with a intersection c

so this is equal to n of a intersection b plus n of a intersected with c minus a intersection b intersection with a intersection c is nothing but a intersected with b intersected with c

so we had this equation and then we have equation two

so if you put the value of n of a intersection b union c from equation two in equation one what do we get n of a union b union c this is equal to n of a plus n of b plus n of c minus n of a intersection b minus n of a b intersection c minus n of a intersection c plus n of a intersection b intersection c right

so for the union of three sets the number of elements you first add the number of elements in each of them n of a plus n of b plus n of c then you take the intersection two at a time and then you subtract the number of elements in them and then you look at the intersection of all three then you will have to add them

so here when you subtract the number of elements in the intersection of two at a time then you have subtracted the number of elements in the intersection

so you will have to add that to get the number of elements in the union of three similarly one can derive formula for the number of elements in union of four also but we will not do that

so let us look at an example

so suppose there are 400 people and they speak either english or hindi or there are some people who speak both and what we know is that 250 people they speak in the out of these 400 people and out of this 400 200 people they speak english then what we want is that how many speak both languages ok

so what we have to do

so let h this is the set of people speaking in the e is the set of people speaking english

so what is given is n of h is two hundred fifty n of e is 200 and n of h union e

so h union e is the set of people who speak either hindi or english this is given to be 400 and what do we want to find what is n of h intersected with e how many people are who speak both in the n english

so we know that this we can calculate by using this formula n of h union e is equal to n of h plus n of e minus n of h intersection e

so this implies n of h intersected with e is equal to n of at union e minus this plus n of h plus n of e

so this is equal to minus four hundred plus 250 plus 200 which gives 50.

so there are 50 people who speak both hindi and english ok

so let me do some problems suppose we have sets a b and c such that a union b is equal to a union c and a intersected with b is equal to a intersected with c you are given that a union b is equal to a union c and a intersected with b is a intersected with c prove that b is equal to c

so so let me show how to prove that two sets are same

so to show that b is equal to c what we will prove is that b is a subset of c

so to

so b equal to c it is enough to show that b is a subset of c and c is a subset of b

so lets see why b is a subset of c

so let x be any element in b we have to show that x is also in c

so we know that b is a subset of a union b

so if x is in b it is also in a union b but a union b is equal to a union c

so this implies that if i take x in b x belongs to the union of a and c

so this implies x belongs to a or x belongs to c now what we have to prove is that x is in c

so if x is in c then we are done x is in c then ok if otherwise if x is in a then x belongs to a intersected with b because x was already taken to be in b but a intersected with b is equal to a intersected with c

so x belongs to a intersected with c this implies that x is in c

so in both cases

so we have x belonging to b implies x belongs to c therefore b is a subset of c and similarly you can show that c is a subset of b if you take any x in c then by the same argument you saw that it also belongs to b this implies that b is equal to c ok second problem let me do this

so we will show that if the power set of a is equal to power set of b then a is equal to b

so let me recall that power set of a recall power set of a is equal to set of all subsets of a

so this is equal to all c such that c is a subset of a

so if we know that the power sets of two sets are same we want to prove that the sets are same

so so we want to prove that a is equal to b

so note that since a is a subset of a a belongs to power set of a right power set contains all subsets

so in particular it contains a also but power set of a is given to be equal to power set of b

so this implies a belongs to power set of b which implies that a has to be a subset of b right and similarly power set of b is equal to power set of a

so b belongs to power set of b which is equal to power set of a this implies that b is a subset of a hence a is equal to b

so let let me introduce some notations which we have been using

so so if i write some statement one implies statement two this means if statement one is true then statement 2 is true right

so instead of writing this if then statement we use this implication sign to say that statement one if statement one is true then statement two is true and another one we use statement one this both sides implication statement two this means statement one is true if and only if statement two is true

so that means that if statement one is true then statement true has to be true and also if statement two is true then statement one is true and there is also a shortcut notation to write if and only if we write i double f this means if and only if

so instead of this double implication we sometime write i double f to mean this

ok let me give one more example

so suppose a intersected b is non empty b intersected with c is non empty and a intersected c this is also non empty is it true that a intersected b intersected c is non empty

so if we have three sets such that pair wise they are not disjoint we have something in common then is it true that there is something common in each of them

so the answer is no because let us take this example let a is equal to set containing only two points zero and one b is equal to one and two and c is 0 and 2 right

so 0 belongs to a intersection c 1 belongs to a intersection b and 2 belongs to b intersection c

so all these pair wise they are not disjoint but is there any element which is common to a b and c 0 is not in b 1 is in a and b but not in c 2 is in b and c but not in a

so a intersection b intersection c this is empty ok

so we will stop here today and in the next class i will do some more examples of sets and that will finish the chapter on sets thank you you