

In the last class we created equations for projection motion and looked at all the different relationships for motion in two dimensions, let's try today we will solve some kinetics problems and let me start with abbreviation of projection motion as we recall a projectile. Speed is the motion of a particle which is released at a θ angle with a velocity v_0 and when we look at this particle it is released into the air because we see that this velocity is curved and if we draw our x and y coordinates like this initial velocity v_0 . And v_0 got both elements

so it's a two dimensional motion and the way to look at it to deal with this kind of problem as we have seen is to separate the motions in the horizontal and vertical directions. Here if we look at the horizontal motion then the acceleration of the particle on the horizontal side which I write as $a_x = 0$. The initial velocity which I write as v_{x0} will be $v_0 \cos \theta$ and if we look at the vertical motion which is in a sense doubled from the horizontal motion. If here we see that the acceleration is equal to the acceleration due to gravity and because y is pointing upwards then a_y is equal to minus g and if we see that the initial velocity towards y is equal to $v_0 \sin \theta$ so now we get from this and if we look at this we get 2 straight forward relations

so we write more equations for v_x and v_y and displacement along x and y and before as we have divided the horizontal and vertical elements separately so any time t will be given next $v_x = v_0 \cos \theta$ and then we have a term plus $ax t$

so it's plus ax times t but because $ax = 0$ so v_x on x will be the velocity x element of velocity will be constant the vertical element of velocity we can write v_y is equal to $v_{y0} + a_y t$ let's write v_y is equal to zero $\sin \theta$ minus g times t. Because g is equal to the acceleration of the y direction

so if we write that the distance traveled to x

so let's make this small image again we start from the coordinates of $(0, 0)$ and then the projectile travels a path that we want to find in any position x. What is a comma? The coordinates will therefore be given x coordinates by $x = v_0 \cos \theta t$ and y coordinates $y = v_0 \sin \theta t - \frac{1}{2} g t^2$ and from these 2 equations we get the equation by writing y as the function of x and we get it by subtracting time and if we do that, what we see is x is equal to $x_0 + v_0 \cos \theta t$ and this is the equation of t and then we put it in the expression for y

so we get y equal to $v_0 \sin \theta t - \frac{1}{2} g t^2$ times $x = v_0 \cos \theta t$ and it gives us the expression when we simplify it to get y equals $x \tan \theta - \frac{g}{2 v_0^2 \cos^2 \theta} x^2$ which we explained last time is a parabola equation

so now that we have this equation the distance we find in the projectile speed is the distance that the projectile travels before hitting the same level or the ground if it is thrown from the ground

so let us let the coordinates of this point be x equal to x and y equal to 0. The starting point is zero comma zero. All we have to do to get the expression of pressure for this range is y is equal to zero here and it will give us the value of x and we also see that what we do is if we want to calculate the flight time of the projectile if we call it t this time. For this time we have to go to the velocity equation of velocity equation we have v_y is equal to $v_0 \sin \theta - g t$ we see that the expression of x is equal to $x = v_0 \cos \theta t$ squared we put y is equal to 0 here

so when we have y here 0 of Equal then we get 0 is equal to $v_0 \sin \theta t - g t^2$

minus half gt^2 and from here we get the expression for the flight time which is equal to $2v_0 \sin \theta / g$ and then we know once we get the range

So $v_0 \cos \theta$ in t

so it is equal to g in $v_0 \cos \theta$ to $2v_0 \sin \theta$ and we can write it as $2v_0^2 \cos \theta \sin \theta / g$ and it can be simplified

so these are some of the things we can get now. Let's look at this expression again and see this expression. Why the orbital path that y coordinates that travels is given by subtraction of the $\theta \sin \theta$ semi- gt^2 square

so now we understand that when this particle travels at a projected speed it is equal to 2 times t_1 and t_2 . And if we look at this equation, it is very clear that we get a quadratic equation in t for y for the same value and the two main

stars of this quadratic equation give the values of t_1 and t_2 . Finding that Δt is equal to if we need the expression for Δt which is t_2 minus t_1

then we have these two roots t_1 and t_2 and we can get if we solve for the two roots then we can subtract them and t_2 subtraction can get the expression for t_1 and it will give us the time it takes for the particle to cross the distance

which is now here at 2 different places while crossing the same height. By subtracting B is equal to and root G_u The product of c is a and where the equation is given by $ax^2 + bx + c = 0$.

The two roots are α and β . If I see $\alpha + \beta$, the whole square minus 4 times $\alpha\beta$ will give me $\alpha - \beta$ whole square. So if I need to find this Δt , then I don't need to solve the two roots. I can express this amount

as $ax^2 + bx + c = 0$ and then from here I can get the difference of the two roots because $\alpha + \beta$ will be given a . The subtraction of b by $\alpha\beta$ will be given by a by c

so I can feed these things here and from there I can get the root difference so I will leave it to you to work out in terms of parameters our problem I

told you the basic method how to do it because some problem you may be asked at this time or there may be a problem where you may be asked to find Δt_1 and Δt_2 where Δt_2 will have time for another height and maybe This height

difference will be given to you and then you can express this height difference in terms of these variables

so that this kind of problem can be easily created and worked on and you can try it for yourself

so now let us see some more things in projectile motion max. Height. This is what we showed in the last class that h is achieved for maximum h at a

projection speed when v_y is equal to zero

so when you apply v_y it will be equal to zero the time v will be equal to zero $\sin \theta$. Zero on G and you realize that it takes half the time it takes to

complete its motions to reach the same level and this should be expected because the time it takes for a particle to go up $1d$ takes the same time to go down at $1d$ speed and $1d$ perpendicular to a projectile Speed is decoupled from

horizontal speed

so when we work the height if we want to work then time $v_0 \sin \theta / g$

so now we put the expression in the formula for y

so it becomes equal to $v_0 \sin \theta$ In the period of θ which is $\theta \sin \theta / g$ minus half times g multiply t^2 square

so $v_0^2 \sin^2 \theta / g$ and when we do this

so this y is equal to h and it becomes v equal to $\theta \sin^2 \theta / 2g$

so this is the maximum height achieved by the projection and it takes time to achieve t equal to 2 where t is the total time of flight

so what we have taken is the total time we saw it $2v_0 \sin \theta / g$ equals g and reaches the maximum height $v_0^2 \sin^2 \theta / 2g$ your g now

from here if we t We see that t^2 becomes equal to $4v_0^2 \sin^2 \theta / g^2$

square theta zero on G square and we see that it will be equal to 8 hours^2 and also if we make the expression of h in terms of range then what we get is $h = \frac{v_0^2 \sin^2 \theta}{g}$. The range of over 4 is equal to the time tangent. This again I will tell you to work. You have the formula for the range of work. Find this out and it will also be equal to t^2 divided by eight. So based on what we have done, I hope let them work. Now let us also look at the trajectory formula of y. The range terms and for that we see the expression $y = x \tan \theta - \frac{g x^2}{2 v_0^2 \cos^2 \theta}$ and so we can rewrite it as $x \tan \theta - \frac{g x^2}{2 v_0^2 \cos^2 \theta}$ so we multiply the sine theta by θ and divide the sine theta by θ so it will be equal to $x \tan \theta - \frac{g x^2}{2 v_0^2 \cos^2 \theta}$ then we see that there is a $2 v_0^2 \cos^2 \theta$ which is equal to the range so that it is equal to $r \sin \theta$ over $r \cos \theta$ so that $x \tan \theta - \frac{g x^2}{2 v_0^2 \cos^2 \theta} = r \tan \theta$ so let us now look at the vertical distance traveled by the projectile in terms of range. Explain ah because sometimes it can be useful and looking at some problem so let's write y is given $x \tan \theta - \frac{g x^2}{2 v_0^2 \cos^2 \theta}$ so we call it $x \tan \theta - \frac{g x^2}{2 v_0^2 \cos^2 \theta}$ and we multiply and divide by $\sin \theta$ so it is equal to $x \tan \theta - \frac{g x^2}{2 v_0^2 \cos^2 \theta}$ and then what we get in the second term is a $2 v_0^2 \cos^2 \theta$ has a $\sin \theta$ The theta has θ This product is equal to the range so we get $x \tan \theta - \frac{g x^2}{2 v_0^2 \cos^2 \theta} = r \tan \theta$ and then we have the tangent theta is zero so this quantity can actually be further simplified we can write it as $x \tan \theta - \frac{g x^2}{2 v_0^2 \cos^2 \theta} = r \tan \theta$ this x property as θ to $r \tan \theta - x$ so r can express this way so what we are doing is we are interplaying between different variables we have r we have h we have t and we need Depending on we can express other variables in terms so let's solve a problem for example given that a projectile is released with a velocity v_0 angle θ as we specified and its position at any position x, y The angle with which the projectile is formed originates with the starting point alpha and if it is the end point r if I add it with a straight line then this angle is beta and what we need to find is a relationship between alpha beta and theta zero So let's look at this problem. At this altitude this height y is given by coordinates x, y so this height is given by y so the alpha tangent is given let's say this distance is x so the alpha tangent y is given by $x \tan \alpha$ Is given by dividing by y sorry y divided by $r \tan \alpha - x$ which is the tangent of beta if we add two then we get $\tan \alpha + \tan \beta = \frac{y}{x} + \frac{y}{r \tan \alpha - x}$ and when we work it out so It is equal to y times so it is the tangent biter of the alpha P. The loss tangent is equal to y times $r \tan \alpha - x$ plus y times x times x times $r \tan \alpha - x$ so it is equal to y times r divided by x times $r \tan \alpha - x$ and if we Here is the general expression of y that we just got $y = x \tan \theta - \frac{g x^2}{2 v_0^2 \cos^2 \theta}$ times $r \tan \alpha - x$ so so y times $r \tan \alpha - x$ times x is equal to the tentacle of theta zero

so it means tangent is equal to tangent of alpha plus tangent beta theta zero so such a problem can be easily solved One of the problems you need to look at is where the given variables are and in terms of which expressions you can find them and in case you don't need to memorize any formula, you just have to remember that x direction acceleration is 0 with x and y combination x so v_x is always $v_0 \cos \theta$ and traveling distance will be $v_0 \cos \theta t$ in y direction acceleration is equal to $-g$ so t

so vertical velocity will always be equal to $v_0 \sin \theta - gt$ and y is positive The distance traveled will be $v_0 \sin \theta t - \frac{1}{2}gt^2$ Now there is one more thing one can see at projection speed and all we have is to look at the range formula we say range R is equal to $\frac{v_0^2 \sin 2\theta}{g}$ so Now if ours The value of v_0 and θ is given to mean a certain velocity and θ and if we see another projectile that is thrown with the same initial speed v_0 but 2θ at an angle of $\pi - 2\theta$ when we see these two projectiles This means that a projectile is thrown at an angle of θ to another at the angle of $90^\circ - \theta$. Now if you calculate the range, its range will be equal to $R_1 = \frac{v_0^2 \sin^2 2\theta}{g}$ and $R_2 = \frac{v_0^2 \sin^2 (\pi - 2\theta)}{g}$ will be equal to the square square $2 \times \pi$ by 2 minus 2θ and it R Nothing will happen. The sign sign of this angle is $\pi - 2\theta$ which is the same so it will be equal to R_1 because $\sin(\pi - 2\theta) = \sin 2\theta$ is equal to the theater sign

so for the given initial velocity the same range will be covered by two angles and the sum of these two primary angles is ninety degrees. Will be equal and from this we can clearly see that the range for a given v_0 will be maximum when θ will be equal to zero four or forty five degrees

so a body angle which is thrown at an angle of 45 degrees covers the maximum range but notice when We are talking about range here which distance does not mean that time taken by t_1 and time t_2 will be the same time will be different and it is clear from this fact that this time of flight depends on initial vertical velocity and when θ is different Of projected one and of projected two The initial vertical velocity will be different though the ranges will be different and similarly the maximum height that they clear or that they reach h_1 and h_2 will also be different

so the range will be the same but these things will be different and sometimes you have t_1 and t_2 or h_1 To find the relationship between h_2 and these things you can now do by playing around with things here. As an end let's take a case of a projectile at an ah projection speed. In fact the equations we created earlier are valid here but when We can talk about a curved plane as we can see the problem

so let's say we have a plane here ah plane it has an angle α and we throw it a coordinate θ and we throw a projectile with a velocity v_0 at an angle θ from horizontal

so θ is the angle that makes the projection relative to the horizontal and it is the curve with which the projectile is thrown now it will go up and hit it and it will come back and hit here Now let's say that this was the original point B and let's say this distance now since the range has now noticed this range is different from the previous problem in the sense that in other cases where we hit the ground back in the previous case it was at the same level now. The projectile is hitting a different wire level than the starting point

so now if we want to find this range in terms of these parameters given to us then what parameters do we have now? There are angles of inclination and while

doing this we now want to find the range can be easy because now if I choose for this particular problem then I choose if it is a slope and this is the perpendicular direction I choose x and y perpendicular to the curve. And so the projection is going on

so now if I look here we start from the original $\theta = 0$ and the coordinate of the final point I want to find is r, θ because now with y bend Perpendicular and x along the slope

so now when we see it the projection feels an acceleration equal to g now this vertical direction is not parallel or perpendicular to x and y

so what we do is we solve the acceleration along x and y direction So what happens if I choose x and y as I have chosen,

so what we can see now is that if I look at this side of the vertical side, this side is parallel to this plane, this angle is alpha, this angle is 90° minus alpha,

so here if the element is this g This element will be $g \cos \alpha$ in this direction and the element in this direction will be $g \sin \alpha$. This is actually $g \cos(90^\circ - \alpha)$ which I can write as $g \sin \alpha$. I have taken x along this direction g then its subtraction and the y component of acceleration as I have chosen this x and y will be minus g cos is equal to alpha

so we have to realize these two points

so now the difference is when I use this formula of x and y which What I get when leaning in compared to the previous case is x There is an acceleration on the side and there is an acceleration on the y side where before the acceleration was only along the y direction and this difference is coming because the way we have chosen our axis

so if it is

so we write it like this

so now all we have to do is look at the axis Minus g sin equal to alpha and ay equal to g equal to alpha $v_x = 0$ now we need to see the x element of velocity

so velocity is $v = 0$ makes an angle now this $v = 0$ velocity is making an angle with x axis $\theta = 0$ minus alpha and So what we have with the perpendicular element is that $v_x = 0$ theta zero minus alpha will be equal to $v = 0$ cosine and v_y will be zero theta zero minus alpha is equal to $v = 0$ sine,

so once we have this we can only proceed we v Let us write our equation for v_x and y. What we have is v_x is $v = 0$ cosine of $\theta = 0$ minus alpha minus $g \sin \alpha t$ and for v_y we get by equal $v = 0$ sine of $\theta = 0$ minus alpha minus $g \cos \alpha t$ and then we write x element x component $\theta = 0$ minus alpha t minus half $g \sin \alpha t^2$ In the alpha product t will be equal to $v = 0$ cosine of the square and what we get from here is that y is equal to zero v is equal to the zero sign. $g \sin \alpha$ get this extra term because of alpha

so now when we get this sum for y now r, θ is equal to $\theta = 0$ y

so it means for this value zero $\theta = 0$ v zero sine minus alpha multiply t minus $g \sin \alpha t^2$ because by alpha t square t two and from this we can get the value of t so we get t is equal to two v zero sign of $\theta = 0$ minus alpha by $g \cos \alpha$ so this time the projectile takes time to get back to the plane now we need to find the expression for our range now to do this One way is that we can plug the value of t into the expression of x which is equal to $x = \theta = 0$ minus alpha t minus half $g \sin \alpha t^2$ v zero cosine and it will give us the value of r but we can look at it differently and it is numerical. Will be a little easier

so let's draw this picture again This was x This was y This is the range we see This angle was alpha Now what I say is that I call the horizontal side as x star

so the distance we have is the distance that the projectile travels when it travels a distance and the horizontal distance $r \cos \alpha$. Let $r \sin \alpha$ be the vertical distance. So now if I want to find it $r \cos \alpha$ is the horizontal distance that has been traveled and it is nothing more than $v_0 \cos \theta$ in t_b because if I look at x star coordinates I still have the old equation $x = v_0 \cos \theta t$. The initial velocity traveling the distance is nothing more than multiplied by time and when it comes to its position b the time given as $r \cos \alpha$ is equal to the capital t and the capital t which we now see here is equal to $\frac{2 v_0 \sin \theta \cos \alpha}{g}$.

so I can get $r \cos \alpha$ equal to $v_0 \cos \theta$ multiplied by $\frac{2 v_0 \sin \theta \cos \alpha}{g}$ minus α divided by $g \cos \alpha$.

so this way I can get the value of $r \cos \alpha$ and what we understand that is, the $r \cos \alpha$ range is nothing but equal to $v_0 \cos \theta$ from this figure. It is clear that $r \cos \alpha$ is equal to $r \cos \alpha$.

so what I have found is that the range $r \cos \alpha$ is equal to $v_0 \cos \theta$ to $\frac{2 v_0 \sin \theta \cos \alpha}{g \cos \alpha}$ minus α divided by $g \cos \alpha$ and it gives me a r . Gives the expression which is equal to if I divide $2 v_0^2 \cos \theta \sin \theta \cos \alpha$ by $g \cos^2 \alpha$.

so I can find it and I get the same expression if I plug the value of t into the expression of x and then I would simplify the expression using trigonometry but I get the same value of r but here I have used this expression and used horizontal elements of range and using trigonometry its relation to r . I probably have the same relation in less time or whatever. So when you have a problem where the situation is different from what you saw you should. For example in this particular problem we chose x and y along the curve so that the expression of r becomes quite simple. It only becomes y coordinate 0 but when we I do then we realize that we need to split the acceleration along the new x and y direction similarly for example you might have a problem where some of your curves are thrown down but all you can do is if you choose your x this way you now have your y . Choose when you solve your gravity element this element will be positive to the x element it will not be negative because your x is now going down.

so depending on whether your problem is x up or down you have to solve a certain value of acceleration along x and y and then they have to be solved. Be careful about the symptoms. If you take the y above, there are some negative signs.

Consider all the positive signs. Before you complete your problem in these symptoms we have looked at various concepts of projection speed in detail. Let us look at another problem which uses the concept of relative velocity and sometimes these kinds of problems are also referred to as pursuit problems which is given to us by an aircraft. There is an initial position if someone says that this is a distance b .

so if I look at the plane in terms of xy the plane is primarily a comma b and a missile l_a to the aircraft.

Which is also given as constant and the missile is always directed towards the aircraft.

so the aircraft and the missile based on the initial position and given that the plane is traveling and the plane is traveling along a straight line so v_a is constant and we can also say that the velocity of the aircraft we can reduce it. Equally given that the velocity of the missile is constant speed constant and it is always directed towards the aircraft which This means that when the aircraft moves to the next position, the missile will change its direction accordingly.

so that it always moves towards the aircraft. We have the plane in this

position. The missile is here and it is traveling with the v_a . The velocity of the missile is directed towards the aircraft so it is v_m . The plane would have moved to a position other than where it started, so the plane would have moved from a comma B to its position now given by x and since it is traveling in a straight line it will always have a y coordinate b and now the missile is somewhere here. We do not know the exact position at any normal time t but the speed of the missile is directed towards the aircraft so what we do here is that we Then we put a coordinate system that we see things with respect to the plane so now if we look at this we can say this angle at any common moment between the line of the plane and the line of the plane. Missile I call s angle as ϕ so let's draw it bigger this plane this is a missile in a normal position I am drawing this this I call this angle ϕ so what we get is the rate of separation of the aircraft and the rate of this separation of the missile I mean, if I call it r , the dimension of dr will be equal to the minus velocity of the missile in the direction of r , so what will be our separation? What is the rate of change of this separation? Actually I shouldn't have put the vector marks here so I'm going to cut them out here now and if we look at this figure what we get is the velocity of the plane towards r this angle is also ϕ so it will be equal to $v_a \cos \phi$ so the rate of change of separation $v_m \sin \phi - v_a \cos \phi$ is equal to dr/dt and if I combine it, what I get is dr is equal to the integral of $v_m \sin \phi - v_a \cos \phi dt$ from zero where t is the capital t when the missile hits the plane and this dr when I combine it Then No this is equal to the sum of these two because this dr when we see it is nothing but the total division that was there and this separation is the square root of a square and B^2 so what we get is we get an equation which is called our square root a Square plus B^2 is the segment of the square which was the first and finally the segment is equal to zero so what we get when we combine dr is the square root of a square plus B^2 it will be equal to the whole of zero to t $v_m \sin \phi - v_a \cos \phi dt$ Now what we understand here is that angle ϕ is changing with time so it is not easy to find the integral part of $\cos \phi dt$ because angle ϕ is changing in each position so what we can get from here is that its core is a square plus b^2 Equal $v_m \sin \phi t - v_a \cos \phi \int_0^t \cos \phi dt$ then we ask if we can find integral $\cos \phi$ from some other information which is our problem and if we look then we can understand what then x direction and since we have We have placed our coordinate systems in the reference frame of the plane so that the relative velocity of the missile relative to the plane towards x it will be equal to $v_m \cos \phi$. I can write by dx/dt which is the separation of x distance in the frame of the reference plane is equal to $v_m \cos \phi - v_a$ and if I combine it then what I get is integral dx will be integral $v_m \cos \phi dt - v_a t$ and this x distance which has been moved in the reference frame of the plane This is nothing more than the initial separation because it is the d distance that the missile has to cover the plane's reference frame with x direction so what we get from here is the second equation that gives us a is equal to $v_m \int_0^t \cos \phi dt - v_a t$ I can call it as equation number 2 and the equation I got earlier I can call it as equation number one from one we can get the value of integral $\cos \phi dt$ from zero to t and we get 2 And when we do everything max then finally what we get from this substitution is t

comes out a factor $v a$ plus a square root of the square root of the product $v m$ plus b square divided by $v m$ square minus square. Now let's move on to another aspect of dynamics. Let us look at this quantity $\mathbf{v} \cdot \mathbf{a}$ and see if we can get anything from this scalar as we see that \mathbf{v} is velocity vector and \mathbf{a} is acceleration vector. Please note that all the formulas we use as a function of v . These formulas are valid only when the acceleration is constant. If the acceleration changes then these formulas are not valid. Now we see this dot product of two vector quantities \mathbf{v} and \mathbf{a} and v and a . \mathbf{a} is both common meaning they are not constant they can be they can change over time and they can be in any direction. That is, they are each other. The angles between the vector and a vector are either zero or π , but at two different speeds we can have an angle \mathbf{v} and an angle that in general between \mathbf{v} and \mathbf{a} instant this will not be equal to 0 or π it can be anything between zero and π and such a body in general will be moving along a curved path just to recap what we had seen was when a body moves along a curved path then the velocity is always tangent to the path and the acceleration has two components a component which is tangent to the path which is nothing but the rate of change of speed and the second component which is perpendicular to the path and which is pointing the second component points to the center of curvature of the path which means because this is a curved path locally we will assume if this body is moving like this that if this is moving in this way that if it is moving in a circle then a second component of acceleration will point towards the center of that circle and this is what we can call as the normal component and this is given by speed square divided by the radius of curvature so this is what will always happen when a body travels in a curve path now what I wanted to discuss was if we look at this expression of $\mathbf{v} \cdot \mathbf{a}$ now looks like a very mathematical quantity there is some velocity there is some acceleration but let us look at this we can write this as $\mathbf{v} \cdot \frac{d\mathbf{v}}{dt}$ because acceleration is $\frac{d\mathbf{v}}{dt}$ and this we can write it as $\frac{d}{dt} \left(\frac{1}{2} v^2 \right)$ so this will be equal to half times $\frac{d}{dt}$ of v^2 where v now is without the vector therefore this is equal to speed so now from here what we can see is if we look at this quantity $\mathbf{v} \cdot \mathbf{a}$ if this is equal to 0 then what this means is if $\mathbf{v} \cdot \mathbf{a}$ is equal to zero that means $\frac{d}{dt} v^2$ is zero so the physical meaning of that is that speed is not changing with time so speed is not changing with t and if always if at all times $\mathbf{v} \cdot \mathbf{a}$ is 0 so let us write this if for all times $\mathbf{v} \cdot \mathbf{a}$ is 0 this will imply that the speed with which the particle is moving is constant so if the acceleration is perpendicular to the velocity then the particle has to travel with the constant speed also we can have a look at the sign if $\mathbf{v} \cdot \mathbf{a}$ is greater than zero now physically when will this happen if this is the velocity vector if this is the acceleration vector $\mathbf{v} \cdot \mathbf{a}$ will be greater than zero if this angle θ between \mathbf{v} and \mathbf{a} this is if the angle θ is between 0 and 90 degrees because it is $v a \cos \theta$ if the angle θ is between 0 and 90 degrees then $\mathbf{v} \cdot \mathbf{a}$ will be equal to magnitude of v times magnitude of a times cosine of the angle so that will be equal to this will be less than 0 . so now as we have seen $\mathbf{v} \cdot \mathbf{a}$ is equal to the rate of change or half the rate of change of square of speed so if $\mathbf{v} \cdot \mathbf{a}$ is positive this implies $\frac{d}{dt}$ of speed square is positive and this will mean that speed is increasing and if $\mathbf{v} \cdot \mathbf{a}$ is less than 0 this will imply that during the path of the particle the speed of the particle will decrease

so sometimes when we have to make some conclusions about quantities we can use these mathematical facts to make some conclusions about the qualitative interpretations using mathematics is what we can very easily do

so now what we have done

so far is that we have studied the equation of motions for one dimensional motion for two dimensional motion we did them for the case of constant acceleration we also derived the case when the acceleration is not constant then or then we you write the position vector velocity and acceleration as derivative that is acceleration is the derivative of velocity and position vector the derivative of position vector gives us the velocity vector so and we have seen how to differentiate the vector quantities in detail so we have therefore we have completed the study of motion of a particle of how to explain the motion of a particle this is what we call as kinematics so far and also one more thing which i expressed to you very clearly is that when we look at $\frac{dr}{dt}$ we went up to the second derivative that is up to acceleration ah sometimes the question is asked why don't we go to the third derivative or the fourth derivative and the reason for that will become clear when we study what causes motion

so far we have just studied the details of the motion without trying to understand what is causing the motion now that is what is going to follow next what we are going to do is we will try to understand why a body starts to move and what we will see is that there is a quantity called force and when force is applied on a body that is what causes the motion of a body to change and this is what will be covered under newton's law and that is what we call as kinetics and together kinetics and kinematics are referred to as dynamics so now having studied the motion and kinematics of a particle now we will focus in our next module on the dynamics of a particle that means we will look at a particle and then we will relate the force to the rate of change of momentum of a particle which we will define and that is what newton's laws tell us about newton's second law particularly tells us about this which we will cover in the next class thanks you