We will continue our discussion of vector activity in the last class. We saw how a vector can be solved with its Cartesian elements and we used ij and k as symbols of unit vectors where I was a unit vector along the x axis j. A single vector along the y axis and k is a single vector along the z axis and what we have shown is how a vector a can be written as axis and ayj plus azk. Similarly if we have a second vector b then this can be solved along vector b Components so that vector b can be written as bxi plus byj plus bzk One advantage of resolving vectors along the direction of the Cartesian axis is that once we solve the vector with these directions the addition of vectors can be done very easily if we express the vectors. a and b this way and we want to find their sum

so if we want to find a sum b then if we can do it easily add bx times i plus ay add times j plus az plus bz times k and simple if we add up If e vector is multiplied by a scalar or something is subtracted by a scalar then we can follow the same way

so for example 2 times subtraction three times b. The three would be equal to times bxi plus byj plus bzk

so it would be two cells minus 3 bx times i plus 2 ay minus 3 times 3 plus j plus 2 az minus three bz times k

so ah

so to help us solve a vector with elements along the Cartesian axis We will also look at the products of vectors to simplify the next few activities .
We have seen the addition of vectors, we have seen the subtraction of vectors, and we have also seen how we deal with the next when a scalar is multiplied by a vector .

 Now we are talking about the product of vectors. What we see here is that the sum of two vectors is a vector but we cannot say the same thing for a product and in fact you can see that at this level we can say that there are two ways to define a product. Now let's define different types of products

so we define two different types of vector products and what we will see here is that both these products satisfy the distribution rule of multiplication which is the product of a and b plus c

so we multiply by b plus c it a and c will be equal to the product of a and b plus and

so let us first define the first type of product as the first type of vector product which we define as the product of two vectors is called a scalar product or because we use a symbol for it is also called dot product now How do we define it

so we have two vectors a vector a and a vector b and we say that when we put two vectors together with the tail the angles between them are theta and this angle is the smallest factor between the two angles. The angle between a and b One can also see the angle of refraction formed between a and b but we see the smallest of the angles

so this angle will be between 0 and 180 d Greece

so we now see that the scalar product of vector a and vector b is defined by it
 The first is the dimension of vector a, the second is the dimension of vector b and the third is the cosine of angles between a and b which is the theatrical cosine and

so what we write is that we mark it as vector a and we have a point with b Let's use dotted symbols. This is equal to the dimension of a factor B is equal to the cosine. Theta where theta as I explained the angles between a and b

so this product because they are all scalar this product is a scalar
so dot product is always a scalar

so now we see some features of dot product

so the first feature we see is that a scalar product is variable which This means that a point B with a point B is a point B with a point and this is clear from the definition of its product because a point with b will be equal to the dimension of a. Of cosine theta,

so these will be ah these will look exactly like this dot product now when a point B is a scalar

so the second feature we see when a point B is a scalar it can be positive or negative

so it can have a sign and it will depend The angle of a will always be positive on the angle b The dimension of b will always be positive

so the sign will depend on the angle theta if the angle theta is between zero and pi then a point B will be positive and if theta pi by 2 Bong pi contains but a point B would be negative

so now we look at a dot product. There are a few more things that we see in a vector with a dot a

so that it would be nothing more than a multiplication of cosine multiplied by zero degrees. This will be nothing but the square point product of the dimension of a square or the dimension of the vector. Let's see the second thing with the vector itself. If a d is equal to ot b 0 then what we see is that the dimension of a must be 0 or the dimension of b must be 0 but these If we can say in trivial cases but if it is not then the theatrical cosine must be equal to 0. They are not a 0 and sometimes it is used. Prove that 2 vectors are mutually perpendicular. If you have to prove this, you take the dot product and if you get the point multiplied as 0, you can show that the perpendicular of ab and we sometimes A special word We use what we call vectors. A is the orthogonal of b instead of perpendicular,

so this is a term that we now use. The third thing if we look at the scalar product of two unit vectors, the angle between them will be cosine. Is the distributive property that we talked about that was taken care of if we look at dotted with b plus c then it will be vector a dotted with b plus vector a dotted c now let's see what this dot product looks like in terms of unit vector I look at the unit vector ijk along the axis

so when we look at it clearly if I see i dotted with i then it will be 1 in the same way j will be dotted with 1 and k dotted with k will also be 1 but if I look with j i dotted is equal to the cosine of the angle between i and j so it will be 0 dotted with ik 0 and j dotted with k will be equal to 0. So now it can be commonly used to extend and write a dot product

so for example if we have a vector equal to a then let's write this if we have a vector equal to a equal to axi plus ayj plus azk and vector b equal to bxi plus byj plus bzk then if we want to write the dot product then we will write a dotted with b it and

so we will use the distribution law to extend these

so that it will be equal to bxi plus byj plus bzk with axi plus ayj plus azk dotted and

so now we have this Let's expand each of the terms again. The first word gives us ax bx bar i dot i plus you get axbyi dot j plus axbzi dot k and then we can expand it. Now we will have a total of 9 terms in these 9 terms. It will be 0 in the same way i will be d. 0 This second term can be given to me to expand completely. Dotted with j plus azbzk dotted k and us Continuing these things with ik dot will be dotted with 0 k dot j will be 0 j dot k will be 0 and each of these i dot ij dot j and k dot k will be 1

so finally what we get is in terms of a dot b elements equal to axbx plus ayby plus azbz

so we can now work this way if we look at the triangle formula of cosine
so that we can easily sh ow using this dot product if we have a vector a here a
vector b the angle between them is theta if i Now let's plot this vector this
vector is nothing but a minus b and let's call it c because when I add b to the
vector I get the third side of the triangle which gives me one
so this vector which I call c the third vector is a minus b
so Obviously we can write vector c is equal to vector a minus b and if I write
it in terms of its dimension then the measure of c is equal to the dimension of
a minus b
so what we will do now is take this first expression let's take a dot product
of c on both sides Or
so we will write the point product of c as a subtraction equal to c is b Dotted
with c again which is equal to a minus b
so we write it in this form and we have what we have here
so if we extend it then we get c dotted with c plus a plus b dotted b twice
vector a Dotted b where we used commutative and distributive properties
so now if we write th c can be dotted with c it is the dimension of the square
of c equal to the dimension of a square and b is twice the dimension of the
square of b and the b is the cosine of the angle between a and b Theta and
dimension are nothing but these lengths of the arm
so we get c squared is equal to a square plus b squared minus two ab cosine
theta which is the cosine formula for the triangle and what we understand is
the angle cosine between ah if there are two vectors and if they If an angle
is theta then the cosine of the angle between the a and b vectors can also be
written as dotted by dividing b by the magnitude of b
so if we have to find the angle between the two vectors then we have dot c of
two vectors. We can take the product and divide it by the dimensions which
will give the cosine of the angle between our vectors which we use the dot
product in many situations and a quantity where you will see the dot product.
Willpower When we talk about mechanics and when we talk about working due to a
force or force acting on a point, when we talk about these quantities acting on
energy we see that these quantities are nothing but two vector dot products.
For example, the work done will be defined as the force product of a point with
the force and displacement of the point at which the force is being applied and
similarly the force will be defined as the point product of the point at which
the force is being applied and the point of velocity. Is applied
so we will use the dot product and the other amount involved in the dot product
will come in the course. Now suppose we have a vector a given by oxy plus ayj
plus azk and we want to find the unit vector which we represent as e. Now the
unit vector we know will be given a vector with a hat along sub a vector which
is equal to 1 dimension a
so this single vector e will be equal to sub a vector a divided by the
dimensions of vector a and to find the vector a wha Dimension t we will do as
we have seen we will take with ourselves the dot product of a to give us the
dimension of a square and the point product of a will give us ax square plus ay
square plus az square
so we can easily say that the dimension of a will be x square and square root
of ay square plus az square
so a single vector along a will be the axis plus ayj plus azk and it will be
divided by the dimensions of a
so we divide it by its square root ax square plus ay plus az square
so any vector Suppose we are given a vector a and we need to find the element
of a along a direction eb which means that where some direction is given eb is
a single vector along that direction or we can say we have a b along a To find

its components So this component is dotted with eb by vector. It gives us a component of b along with a. You can have a lot of trouble finding the element of a which is perpendicular to eb

so we have to find the component of eb perpendicular

so The first thing we will do is we will take a dotted vector with eb. It will give us a component along eb. Now this is a scalar. So what we will do is we will take a dotted vector with eb. And now if we want to find the perpendicular element of eb, vector a minus a dotted eb with vector eb it will give us an element that is perpendicular to eb

so it may need some problem but one thing we need to understand is that if we Speaking of the material of a vector along a certain direction but we can get it by taking it we get the dot product of the vector with a single vector along that direction. Now if I move the material of the same vector along the second direction then We can do this with a unit vector with the second product of two multiples. Direction Now the sum of these two elements ents will be equal to the original vector only if the directions of the components are perpendicular if they are not perpendicular then we will not get the original vector which is actually 0 to 2 times the original vector

so Occurs when the elements are perpendicular to each other. Now look at the first type of product. We see the second type or the second type of product which is called vector product of two vectors. This product is now also called cross product. The vector product of two vectors The amount of a vector is not a scalar and in the advanced course you also see that it is a vector in a rather limited sense but for our purposes we will take it in such a way that the vector product of two vectors is a vector now how do we write it ih? ave two vectors a and b before us were the same vectors a and b vectors

so we define a product that we now use the x sign or cross b which we say is defined as a vector which we first point to The vector direction is normal in a plane consisting of vector a and vector b   which means that the two vectors a and b above it always make a plane a cross b a vector which is perpendicular to this plane. The aspect will be explained in a moment

so the dimension of a cross b is equal to the multiplication of the angle b between the angles between a and b is equal to the dimension of the dimension of b. The dimension is equal to the dimension and I call it a vector c

so now what can we say vector c can be written as a single vector along c times ma multiplied by a multiplication of sin theta we still have to explain what is the aspect of c that will give us ec The side of c is the thread of the right hand Is determined by the law of and is a specific rule that we use

so what we do is vector a we have b vector

so we rotate the vector a to b by the smallest angle between them again we see the smallest angle between a and b another The angle will always be a refraction angle

so we see with the smallest angle between a and b we can rotate a to b. Take it and the screw has a right hand thread then the direction in which the screw will move will point us in the direction of a cross b

so we will rotate a b in the direction of b and the right hand screw will move in the direction that points in the direction of a cross b or c Now there is another way we can see this is what we call right hand thumb rule and which is easily seen by everyone

so all we do is we place two vectors this vector and this vector we replace the two vectors together with the tail and Lock now We have to run this exercise with our right hand while we are doing this

so if most of you are doing this if you are right handed then if you are doing this exercise then drop your pen if you are holding your pen then maybe you do

it with your left hand This exercise will have to be done with your right hand. You take the right hand and all you have to do is bend the fingers of your right hand in the same direction when one turns. Through small angles towards b

so in this case a and b are like this we rotate the right hand from a to b and the thumb side gives me the direction of a cross b

so here ah

so we put two vectors together with the tail and curl the fingers of the right han. D to a to b and when we do that the direction of the thumb points to a cross b

so now you notice if i am making a cross b it means I am turning from a to b my thumb inwards or downwards Pointing you will now see if I cross b it means I put my fingers on b turning them to a now you will notice that the thumb is pointing upwards

so b cross and a cross b they point in opposite direction a cross b b The point on the opposite side of the cross a which means we can clearly say a b is equal to b is equal to the subtraction of the cross a and it also tells us that this product is not variable dot product was variable but it is not the second thing If we have a cross b plus c Looking at this but the distributive property is still working it is equal to a cross b plus a cross c then if we see a cross then it will be equal to the dimension of a multiplied by the dimension multiplied by the sine of the angle between an an da now because a and a The angle between the same vector sine a and a of this angle will be zero

so a cross a will always be zero not only if we have two parallel vectors a and b but also if it has one vector a and another vector b which is parallel to a You will still see that in this case a cross b will be equal to zero because the angle between these two vectors is zero now the Cartesian axis which we are drawing which i have shown you which we used in the last discourse also if we call it as x axis it is y axis it as z axis But you can see that they are always in the shape of the right hand that if we see that if we can rotate from there x to y then the third axis z axis will point towards the thumb

so when we draw these in Cartesian axis we draw in 3d They always said I say draw like a hand and one of the consequences you will see is now if we draw a single vector ijk then obviously i cross ij cross j and kc ross k they will all be zero but if we look at the products in the middle of the cross direction it means we have i cross j If I look at it it will be equal to k and then if I look at j cross k and for this I look at i if it goes from x to y from y to the same way if I go from y to z then if I wrinkle my fingers So I 'm pointing upwards which is x direction and if I go from z to xi am plus y direction

so we have j cross k is equal to i and k cross i is equal to j and if you notice these then this thing Makes a cycle that if we put ijk in one order then i cross j is equal to kj cross k is equal to i and k cross i is equal to j means if we follow then we follow this ijk in a circular order we get the third thing Would be positive but if we don't go in circles then we are going to reverse . Direction and obviously the subtraction of i cross j is equal to subtraction k and the subtraction is equal to i cross j and j cross i

so we have j cross i equal subtraction key equal to sk equal subtraction j and k cross j equal subtraction i

so when we do this its I mean we're anticyclic

so this is a way of remembering them. You can do it this way. So we let this cyclical sequence look positive and anticyclic means we go in the opposite sense. The anticyclic sequence is negative. We can use the rules of full expansion and evolution if expressed in terms. The cross product of these

vectors

so we can write a cross b ax i plus ayj plus azk is crossed with bxi plus byj plus bzk and it can then expand

so it   will be equal to aybz minus azbyi plus azbx minus axbz times j plus xby minus aybx times k and follows these rules that i cross i is 0 i cross j is k etc. Another way of looking at what we have just seen is through the concept of determinants which if you have seen in your math course then if we x we have a cross b as determi We can write Nant of ijk axayaz. This is the first of the cross products. We write the second as the third row. So now it can be explained. Let's look at some applications of cross products. Let us see that a is formed by a cross b then the dimension of a cross b is given by the dimension b of the angle b between the angles a and b is given by a dimension of the sine Now if you look at this parallelogram this is also the mother of a cross b equal to the area of this parallelogram Tra gives us the area of the parallelogram formed by the vectors of fields a and b

so it is equal to the area of the parallelograms of the sides of magnitude a and b. We complete this parallelogram and we complete it with c. We see this figure which is ah parallel pipette whose arm is ab and c

so then the volume of the parallel pipe is given as we say it is v then equal to a cross b dotted with vc and since these If we move three sides of a parallel pipeline in a cyclic order we get the same thing

so it will be equal to a cross b dot c

so it will be equal to b cross c dotted a and it will be equal to b with c cross a dotted

so we We can use dot product and cos product cross product to find the volume of parallel pipette then in mechanics we can see the concept of torque or the movement of a ball about a point

so suppose we have a point o and a force f is working

so there is mango What we do is when we draw a vector from o we draw a vector from o to the verb line of f ball and let's say I call this vector r then we define the moment of f ball around o it is defined as cross prod between uct of r and f Sometimes we also call it torque

so the moment of the ball will give us torque and cross product will be used there if we also find the use of cross product when we talk. A point chart When we   talk about the speed of a point charge with bv velocity in a magnetic field the force of this charge is towards the v cross b

so this again we use the cross product to show it if we have to show that two vectors are equal If we show that the cross product is equal to zero then two vectors will be parallel to each other. Now let us take a small example of these vector activities. Two i plus yj Now we need to find x and y

so that b and c are perpendicular to vector d equal to fi fi plus six j and the second part states that for these values show x and y vector b parallel to vector c All we have to do for the first part is we have been given a vector d and we have been given a vector b where there is an unknown where we have to find x and y such as b is perpendicular to d

so perpendicular to a vector bd for part It means Now d is 5 phi plus 6 j and vector b is xi plus three j

so since bd is perpendicular, it means b dot d is equal to zero

so it will give us five x plus eighteen equal to zero

so x will be minus eighteen minus Divide by five and similarly we have dotted with c d must be zero because cd is given perpendicular and c is given as 2 i plus yj

so c dot will give us when we do c dot

so give me 2 i plus 5 phi So that gives us 10 and when we take y plus 6 then

10 plus 6 is equal to 0
so y is equal to minus 10 by 6 or it is equal to minus 5 by three
so what we have now is if we have vector b equal to these values Is
subtraction eighteen by five fi plus three j and vector c is equal to two i
minus five by three j now to show that b and c are parallel
so b to show b is parallel to c we take the cross product well there is a lot
to do One way is one way is cross product
so let's see this first
so we take b cross c
so that this determinant ijk minus eighteen by five three zero two Subtraction
is equal to three zeros by five and it gives us k times minus five times by
eighteen minus five three minus three times by two
so it will give us basically equal to zero k
so since it is zero b and c is another way to do it parallel We can write unit
vectors along b. We can write unit vectors along c and if they are either
parallel or parallel then we get the same thing
so we can just divide b by the dimensions of b and c and if we have the same
unit vector of each other or Negative pie but we can also show that parallel
to bc
so it's another way now it's just
so suppo he has given us again let's take a small example again, let's say we
have been given a vector which is equal to phi phi plus 10 j is now a vector b
component Find where b is equal to three i plus four j and the second part.
We say find the element perpendicular to vector b
so here since we have to find the element of a along with b
so to find the first for a part I will just find this first eb and then we take
the vector dotted with eb We explained earlier then ah it will be the element
of b along a and all we have to do to find the second part is we will take the
vector a minus a point eb and with a single vector it will be a perpendicular
b
so that one can find it These are simple numbers and you can find these answers
 ah and another thing if you check the answers you will find two vectors which
if you take a dot product of those vectors then you should let them be 0 because
they are mutually perpendicular
so we had A little bit of kinetics because we started to see vectors in the
next class. We would see planar motion. We would see motion in a plane and we
would find the expression for velocity and acceleration. We say projection
motion where we have a body in the action of gravity you