i am yami satya narayana of the department of physics iit madras todays topic for   discussion is systems of particles and rotational motion let me write the title topic systems of particles and rotational motion at the 11th standard and 12 standard level in the cbsc curriculum with respect to physics usually   one starts with units and dimensions then motion in a straight line then motion in two dimensions   then you discuss certain important concepts like work power energy and after that   one goes towards this very important topic systems of particles and rotational motion   let me give now ah the motivation for studying this center one studies in the beginning the motion of point particles even if one has extended objects like a football you study in kinematics   you assume that this football is represented by a point particle a motion of a car it is the car   is represented by a point particle but you know but you know it is not going to be terrible at a   certain level the reason is very obvious we cannot rather neglect the size of the actual objects now motion of a solid sphere motion of a solid cylinder or we can think about i will give you an   important example corresponding to motion of systems of particles it is very customary for   us when you look into the sky on a nice evening you will see a group of birds   flying this group of birds will be like this they will have in different shapes for that   matter but as the birds keep moving the shape of this contour also keeps changing and suppose we empty a glass of water on floor then the water flows the flow of water is nothing but   the motion of water molecules there are billions of them trillions of them therefore   this is the study of system of particles becomes important when we consider a cylinder we will   come to it very solid cylinder when it rolls the cylinder consists of several particles this also   an assemblage of collection of particles and right now each is a collection of each is a collection of particles or   assemblies of masses that is what we call about a rigid body   from a system of particles to on one hand like this example to rigid body

so both these kind of   systems can be put into ah the simple definition of motion of an assemblage of particles and now what are the various questions which we are going to study   in your earlier lessons we would have seen you would have seen for that matter   that what are the various conservation laws which are involved conservation of momentum conservation   of energy conservation of angular momentum things like that

so one needs to apply these notions   ah how to extend them or whether we require some additional notions in the case of systems of   particles and a rigid body and now let me give a simple definition of a rigid body what is a rigid   body a rigid body is when for example you have a metallic sphere and roll it

so the metallic   sphere would roll on this table and each of the particles is also moving we will come to it and so   in a rigid body what happens the distance between two the linear distance between any two points of   that object remains constant it does not vary on the other hand if i allow for a some amount of   water to flow on the floor the distance between two particles it is not going to remain same   this is an example of a non rigid motion motion of flow of water and when i leave it on floor   now now there are certain ideal rigid bodies what is an ideal rigid body during its motion its shape   its shape remains the same it does not change at all on the other hand if i have a mashed potato   and i depending on the amount of force i apply the mashed potato will the shape its shape gets   changed render

so an ideal rigid body is one which does not deform or the ch and   the change of shape if at all they are any they are negligible and there are two kinds of motion that is possible for systems of particles   or for a rigid body one it

is what you called as a translational motion the translation motions the simplest you have already we have already discussed it in the case of motion in one dimension two dimensions with respect to point particles and if i have an and which is moving on this plane the ant goes from a particular point a to some other point b the ant has what we called as a displacement right and now i will give you another example this is very standard suppose i have a wheel this is very ah this happens in our day to day life this is what we call it as sleeping another name which another terminology to use in this country is skidding it is a common experience in indian roads on a two wheeler or a cycle when you move on roads at certain points you know we will not be knowing some oil would have been spilt

so when your vehicle moves on it till the vehicle reaches that particular point actually the wheel would be rotating about an axis and moving it has got both rotational motion and translation motion but however when it reaches that slippery surface what happens after that the wheel moves like this in a straight this is what you called as skidding or slipping so when this happens it has only translational motion and rotational motion is not there this is one of the reasons why when ah the skidding can be very dangerous and because the front wheel which enters suddenly it has only translational motion and the rear wheel which is yet to enter that particular region which has rotational and translation therefore it is some kind of a very inconventional situation then the vehicle can skid or slip and ok another standard example which you will find in text books is i have an inclined plane and i have a let us say an object and and this slides this is what you called as sliding now all the particles of this body have the same velocity v remember velocity is a vector therefore better you put it there with a symbol ok this is also an example of a translational motion

so all particles have the same velocity at every instant of time this must be kept in mind the next example is suppose i have the same inclined plane but i keep a sphere on it it has got it makes an angle inclined plane its not important for us now in this study of this now ah if i take a point here this is the center of the sphere

so if i take a point here the velocity is like this this is the direction of velocity if i take a point here the velocity is like this if i take a point here velocity is like this whereas this the center of this sphere which we will call it as a center of mass we are coming to it in a minute it will have a motion like this

so the different points on this sphere suppose i take an inner one it will have a different direction the different points of the sphere have different velocity now with respect to this this point of contact this is ah has got it is at rest

so is the the point of contact at the point of contact if you calculate the instantaneous velocity it is zero

so different points have different velocities and now we come to ah rotation

so i have been trying to give you motivation for systems of particles and different kinds of motion which is different from which will not have that kind of situations in linear motion of a point particle center i consider the motion of a top this is very standard a top what you do the top is an object like this what you do is it is made of wood or different material then you you wind around it ah a thick thread rope rather and then you twirl it then it begins to rotate there is an axis about which this top begins to rotate it is an example of a rotational motion now when we we we when we consider the motion this kind of rotational motion of a top we need to extend all those concepts which we studied in the case of a linear motion of a

point particle   and then see how well they work and then rotation of rigid body about a fixed axis about a fixed axis that is one axis axis is fixed so i have a rigid bod sorry i draw a better diagram

so it is rotating about an axis now you will find that all the points on this axis   they are at rest whereas different points if i take a point like this i will call this as r one   it is having a motion like this its motion will be like this i consider   a smaller one its radius is small then its motion will be like this its radius is r2   right therefore the different points on this rigid body they are having different   linear velocities and they go around and right   however all the points on the axis are fixed

so this is what you call this these planes   are perpendicular to this this plane and this plane are perpendicular to the axis of rotation   now you may ask me sir is it the only way in which it ah top would rotate about a fixed axis   is this point going to be fixed always from our practical experience we would have seen   ah we will have slightly complicated motion then motion about the fixed axis and examples we have   considered here

so i can have a situation this is from our common experience what happens is the top this little

so the original axis was like this let us say i call this as the original i axis this as the original i have this original axis here now ah originally it was very vertical   initially the top was vertical then its slides and then it goes round it wobbles its head then you will have a you you have what you call it as you have a actually a cone being generated right this is this kind of ah motion is what is known as   precision you say that access of the top processes about the vertical line this is known as precision there are they can be little more complicated objects also   now let us consider an example of a situation where both are there   suppose i have a football we have we all have played football at some time or other i kick a football like this i have a football it all depends on the point of application right i when i kick this what happens is even though what i am having is unlikely instance the ball as it is goes without rotating about any axis the ball the ball bodily on its own it moves and comes even though this is kind of a rotation is this kind of motion is   somewhat very rarely happens this is what we call it as pure translation this football when i kicked it it does not rotate about any axis in whatever way this   is pure translation on the other hand i can have a situation where in the way i kick it it depends it keeps rotating the ball ah in its along its trajectory it keeps rotating about in all   possible ways there may be it may rotate about one fixed axis or it may rotate about two axis   or whatsoever we have seen very often when in a football match people kick the ball   it has a very aesthetic trajectory especially during free kicks and so   and you can see that the ball rotates in its motion when it reaches its final destination now

so far in the last 10 minutes 10-15 minutes also have been giving you a motivation for ah   different kinds of motion one is translational motion another is rotational motion ah you   can have rotation about a fixed fixed axis here you have more complex objects   rigid objects which can rotate about two or more ah one or more axis we will come to it   little later and now i will give you one ah a simple example suppose i have a door like this there are hinges there is one hinge upwards the there is one hinge here in the lower portion   now when i ah this is the floor now how do we rotate a door very often you have to apply the   force normal to it i indicate it like this the force is applied in normal way that is the easiest   way to close or open a door on the other hand if one way to apply the force at the hinges the door is not going to rotate at all you cannot open it in or close it you can do

so here applying a normal force or some side force which is making an angle to the door whichever way you will find that among all way among all possible such forces if you apply normal forces normal to the door which is which makes our life easier to open or close

so it tells you that the rotation is really about the the point of action of a point of application of a force where you apply becomes very important you may say that i have missed out a standard example like fan when you put on when they put the switch on the blades of the fan they rotate there are you would have seen even pedestal fans in a pedestal fan what happens a fan would rotate a fan would rotate it has a it has a and then from here what you do you have here you head down here

so it can also rotate ah once this blade rotates the blades rotate and you get the air and it has got an oscillation and then this oscillation is done so this moves and provides air to in its range okay there are different kinds of motion are possible and now one now i am going to introduce next one important concept which is called the center of mass in the case of motion of a particle moving on a one dimensions or two dimensions the mass of the particle because is a very important notion we need without that we cannot do anything further either that or momentum now we have to introduce what is called the centre of mass it is a key concept today i am going to introduce center of mass and do the and tell you how to calculate center of mass in various situations later tomorrow i will tell you how this actually arises in a very natural way let me give you the center of mass definition ah i will consider the simplest is simplest of several particle systems is two particle systems

so i have two particles i have two particles of masses m one and m two this is a distance of x one and this is a distance of x two the the center of mass of the system is denoted by capital x this i will call it as x axis this i i will call it as y axis i know even though i do not need it right now

so the center of mass it is defined as m one times x one plus m two times x two that divided by total mass of the system this is the center of mass under ok we will see how it is going to arise now if m 1 is equal to m 2 then automatically center of mass is in i can x one plus x two by two very elementary simple calculation you have to do that

so two objects are there m this is at m one which is located at x one with respect to a coordinate system a mass m two located at x two with respect to same coordinate system then its center of mass is the middle of x one plus x two right now we will extend this to several particles extending this to several particles what is that we have

so i have ah along the same straight line i can take m one located at x one m two located at x two with respect to coordinate system please i am not indicating anyway here i will do it then similarly m n x n then x center of mass the center of mass is located at m one x one plus m two x two then m n x n that divided by some of the masses which is nothing but the total masses we should write these things in elegant way it is written like this summation ah m i x i i runs from one to little n divided by sum of all masses i running from one to n this is the centre of mass ok now what happens in the case of in space what i mean is that this is we are considering all the particles are on a straight line and then our coordinate system is center of the coordinate system is here with respect to that we are measuring distances your particles can actually be distributed in space then what do we do i hope you are aware of the concept of position vector this is what you call a position vector

so a position vector is has got it is x component plus y component plus z

component   this divided by this is how you denote the position vector of a particle

so sometimes   we also have this notation ah the the unit vectors are denoted by e x e y   and e z

so this should not confuse you when we use different kind of   this also very standard ok in that situation that we write z equal to x e x   plus y e y plus z times the unit vector along z direction this is the concept of position vector

so now we have particles m one this position vector is r one and another particle m two mass   and you say the position vector r n etc like this you have and then you have a situation this is the   r n r sub n unit vector corresponding to the nth particle then its center of mass is given by then its center of mass is a vector quantity   what is that you are going to do you are you are going to let me write it then explain m i r i   mass into the position vector of the corresponding particle some over this that divided by some m i this is what now this is a vector quantity therefore r i must write a vector for sure   then denote is the centre of mass what is that we have done sir what we have done is we have done   a similar calculation here what we have done here and we have done it the same calculation for each   of the axis for the x axis for y axis and z axis we have done that that is what we have done

so   now you may have a situation where in ah   this is if the particles are systems of particles on the other hand if you have rigid body what   happens if you have a rigid body what will how do we extend this definition of center of mass

so this particular lecture we are more focusing on this particular this very specific definition   of center of mass and we will see how to calculate them suppose i have here let us say this is ah   it is like a mass distribution i have a rod like a scale meter scale or a foot scale right now   i take an element here this is let us say this is x this is x one dimension therefore and then the mass that is this is the position i x n i i divide this into smaller and   smaller i consider this is x i division now the mass that is here is delta m m i ok at the position x i this is same as what you had done earlier the infinitesimal mass   that is available is a small mass that is ms delta m i

so i need to multiply that times x i   summation over all i divided by delta m i i runs from whatever this is how this center of mass   in one dimension this is exactly same as what we did earlier only thing is at the particular point x i there is a small mass that is delta i

so you may see you may think that here we have   denoted ah ah there is a point here there is a line here this x i actually represents this   if you want you can take it as a center no problem

so it is a delta m i now if number of   such divisions becomes very large infinitely large now in the limit as capital n tending to infinity   the limit as capital n time to infinity what happens this will go over to m dx divided by integral integral ah d m x y into x d m divided by d m the limit as capital n tend to infinity delta m it will get convert to   the differential dm here this will convert to differential dm

so in three dimensions what happens the same thing if the mass distribution is there in three   distribution and three dimensions then our center of mass is equal to integral you are going to do   one such calculation for each of the dimensions one for x axis one for y axis one for z axis and   its a vector therefore no longer it will remain ah one coordinate alone therefore it will r d m divided by d m where r is the position vector of ah a typical point around which   the mass distribution is dm ok

so these are the various ah cases which have seen now we will see few illustrations of this concept center of mass and one typical calculation that is you will find in most of the books is about let us take our solar system will take sun earth system you know earth belongs to sun in the sense that ah it is a it goes around it one of the planets and i will give the typical numbers suppose i have ah sun here i do not need a diagram just for sun and its mass is 2.0 into 10 to the power of 30 kilograms this details you can get from standard literature and then you have earth it is very small right here small compared to sun its mass is ah its mass is 6.0 into 10 to the power of 24 kilograms you can see that this of the order of 10 to the power of 30 this of the order of 10 to the power of 24 therefore 6 orders higher the mass of the sun right now in between distance is 1.5 into 10 to the power of 11 meters from the center of center center of the earth these values we can get from standard tables now we want to calculate the center of mass of this system now what we will do we need to choose a coordinate system i am going to choose the center of the sun so center of the coordinate system x center of mass choose the origin choose the centre of sun as the sorry as the origin ok then if i do that what will i have i will have mass of sun into i am calculate i am i have to multiply by the ah corresponding distance it is located here i am now what i am considering is entire mass of sun is located here therefore that is 0 plus mass of the earth times the in between distance 1.5 into 10 to the power of 11 meters that divided by total masses six point blue into ten to the power of twenty four kilograms plus other one is two point zero into ten to the power of thirty kilograms this is corresponding to the mass of the air this correspond to mass of the sun
so i need to substitute for the ah mass of the earth here six point zero ten to the power of twenty four and you can do the calculation it will turn out to be four point five into ten to the power of five meters now we are getting some number how did we get this we are using the definition of standard definition of center of mass and we are getting a number now we need to compare how big is this or how small is this
so one way is to look at the radii of sun and earth respectively what is the radius of sun the radius of sun radius of sun is radius of sun this is the symbol the radius of sun is given by 7.0 into ten to the power of eight meters you can see that this is of the the the center of mass is located from the sun at a distance of four point five ten to the power of five this is the x center of mass it is not lying here or here or here or inside the earth the centre of mass lies well within the sun at a distance of 10 to the power order of 10 to the power of 5 this much distance is what 10 to the power of 3 7 into 10 to the power of 8 therefore this is much much less than the radius of sun ok one did not get shocked about it we are not inside the sun only the center of mass of the system is inside the sun now you may say is it always the case sir whenever we have two bigger planets it need not be the point is it all depends on how dense is the mass of the sun what is the mass distribution here suppose the same amount of mass is located in a distance of 3 into 10 to the power of 3 meters suppose in some other situation where the same amount of mass is located within a smaller radius then obviously the center of mass is going to lie outside it will not lie inside the sun it will lie outside it it will lie on somewhere here and
so the center of mass is a statement about it is also is
so we can conclude the conclusion that centre of mass is also a statement about how massive is a when we are dealing in a particular case problem now i will consider one more example you can do the similar calculation in the case of earth moon system now we will take a two dimensional example this is

example one i will consider another example in this example i will consider
two dimensional problem i will consider four particles i hope i will draw a
nice diagram looks fine x axis y axis
so they are on the vertices of a square four masses
so this i put here one kilogram   this is this coordinate is one comma minus
one comma one sorry   this coordinate is minus one comma one and this is two
kilograms i put it here   and the mass is one comma ah here the coordinate is
one comma one   now here it is one kilogram what are the coordinates of this x
axis one y axis minus one here i place a mass of two kilograms   what are the
coordinates of this x is minus one y is minus one therefore   the centre of
mass what is the definition we need to apply straight away the definition
the mass one kilogram multiplied by the position vector position vector is
actually one comma one   what is the meaning of minus one comma one minus one
comma one it actually represents the vector   minus one into unit vector i plus
one into unit vector j
so this is nothing but   minus i plus j
so do not get confused that how this denotes a vector this is a another way
of   denoting this vector which you would have learnt then here it is two
kilograms into one comma one   plus come to this vertex one kilogram into one
comma minus one plus here it is two kilograms into minus one comma   minus one
divided by i have to add all the masses
so four six ok
so what will happen to it you will see here this one into minus one is minus
one one into one   plus one and then here it is ah two kilograms i need not
reach just
so i am not writing the units   here plus two minus two therefore it will be x
coordinate is zero similarly y coordinate is zero
so therefore in this case the centre of mass or centre of mass is at the origin
right this is   how you will be doing this various problems we can also ah i
can consider this as a lamina and do it   what do i do it no i now i am doing
the other problem
so i am shading this   here the mass is 1 kilogram here the mass is 2
kilograms   here the mass is one kilogram and here the mass is two kilograms i
am doing the next example   then what i need to do the center of my this
lamina is actually divide into four parts so   i take this particular lamina
its center of mass i will look for a different colored chalk its around yeah
now the center of mass of this square is ah this is half   comma this will be
again half because the side of each square is half
so like this
so i can find the center of mass of each of the squares and do the similar
calculation   ok fine now there are some problems in in center of mass which
we can do by inspection what i mean   is there is an inherent symmetry we can
exploit it we will do one or two problems illustrations
so let me consider ah i have a this is by center of mass   this another
example i will say example four involving symmetry symmetry is a big term in
physics you will come across it very often   suppose i have a triangular
lamina of uniform thickness ok i take a uniform cardboard and cut it   i want
to find the center of mass now in such cases i can make use of symmetries to
tell you   what i mean is i can make use of the geometry and straight away i
can one can calculate it   what we do is i divide into smaller and smaller
strips of infinitesimally small thickness
so if i   take a infinitesimally small thickness its center of mass is going
to be at the center
so next strip   will take it will be at the center therefore all the center of

masses will lie in this is it right now i do the similar thing therefore i have this is what is this line corresponding to the center of mass of such a strip all of them are right therefore now you can find out if i do this for this side also i infinitesimally divide and join the center of mass obviously the center of mass this point this centre of mass is what we call it as where all the midpoints of each side which point of each side is joined to the opposite vertex such a point is known as centroid

so by similarly when i have a solid sphere its center of mass is going to be located at the center solid sphere of uniform mass distribution i can also have a situation where the mass distribution is uneven in various regions not like that its a uniform its a metallic sphere of uniform distribution of mass then its center of mass will be at this point now there are situations where one needs to make use of integration you cannot escape that as a physics student you need to allow and its a tool not very complicated integrations are required suppose i have what i will call it as a uniform mass distribution

so i have a rod which is ah let us say that this is at a d dm is the mass which is located at the x trans x

so suppose i know that in this k is the linear density corresponding to an infinitesimal distance d x this is d x this thickness is there no this is d x the mass that is available here is d m so this is whatever length i take dx whatever infinitesimal bit i take the mass distribution is k times dx k is some constant ok then i need to calculate the center of mass the center of mass by definition is nothing but integral at the distance x there is a distribution of mass dm that divided by total mass of the system this is equal how to integrate from x is equal to zero this end this is x is equal to l this integral is x square by two i need to evaluate between zero to l which i forgot to write zero to l this divided by total mass is m let us say then this is i can calculate this because d m is k d x k d x i forgot write a k here sorry because this d m ah this is d m times d x d m is k d x right and so i when i do that this will become d x

so i have k d x this will be equal to l square by two k and k will get cancelled all that i will have is l by two therefore if i have a rod of uniform thickness if the mass distribution is uniform then when i calculate it the center of mass will be exactly at the center this is l by two measured from here right now let me summarize what are the various things we have seen before i do that is it possible such the center of mass of a system lies outside it simplest example is a two particle system centre of mass is not going to be with x one or x two its a point mass suppose i have an object like this by symmetry arguments i can say that the center of mass where it going to lie it will be somewhere here it will not lie on this and this is another standard example which people use it in gymnastics i will not go about discussing about it

so let me summarize what are the things various we have seen in this ah first we started with the motivation that is required to move on from the kind of ah problems you study in one particles moving in a straight line in one dimensions or motion of a particle in two dimensions two several particles moving in general in three dimensions and we saw that two kinds of motions are possible one is translational motion and it is rotation

so a general rigid body is one in which the two points are fixed and okay the motion of general motion of rigid body is one translation followed by rotation one important concept that is required we are going to use it i will tell you how this concept comes tomorrow but we have given it and used it is the concept of center of mass and and also given different examples of

rotational motion possible   we have calculated a few illustrations we have seen sun earth system then four masses   which are distributed on the points of a square then the laminar problem i did not complete it   hopefully you will do it otherwise tomorrow i will finish the calculations for you and last one is how you calculate the center of mass using the symmetry that is involved in the problem and there   are situations where the center of mass has to be calculated using integration

so just to remove   that fear i have taken a simple illustration of a one dimensional problem where in a in a small infinite symbol location d x that masses that is available is d m suppose i know this law   ok the mass that is available is proportional to d x it is called linear mass density   then you will find that it is l by two you can have some other mass distribution also then this if it is going to be mass is more and more distributed away the center of mass will move   away with this will stop for today and tomorrow we shall go further tomorrow we will be talking about   what are the conservation laws that are required how we how the conservation   velocity was used in kinematics of one and two dimensions can be put to use   using the centre of mass concept in the case of several particles and rigid bodies thank you
so
 do you