In the last class we saw vector operations and we saw vector dot products. We saw vector cross products and we saw how we could perform vector operations. Which I am drawing here and in a plane it will be a curved path,

so let us say the particle is in position p at t we are studying it in case of some reference frame where we have fixed the coordinate axis

so the position of this particle is vector t at time t by vector r Suppose t plus deltas at a time means that the particle reaches the p prime position after time t in the delta which we identify as p prime as t plus deltas and here the position vector is now in this coordinate frame as t plus deltas r. Given that let's say xyz since we 're talking a planner. The system z axis will always have a z coordinate always zero

so we see the coordinates of xy in this position p x xy coordinates in x x y and p coordinates in position x plus delta x and y plus delta y and vector pp this vector is prime. I will say this is the displacement vector which we represent by delta r

so now what we have seen is that if we write the vector pp prime it is equal to the vector delta r and it is equal to r in t plus delta t. Minus r at t and what we have seen is the instantaneous velocity of a particle at the point p . Is equal to the derivative

so it is the instantaneous velocity of the particle in p and it is the same as one dimensional motion but now that it is a two dimensional motion we have a difference between the two vectors and we can also realize that r at plus delta t it x plus delta xi plus y plus delta yj And r is equal to t xi plus yj

so delta r can be written as delta xi plus delta yj which means we are taking the scalar component of vector delta r and

so vector v then becomes equal to delta x by delta ti plus delta y by delta tj And we can also write it as vxi plus vyj where vx will be the x component of velocity and vy will now be the y component of velocity. Here we have defined it in the instantaneous case. In that case the average bay We will say g as v and keep the average sign and it will be equal to delta r by delta t and in this case when we talk about the average velocity that the boundary does not go to delta t 0 then define it for any delta island. Now we can also understand that its direction velocity is the direction of the delta r and if we go backwards then the same in our image here is the delta r in this direction

so the velocity will be towards the delta r and at the boundary delta t 0 this direction will be tangent to the path. So let us write that the delta r side of the range is tangent to the path

so this means that the velocity side is always towards the path and if you think physically it would also make sense because something traveling along a certain direction sticks to that direction. It has to be on some moving path that it can't go in or it can't deviate from the path

so the direction of v is always tangent to the path and if you look at the dimensions of v then what we saw is this velocity we have it We can write vx i plus vyj if we compare it to the elements x and y Write the vertex and the dimension of v as it will be equal to the square root of vx square plus vy square and sometimes we also write it as v or we can write it as v without the vector sign

so it is a symbol of velocity v and also what we see is if it is This vx is vy and if this angle is theta then velocity vector makes the theatrical tangent the direction given by vx on vy

so dimension is the square root of vx square and the direction of vy square given by tan theta is equal to v by bx or we Write two elements vx and vy

so these two things we want to consider would be the concept of relative velocity in two dimensional motion and it is the same as relative velocity in

one dimensional motion but now since velocity has different aspects we have to deal with them now in a vector. The velocity of a particle a is va and the velocity of a particle b is vb, both of which are measured with respect to some frame of reference, but the velocity of a relative to b can be written as va minus bb or sometimes it is a relative to b Also written as relative velocity so the concept of relative velocity is the same as 1d speed we saw here but now we have to take va and vb as vectors and it will be a vector minus

so for example if raindrops are falling this way and a person is cycling this way In the case of bicycles the velocity of rain will be equal because in the case of bicycles v bicycle plus the sum of rain will be equal. With the velocity of rain

so if a person is cycling this way and the rain is going down vertically on him he will feel that the raindrops are coming at the angle given by the vector which by subtraction of these two vectors then we move towards the acceleration of a particle which we call vector a. And as we have seen acceleration is given as the rate of change of velocity

so if velocity vt at t and velocity at t plus delta t is v at t plus delta t then we can write the delta v equal to v v t t plus delta t The vector v at subtraction t and the acceleration will be given to the range delta t zero v at t plus the delta t minus v divided by the delta t

so we look at the velocity vector at the two points and take the difference between these two and it gives us the acceleration and if the delta t is too small Or if it is a finite interval then the acceleration we get is the average acceleration

so and as before we can add the acceleration to the axis as ayj where ax is the x element of acceleration and it is equal to delta vx by delta t and ay is the family The y component of the acceleration given by the text is vy by the delta t

so we have this amount of acceleration and what we see here is the x component of the acceleration which is the derivative of the x component of velocity d by dx by dt by dt and it can also be written as time and si x As its second derivative, the y component of acceleration is dvyydt and it can be written as d by dt by dt by dy and it will be the second derivative of displacement y element will be given by it. Velocity and acceleration are the same direction that will be given along the line. There must be a negative sign in it which means it may be opposite to that direction but in general the direction of the vector will be the same. The thing I have seen is that the direction of any instantaneous motion is tangent to the path. Now what can be said about acceleration? Let's say the particle is this position p. This is now position p prime. When we see this, immediately the direction of v here is the tangent to the path here and the direction of the p prime in p prime will be the tangent to the path in p prime. Let's plot velocity vector together with tail

so we have vector v here and we have vector v prime here and the vector that connects these two is vector delta v and now we know that acceleration is equal to delta v by delta t

so its direction of acceleration Must be along delta v and which is not explicitly along the tangent and

so what we see is that there is acceleration in a curved path in a curved path due to two elements the first element comes due to change of motion. Particle and it is an element that occurs when a particle moves along a straight line

so there is a change in the motion of the particle and this element is always the same because if the particle moves in the same direction its speed changes then there is an acceleration

so this component vibrates with that direction Where velocity is that it is along et et but besides we have a second element of acceleration which is perpendicular to this direction and it comes because the direction of velocity changes due to the curved nature of the path. v will lead to an element which is basically perpendicular to the direction of et and this element is actually the second element or perpendicular element which is perpendicular to et.

This is a circular motion that indicates that the particle now moving in the center of the circle may not be the actual size of a circle but we can locally assume that it is a circular path then it is pointing towards it. S is the center of that circular path and if it is the tangent direction then this direction is perpendicular to the tangent. The first is proportional to the square of motion

so whatever the speed of the particle at this point it will be proportional to the square of motion and locally if the particle moves in a circle of radius r then this means if we assume at the moment it is a local circle and if the radius of that circle is r then the normal element of acceleration Accelerate it is given by v square over r and r is called the radius of curvature of the path

so this is something we should always keep in mind that when a particle travels in a curved path there will be two elements if it is an element that changes the rate of motion of the particle Cause or other element even if the particle is moving at a constant speed because there is an element of acceleration in the curved nature of the path that is perpendicular to the path,

so now looking at these things what can we do Hall we can look at the motion of a particle uniform circular motion

so we look at a particle which is following a circular path and the word uniform means that it is moving at a constant speed

so if we plot it here it is a circle if we have a The particle is moving along the circle or the circumference of the circle with a constant motion then we say it is in uniform circular motion,

so now we say the particle is moving at a constant speed which is given by v if the particle p is in this position then we see if Radius r then if we see acceleration at point p then the motion of the particle is constant but this does not mean that acceleration zero point p will point towards the center of acceleration r and it will be given a direction towards the center of circle by r by v square

so here is the direction of acceleration Indicates that its dimension is the square of motion divided by r. Now there are some terms that we define with uniform circular motion

so the first thing we see is that even if the motion is constant at a uniform circular motion Acceleration is not zero. Now we say the particle goes from p to p prime and the center angle of the circle is given by the delta theta. Is and if delta t particles take time to travel from p to p prime then we define a quantity called angular velocity. As we have seen it is given by delta theta by delta t. Now if we try to find the motion of the particle, the speed will be given by the distance pp prime divided by delta t Is equal to the arc pp prime divided by delta tee and it will be nothing but delta theta on delta tee

so we can see that the velocity r times is given by omega and if we If we look at the acceleration of the particle then it is given by v square over r

so it is equal to the omega square of r square and it is equal to r omega square. We have seen that pointing to the center of the particle. Now there are a few more terms that we define. One is the time taken for a revolution. It is called duration and the symbol used for it is capital. will be equal to t and

the symbol used for the frequency is the Greek letter nu. Now if we look at omega omega is nothing but angular displacement divided by time and if we want to express omega in terms of period t then the angular displacement for a revolution is two pi radians and time Is t

so omega will be equal to two pi on t and

so we have omega equal to 2 pi on t which we can also write as 2 pi mu because nu is equal to 1 Over t and if we now see that velocity r is equal to omega in terms of omega velocity then it can be written since 2 pi r nu and acceleration which r is equal to omega square 4 pi square nu is equal to square r

so we have velocity and velocity values in terms of frequency and duration Now we can get this period validity to be constant if we have to give uniform circular motion zoo see briefly what happens if circular motion is not uniform means the motion of the particle changes

so now if the motion of the particle changes then it also means that omega will not be constant and we have angular acceleration Can determine a quantity called equal to. The rate of omega change over time and what I will mention here at the moment I will not prove it but we will show that even ordinary circular motion velocities are given by r omega by et direction

so it will be instantaneous velocity since omega is not constant and acceleration is r multiplied by d Omega will be given by dt along et direction and we will have plus r omega square along n direction and we can write this along r times alpha et plus r omega square here

so when a particle travels in a circular path its acceleration has two elements of acceleration An element that is centered towards the same circle as ar is equal to r speed omega square and has a tangent element of acceleration which is equal to r times alpha and velocity still continues as omega regardless of alpha value

so we have seen circular motion now let's take a A particle that travels with a constant acceleration which means that the acceleration is equal to the acceleration and the direction is equal to the constant and we say that the velocity of a particle passing through the constant acceleration is equal to zero at t 0 and This is the velocity of v at t

so what we want to know is what will be the v velocity of this particle at t and if the particle is in a position r is equal to zero and t is zero then what will be its position at a time t if it continues with constant acceleration

so The first thing we do is we know that the acceleration is constant

so if we write this acceleration v minus v 0 divided by time t which is t minus 0

so it is equal to v minus v 0 upo nt and what we get is the velocity of v is equal to v Will be v 0 plus a times t and if we write it as a component, we get our simple equations vx is equal to zero x plus ax times t and vy is equal to v. 0 y plus ayt where the acceleration a is equal to axis plus ayj's velocity v is equal to vxi plus vyj and the initial velocity v is equal to zero v is zero xi plus v is zero yj

so this is the first expression we get and now we do position vectors

so we have vectors at t r Find the position that the position vector r is equal to r zero and t is zero

so to do this now we see that the average velocity between time t and time 0 will be given v 0 plus v divided by two and

so displacement which r minus r is zero. Equal to it will be given by the average velocity t it works because the acceleration is constant

so we can write it as equal v 0 plus v 0 plus 80 by 2 times t

so what we get by putting all this is r minus r zero is equal to v zero t Plus half times t square and it gives us r equal r zero plus v zero t plus half

kt square and again we can write it separately along the x and y directions
so we get it as x x x plus v 0 xt plus half equal to axt square And we write
the component y which will give us y is equal to y 0 plus v 0 yt plus half ayt
square. Notice that these two equations seem to run along the particle x
direction and y direction in an independent way and we can consider the motion
separately in an independent way with acceleration ax and ay and but if we want
to find the particle path equation we have to do The relation of y to our x
which gives way to our particle and all we have to do is to remove the time
between these two equations and this will give us x as a function of y or y x's
now we have uniform acceleration or Under constant acceleration we see a
special case of ah motion n and what is called projection motion
so the projection motion is the motion of a particle traveling under the
influence of gravity
so it is a special phenomenon of motion that we see constant acceleration
motion that acceleration does not change due to gravity So what we are
considering here is what would be an example of this kind of motion: the speed
of a cricket ball or the speed after a ball is hit by a bat or the speed of a
bullet after it is delivered from the hand of a ball. What is the difference
between this projected motion and the ah and a body that is thrown just
vertically? Tell me I have a ball in my hand or I take this pen I throw it
vertically now it is a special case of projected motion but when I project
Speaking of motion, it can also be a case where when I throw the body, it also
has a horizontal element of velocity nt
so what we consider under projection motion is not only a body that moves
vertically but it can also move horizontally because initially the body has a
velocity. There are x elements
so let's try to figure out what we have that can be projected. t time velocity
has an x component equal to zero and then we see because it has some velocity
component t is equal to zero and because let it be defined though we define the
upward direction as plus y and x is the horizontal direction
so first write the acceleration vector as the acceleration vector 0 i minus gj
equals no acceleration on the x side of y and the acceleration is equal to
minus g and we take t is equal to 0 is the starting time and let the particle
stay at 0 which means t is equal to t 0 at the time of origin and its initial
The velocity is given by v 0 which means we can say that this is the origin.
The particle is in this position and has velocity v 0 which we can express as
horizontal as velocity v 0 and angle theta 0 which makes the initial velocity
of velocity
so here we can see that x element of initial velocity will be given by v 0 cos
theta 0 And the y component of the initial velocity will be given by v 0 sine
theta 0
so if we know the initial component of the initial velocity vector of velocity
it means we know both components of velocity and then what we want to determine
is to find the coordinates of the projected or p particle next time Let us now
briefly look at the equations. Before we get into the equations, let us look at
the physical motion of the projections. The particle is expressed when t x is
equal to its x velocity component v 0 because the y component of the velocity of
theta 0 is the sign theta 0 and the acceleration it is experiencing is only in
the minus y aspect of the acceleration. The velocity of x will not change
across which will be equal to v 0 because theta 0 If we look at the y component
of velocity initially it is v 0 sin theta 0 there is a negative acceleration
subtraction
so this element will decrease and it will move to 0 and then We can see that
the acceleration of minus G is felt across the particle

so once the vertical velocity reaches zero the particle will start moving downwards which is in the direction. Subtract y and it will continue to increase the speed of the particle because it started from zero and it will eventually touch the ground. This distance from the origin is called the range and we will find out how to determine the range in terms of these given parameters. The starting velocity v 0 is equal to v 0 cos theta 0 i plus v 0 sin theta 0 j and the starting point is the origin which is 0 0 which means x 0 is 0 and y is equal to 0 and 0. So now if we look at the x element is equal to x because the velocity x direction is constant in it

so next time the x element is given by v 0 t

so it will be equal to v 0 cos theta 0 times t

so that gives the value of x coordinate of our projectile that y is given. Will be v 0 sin theta 0 t minus half gt The square minus comes because the acceleration is minus g

so we have and also if we write the velocity vx next time then it is equal to v 0 cos theta 0 and vy is equal to v 0 sin theta 0 minus gt These are like a one-dimensional equation of the velocities of x and y written separately and from this we understand that the direction of motion of the y particle is equal to the y direction of motion of a particle which is thrown vertically upwards at the initial speed of v 0 sine theta 0 which is the primary of this particle From the vertical motion that I am trying to do, we know that if a particle is thrown vertically upwards at some speed and if it is thrown as a projectile at theta angle 0

so that the two vertical motions are the same, both the particles will return to the ground. So if here at the same time now If we want to find the path of the projectile then we will use these two equations and we will omit the value of t as the function of x will end the time of getting y

so this is the easiest way when we continue it .

 To do this we have to go into this x equation

so we have xt equal to x divided by v 0 cos theta 0 and we put this value of t in the y equation

so we get y equal to v 0 sin theta 0 t which is x by v 0 because theta 0 minus half a gram t square

so x square uare on v 0 square cos square theta 0

so it gives us when we simplify it it is equal to the theatrical tangent 0 times x minus half g divided by v zero square cos square theta zero Multiply x square and it is equal to x plus bx square where the tentacle of a theta is equal to 0 and b is equal to subtract half g by v 0 square cos theta cos square theta 0 and it is a parabola equation which means a particle which is thrown as a projection The path taken by is a parabola. Now let us first calculate some amount. If we want to find the time to reach the maximum height of the projection, all we have to do to find this time is we keep vy. Is equal to 0 and we have vy is equal to vy   zero subtraction

so it would be nice to get back to you.

 So it is v 0 sin theta 0 times we put the value of t

so that v 0 sine theta 0 by g minus half g times t square

so t square is v zero square sine theta over zero square and

so projected maximum height which Achieving equals v zero square sign square theta zero on to g Now to find the range now we first work with the time for flight which means we first find the time now that time is equal to zero

so we put it in the equation

so that y is equal to zero So we get zero v is equal to zero sin theta zero t minus half gt square

so we get this ah this time which is equal to 2 v 0 sin theta 0 on g while

flying and this is what we already said when we worked with time r max The height will be twice its and the range will be x coordinate for v 0 cos th eta will be equal to 0 times tf

so it will be equal to v 0 cos theta 0 times 2 v 0 sin theta 0 g and it will be equal to two theta zeros v 0 square s on g

so we see the range formula should now be an uh Ah I want to emphasize that the formulas we have derived we are guessing when we talk about the range we started from the origin and we want to find the position when the particle returns to the same level and for this we have created a formula for the range but We can have different initial conditions. For example you can have an event where a particle is thrown from a certain height and then you want to find the x distance where it hits the ground. Starting from x 0 y 0 which is not equal to 0

so whe n we have this situation we have to go back to our original equation we need to see that x direction acceleration is y direction acceleration is minus g and we have x not x 0 and y 0 Work with not being I do. 0 0 But the value you start with and then you can create different x and y coordinates as time functions

so these factors should be considered now when we look at the range formula as we did it. In the case then what we see is that the law of range gives us a given velocity for a given velocity when we want to keep a maximum range it will happen when the theta 0 or the sign 2 theta 0 will be 1 which means the maximum range for a given v 0 theta occurs 0 Is equal to 45 degrees

so if you want to go some farthest distance the angle at which it should be observed its speed should be 45 degrees. The effect of gravitational force

so here we have neglected any other force which will work on the body without virtually gravity. The forces are called pulling and lifting force on the body and if these forces work then the projectile motion will not be the type of motion that this body will show and this is what we see when we observe the movement of a feather. T does not follow a parabolic path and this is because these other forces are also significant and they cause some other acceleration on the feathers. Body where the projected motion is what we say when we talk about it we assume that gravity is the only force that acts on the body

so with this we end the discussion of projected motion and kinetic dynamics in a planar. Path We will look at some examples in the next class and then we will move on to kinetics which is the cause of kinetic motion. When we study we only study kinetic motion. We do not study what is the cause of kinetic motion The dynamics will follow you