

welcome students

so this is the first lecture

on sets first let me define what is a set

so a set is a well defined collection of objects a set is a well defined collection of

objects or sometimes we say elements

so let me explain what do i mean

by a well defined collection

so by well defined collection we mean that given an element given an object we can clearly determine whether the object is in the collection or not right

so let me explain it by example

so one the collection of all vowels of the english alphabet

so what is this collection

so we have the

english alphabet there are 26 alphabets a to z and we know that there are five vowels a e i o u

so this is a well defined collection

so given an alphabet you can tell whether the given alphabet is a vowel or not

so this is an example of set this is a set second example

so suppose i say that the collection of eleven best cricketers in the world

so this is not a well defined collection

because it depends on peoples perspective who is a better cricketer

so this will not be said this is not a set ok

so we will see many

examples of sets in due course

so for notations we shall usually denote sets by capital letters for example a b c x y etcetera and the elements we denote the elements by small letters like a b c x y z etcetera another notation

so we write a belongs to a this is read as a belongs to the set capital a or a is an element of the set a to say a is an element of the set capital a

so for example let a be the set of all even natural numbers right which means that a is the collection of all

natural numbers which are divisible by two

so then the natural number two this is an element of

the set a right we write two belongs to a but the natural number three this is an odd number not an even natural number

so two is three is not in the set a

so for not an element of a we cross this epsilon

so now we will see how to represent a set

so representation of sets

so there are two ways by which we generally represent a set first one is called roster or tabular form

so in this roster form what we do is that we list down all the elements of the set

so all elements of the set are listed within this curly braces and separated by commas

so for example they said a consisting of all even natural numbers less than or equal to 10 will be represented by a is equal to

so we just asked

what are the elements of this set a

so we want all the even natural numbers which are less than or equal to ten

so the smallest such even natural number is two then the next one is four six eight and ten sometime we have sets which does not contain finite number of elements then also sometime we can represent using this roster form

so for example the set of all odd natural numbers can be represented by so one is this smallest odd natural number then the next one is three next one is five seven nine and then we put dot dot dot so if we know a pattern then we write the first few elements of the set and then we use dot dot dot to say that their infinitely many elements of this set there is one more the way of representing a set normally

so the second way to represent a set is called the set builder form

so in this form the set is described by the characteristic property possessed by all the elements of the set

so for example if we look at the previous example the set of the set a is equal to $2, 4, 6, 8, 10$ is represented in set builder form as a is equal to

so we just write it is a collection of all n such that n is a natural number divisible by two and n is less than or equal to ten right

so there can be many different way in which you can write the same set

so now let me introduce some of the sets that we use in mathematics some notations which are used

so we will use this capital N to denote the set of all natural numbers

so set of all natural numbers is represented by is capital N with one more vertical line here similarly we represent by Z this is set of all integers

so this includes positive negative and zero all natural numbers will denote by Q the set of all rational numbers then R will be used for set of all real numbers

so set of all natural numbers integers rational numbers real numbers capital C will denote the set of all complex numbers and then we can use if I write R^+ plus this means set of all positive real numbers similarly Q^+ plus will mean set of all positive rationals

so next thing we will explain what is meant by the empty set

so the empty set is the set which does not contain any elements

so the set containing no elements and the notation used for empty set is this ϕ or we use the curly braces without any element inside the empty set is also called the null set or the void set

so if you see these terms they mean the empty set itself

so for example I can describe a set let A equal to

so we have already introduced this notation this means that this is this set containing all n in natural number such that n is bigger than one and less than two

so suppose we write this set

so is there any natural number which is bigger than one and less than two no

so this set is empty because there is no natural number strictly between one and two another example suppose b is the collection of all x in rational number such that x^2 is equal to two right

so you might have seen in high school that there is no rational number whose square is equal to 2

so this set b is again equal to the empty set let me also use \emptyset a notation

so ok i will come to that later so first let me define finite and infinite sets

so a finite set is a set which contains only finitely many elements

so set is called a finite set if it contains only finitely many elements otherwise it is called an infinite set

so example the set a set of all vowels is a finite set because it contains only five elements but all these examples that i give set of all natural numbers set of all integers rational numbers real numbers complex numbers these are infinite sets

so next thing we will say what we mean by two sets to be equal

so two sets a and b are said to be equal if they contain exactly the same elements

so note that while representing a set the order of the elements is not important

so for example if a is equal to one two three and we write b equal to two three one then a and b are the same set because both a and b contain the same three elements one two and three

so while writing sets it's not important in which order the elements come

so next concept is of subsets

so let x be a set we say that a set capital A is a subset of x and this will be denoted by $A \subset x$ if every element of the set A is also an element of the set x

so this is usually denoted as $A \subset x$ if A belonging to capital A this should imply

that A is also an element of capital x

so some examples of subsets of the sets that we have seen

so note that the set of natural numbers this is a subset of set of all integers right every natural number is also an integer set of

integer is a subset of set of rational numbers which is a subset of the set of real numbers which

includes rationals as well as irrational numbers and then once you learn complex numbers you see

that real numbers are subsets of complex numbers now one important point is that two sets a and b are equal if and only if a is a subset of b and b is a subset of a right

so $a = b$ this is equivalent to $a \subset b$ and $b \subset a$ right

so this is very simple thing to

see that because two sets are equal if they contain the same element

so every element in

a must be in b and vice versa

so a is a subset of b and b is a subset of a but this is very useful

when you try to prove that two sets are same you prove that each is the subset of the other

one there is another concept called power sets

so given a set a the power set of a is the collection of all subsets of a and the notation used for power set is P of a denotes the power set of a

so P of a this will consist of all

b such that b is a subset of a

so suppose a is equal to one two three a consists these

three elements one two three can you write down what is the power set of a

so first of all empty set is a subset of

every set

so empty set is in the power set then we list down all the sets which contains

all subsets of a which contains only one elements

so we have this set one the set

containing two the set containing three

so we got all the sets containing zero element

then all the sets containing one elements then you can list down all the sets which

contains two elements

so we have one two two three one three and then all the sets containing

three elements we have one two three right

so this gives me all the subsets

of this set a containing three elements one two three now if you count down count how many elements are there

so here the number of elements in the power set is equal to eight which is equal to two cube

so for a set a we denote by this will denote the number of elements in a

so if we have if we have a set a containing n elements then the number of elements in power set of a this is always two to the power n why is this

so this is exactly like we did for the

special example containing three elements that when you are looking at the subsets

so if b is any subset of a then any particular element of a is either in b or its not in b right

so there are n elements and for each we get a subset by specifying whether it is in b or not

so therefore we have two

choices for each n elements and hence the number of such

subsets will be two to the n

so this kind of counting you will also learn

when you learn permutation and combination

so so now we will learn some operations on sets

so the first one is called union

so given two sets a and b the union of a and b this is denoted by $a \cup b$

consists of all the elements which are either in a or in b and note that when we say something is either in a or in b it also includes elements

which are in both a and b right

so let me write this in notation

so $a \cup b$ is this set of all x such

that x belongs to a or x belongs to b

so for example let a be the set containing one two and three b the set containing two three four and five then $a \cup b$ it must contain all the elements of a and all

the elements of b

so we write one two three and then we have four and five

so maybe you should note that while representing a set we do not repeat the elements for this example you see 2 and

3 are occurring in both a and b but when we write a union v we do not write two twice or three twice

so another basic operation on set is intersection

so if a and b are two sets then a

intersection b this consists of all the elements x such that x belongs to a and x belongs to b right

so intersection this

consists of all elements which are common to a and b

so for the previous example a is equal to

one two three and b is two three four five when we write $a \cap b$ we look at all the elements which are in

both a and b one is in a but not in b so one is not in $a \cap b$ two is in

both a and b

so two is in the intersection three is again in both a and b

so three is also in

the intersection then four and five are not in a not in b

so $a \cap b$ consists

of only these two elements two and three $a \cap b$ is the empty set then we say that a and b are disjoint

so the next operation on set is the set difference

so the notation is this or simply $a - b$

so what does this mean

so the set difference

of a and b consists of all the elements which are in a but not in b $a - b$ this

is equal to all x such that x belongs to a and x is not in b ok

so for example a equal to one to three b is two three four five then $a - b$ we have to look at all the elements of a

which is not in b

so one is in a and not in b two and three are in a but they are in b also

so we do not include this in the set difference similarly if you write $b - a$ you have to

write down all the elements of b which are not in a

so 2 and 3 are in a but 4 and 5 are not

in a

so $b - a$ consists of 4 and 5

so note that $a \cup b$ is equal to $a - b \cup a \cap b$ sorry union with $b - a$ right

so let me explain this by a venn diagram

so if we have this as the set a

and this is set b then $a - b$ consists of all the elements a in a which is not in b

so that means that this is $a - b$ this one is $a \cap b$ and this is $b - a$

so this is $a - b$ this part is a

intersection b and this one is $b - a$

so you can see that $a \cup b$ can be written as

these unions and note that here the unions are disjoint

so there is no intersection

between these three parts $a - b$, $a \cap b$ and $b - a$
so this one here is disjoint union
so this is something important again
to write the union of sets we can write as union of disjoint sets by using
this set difference and intersection some other terminology
so singleton
sets this means set containing only one element u
so the set containing only one set containing
only zero the set containing only the alphabet a these are singleton sets let
me also
introduce what is called complement of a set
so let u be a universal set and a a subset of u the complement of a in u is
so this is denoted by a' this is equal to
all the elements which are in u but not in a complement of a consists of all
elements which are not in a
so to say complement we always have a bigger set
and a subset and then complement is with respect to that bigger set which we
call the universal
set
so for example if u is one two three four and a is the set containing two and
three and a' complement
is equal to the set containing one and four right this is u minus the set a
set difference
with set a
so we will stop here in the next class we will discuss some more
properties of sets and then we will also look at some of the problems
from the exercise thank you