

In today's class we will continue our discussion on motion in a straight line, we will look at some examples of what we covered in the last class then we will look at the concept of relative velocity and some more examples

so what we will do in the last class Is running and has a uniform acceleration a and if it starts with a velocity of v_0 then the velocity at our end can be given in t by the expression $v = v_0 + at$ where v is the distance covered at this distance is the velocity at the time we write by $x - x_0$ it can be given by $v_0 + at$ will be given in half a square and if we know the two velocities the initial velocity and the final velocity and if we know the acceleration and distance but the relation between these goes by $v^2 - v_0^2 = 2a(x - x_0)$ a few more relations can be formed v_0 can be given as $v_0 = \frac{v^2 - v_0^2}{2a(x - x_0)}$ and

so the distance traveled will be half times $v_0 + v$ times t and similarly if we know the distance that we know the final velocity we do not know the initial velocity then the relation takes shape $x - x_0 = vt - \frac{1}{2}at^2$ In all these formulas we now assume that the acceleration is now a half you should realize that the body is going through a negative acceleration which we call retardation and then you have to substitute a by subtracting r from r where r is the writ. The ordination is to calculate that sign as well and if somewhere you have a minus sign and it lags behind then its net will be a plus sign so you have to take care of these kinds of things. Another thing we have to be aware of is that these formulas are valid only if acceleration is constant if the acceleration is not constant we can not use these formulas and also what you notice in each of these formulas for example if we look at this formula we have two velocity acceleration and time missing in this formula so if we look at our case then we have The final velocity is the initial velocity. Velocity Acceleration Time and Displacement or Distance is $x - x_0$ These are our five variables and in each of these formulas you will see that these four variables come from a non- variable

so what is given and depending on what gives you a suitable formula The first three formulas are usually enough to use and the other two formulas fall directly from the first three formulas and we use these and sometimes $x - x_0 = v_0t + \frac{1}{2}at^2$. You can see the use of s symbol for displacement and so people remember that these formulas are equal to $s = v_0t + \frac{1}{2}at^2$

Now let us give an example. Standing in front of a high window, it takes 0.2 seconds for the ball to fall from the top of the window and then it touches the ground one second later and we need to find the height of the building and given that we can estimate the acceleration value of 10 meters per second due to gravity. Equals square, because we discussed in the last class that these problems where a body is falling under the influence of gravity freely which means that the acceleration it observes vertically is equal to g towards the ground, now let us solve this problem then what we have is This is a building from which the ball is falling, let's choose the direction of x downwards

so the acceleration is equal to g which will be 10 meters per second square now what we have now Given that the ball starts to fall where the ball starts to fall Let this position be zero Let the position above the window be indicated by position 2 below the b_1 le t window and the ground position by 3. So now what we have been given is 1 to 2 This distance is given as 100 centimeters which is equal to 1. Meters and we also know that the duration between these times is 0.2 seconds

so now what we understand is that we know the distance we know the time and we know the acceleration but we don't know the initial velocity or the final velocity

so we use the formula that $x - x_0 = v_0 t + \frac{1}{2} a t^2$ so this is the formula

so now we use this formula from 1 to 2 distances .

So now 1 to 2 $x - x_0$ will be 1 meter starting velocity v_0 which we don't know it will be v_1 The time taken from 1 to 2 is 0.2 and then our plus half times the acceleration is g and the time taken from one to two is Now the zero point is two squares. This g we know it is ten

so the only unknown in this equation is v_1 and when we get the value of one is equal to zero point two v_1 plus five times zero point two square and hence we get the value of v_1 as four meters per second So once we know the value of v_1 . Then we need to find the total height of the building

so all we can do is divide it by we say it's s_0 and let this residual height be s_1

so that s_1 includes the height of the window and from the bottom to the ground so what we can do now is 1 The total time from 3 to the total time we know is 1 plus 0.2 is equal to 1.2 seconds

so we use the same formula again that x we use the total distance which is now equal to s_1 now we know the velocity 4 to 1.2 plus half wins 1.2 square so it When we use this we get s_1 equal to four points eight plus seven points two

so it will be equal to 12 meters

so this is the height from the top of the window to the ground now we need to add s_0 to it

so we go ahead with the calculation now. s_0 is the height or distance from i but the height should be replaced by x above the window from the top of the building .

The ball starts at rest

so we know the velocity of zero we know the velocity in one we know these two velocities but we don't know the time that we knew in other cases we did

so we use the formula that we know

so we know v we know v_0 and we Acceleration is known and

so we can use it to find $x - x_0$ distance. So we use the formula $v^2 - v_0^2 = 2 a (x - x_0)$ is equal to v_0^2 plus two times $x - x_0$ is s_0 now v^2 was given as four v was given four

so v^2 is sixteen it will be equal to two times ten times s_1 plus zero

so here From what we get is s_1 is equal to zero divided by sixteen. Twenty was equal to zero point eight meters

so we get the total height is equal to twelve pl. Our zero point is equal to eight twelve points is equal to eight meters .

This distance could be divided by 2 to 3 because we know the time and velocity at 1 and the distance

so we could use it to find v_2 and go from v_2 to the ground. This remaining distance is added to the window height and is added as zero. And

so there are multiple ways to solve the same problem that you need to see is how you can optimize the variables and try to solve the problem in a minimal number of steps. Now let's look at another problem and what we have given here.

That is, it travels from a train station, from a station b , it starts at rest, passes through the first part of the speed of a constant acceleration a_1 , and then it goes through constant obstruction until it rests at b , and l distance $b - a$ and b and assume that everything is in a straight line

so when we look closely at this problem the common mistakes that students make while doing this problem is the first part of the motion and somewhere they assume that it is the first half of the speed it is half the distance somewhere The problem is not given that a the distance from a to b is half the

acceleration and half the specific part where the obstruction is accelerated and the special part is not given to us when it is obstructed so we have to keep this in mind when we deal with this problem. Let's solve this problem so let's see the solution so what we do here is tell us trains start from a and go to b if we see its speed so it goes from a to b so let's L_1 where it gives the point b_c where it changes from acceleration to retardation so if we look at the velocity drawn here the time curve here if we plot velocity as a function of time then what we get is that the velocity increases when the train goes from a to c and it increases because its uniform acceleration will be a straight line and once it reaches c it the velocity decreases so it has an acceleration of one minus eight so it has an impedance of 2 which means the acceleration is minus 8 and the distance is a and there is somewhere between b where we do not know exactly where it lies so we know that this is the total distance l so now if we can write ac as s_1 and cb as s_2 which we know $s_1 + s_2$ is equal to what we know a The velocity at the point c is zero and the velocity at the point b is zero so let v_c be the velocity at b_c then if we work with what we know then v_c^2 will be equal to $v_a^2 + 2 a_1 s_1$ and we also know that v_b^2 will be equal to $v_c^2 - 2 a_2 s_2$ So what we can get from here is that s_1 is now equal to $\frac{v_c^2 - v_a^2}{2 a_1}$ what we know that is v_a is 0 and we know v_b is 0 so the first equation will give us s_1 equal to $\frac{v_c^2}{2 a_1}$ and the second equation will give us s_2 equal to $\frac{v_c^2}{2 a_2}$ we know this is the total distance. So l is equal to $\frac{v_c^2}{2 a_1} + \frac{v_c^2}{2 a_2} = l$ so what we're doing here is we know l we know a_1 and a_2 we have to get our answer so from here we have $\frac{v_c^2}{2 a_1} + \frac{v_c^2}{2 a_2} = l$ is equal to l so we got v_c in terms of l so it gives us v_c^2 equal to l times $\frac{2 a_1 a_2}{a_1 + a_2}$ so you when simplifying you will find that now we need to find out that the question is not to find VC if the question was to find VC then we got the answer now we have to find out the total time this was the problem so to find the time once I know VC which I know VC is equal to $v_a + a_1 t_1$ one where t_1 is the time taken from a to c is a to time c and we also know from here we can see that v_b is equal to $v_c - a_2 t_2$ so a From Khan again we know v_a is zero v_b is zero so what we have is t_1 what we get from here is t_1 is $\frac{a_1}{v_c}$ and t_2 is equal to $\frac{v_c}{a_2}$ in notice a_2 because we had constraint so we have already subtracted here So now we need to find out the total time so the total time is equal to $t_1 + t_2$ so it will be equal to $v_c \times \frac{1}{a_1} + \frac{1}{a_2}$ and we already know that v_c^2 is equal to v_c is equal to $l \sqrt{\frac{2 a_1 a_2}{a_1 + a_2}}$ Divide one by one into two plus one by two so we can substitute and simplify here and we get our final answer once we simplify we get t equal to $\sqrt{\frac{2 l (a_1 + a_2)}{a_1 a_2}}$ by adding two $l a_1$ one by dividing one by

eight now like this The problem is you have to look at what has been given and what has been asked and act accordingly. Now we can do the same problem. There is a second method to solve this problem and that is we can solve it using graphical method. Now let's solve this problem using a graphical method. Let's try to plot and then see if we can find the amount of interest so the problem is given to us that the train travels and its acceleration is given during its journey so if I look at acceleration and plot it as a function of time then our Given that the train travels for the first part of its journey with an acceleration a_1 and the second part of its journey with the acceleration of minus a_2 because it is given that the obstacle is a_2 so the acceleration vs. time curve looks like this is how we use the graphical method now We use the fact that the area below the acceleration curve gives us a change of velocity and the area under the velocity and time curve gives us displacement and Similarly the slope of velocity and time curve will give us acceleration when the acceleration constant n velocity and time curve will give a straight line along our slope which is equal to acceleration so in this problem when we determine velocity and time curve the train starts at station a it b The point of stop c where it changes is acceleration so it starts with a zero velocity and since the acceleration is constant from a to c it will travel velocity and the time curve will look like a straight line with positive slope and when it goes from c to b it will be Another straight line with a negative slope or so here the slope of this curve will be one and the slope in the second part of the curve is equal to minus a_2 and now let us see the problem we have been given if we want to find this total time from a to b. We say this time t_1 if this time is t_2 then we need to find the sum of this time which is t_1 plus t_2 and what we have been given is the total distance traveled. 1 Now the distance is also given as a field under velocity time curve so if I look at this field now let's say velocity at this vertex is v_c so let's use this information if it is v_c and we drop a perpendicular from here so that v_t is below the curve Area it will be equal to half v_c multiplied by t where t is equal to t_1 plus t_2 total time and it is given to us it is equal to l so this l is given t which we need to find out and now v_c is unknown so here we are Got a relation We want to find the second relation The slope of the line here a_1 Now the slope of the line c will be divided by the velocity v_c v_c minus 0 by t_1 and it is equal to a_1 It is the slope of the first line and the slope of the second line which is v_c will be given as 0 Dividing velocity b by minus v_c by t_2 is equal to subtraction of a_2 so what we get from here is that when we write this we get t_1 is equal to a_1 on v_c and we get t_2 is v_c - Is equal to a_2 on top so what we get from here is t_1 plus t_2 is equal to v_c on a_1 A_2 on b v_c which we can write as v_c one over one plus one a_1 to a_1 and now we use the second relation that we have that half divided by 2 of v_c and l we got earlier so what we get from here Is equal to t so we substitute the value of v_c here so we get t is equal to l over 2 times 1 over 1 plus 1 plus 1 and now we can take the other side t means that t square is equal to l is equal to 2 of l Above 1 plus 1 and 2 and this is the answer we got earlier so the graphical method can be effective if we both understand that the slope of the curve of velocity gives us acceleration during velocity and displacement of the area below the velocity time curve now let us consider the relative velocity Let us focus on and what we understand here is that the position of a

point depends on the reference frame from which it is being measured so the position is a frame dependent quantity because it will vary with the reference frame you are measuring and since the position is a frame. No. The amount of velocity also refers to the velocity and acceleration of a point where the derivative of the position and the velocity vector respectively become the frame dependent quantity and this is what we noticed when we sit in a train we are in the reference frame of the train and this is a common experience. That we see the trees that are there and the telephone poles that they look like. The movement of a point becomes a function of the frame from which you are observing it when we are sitting on the train moving when the train is clearly moving to a person on the ground and the telephone poles and trees are stationary .

The point means that the velocity we measure depends on the reference frame. It cannot be that we cannot speak of any absolute velocity in that sense so let us consider a point p. When we talk about motion with curved lines, curves, etc. when we talk about those things

so here we are considering motion along a point p a straight line

so now we have an observer attached to frame a and an observer which means we have a reference frame a. This observer is considering the movement of point p from where and there is a point p

so the position of p is measured by frame a We mean that x_{ba} is

so x_{ba} is the position of p as observed by the observer

so let it be connected to a frame This is frame a. We also have a frame b and the position of p has been observed by vectors such as frame b. Let us write that x_{pb} is therefore the position of p as connected to frame b Observed by the observer and the position of frame b relative to frame a is indicated by x_{ba}

so x_{ba} is the position of b as observed by the observer attached. All these first points are a notice frame. There are two subscripts. The first one represents the point that is moving or the point which is being observed by both frames. The second one represents the frame from which the measurement is being taken. Is equal to

so we write here this x_{pa} is equal to x_{pb} plus x_{ba} now if we differentiate with time then what we get is $\frac{dx_p}{dt}$ is equal to $\frac{dx_b}{dt}$ plus $\frac{dx_{ba}}{dt}$ and we can write that v_{pa} is equal to v_{pb} plus v_{ba} Where v_{pa} is the velocity of p as observed from frame a and v_{pb} is the velocity of p as observed from frame b and v_{ba} is the velocity of frame b as observed from frame a now what often happens in frame a will be fixed and if it is clear that it is the same frame then a can be omitted because when it is clear that frame a is fixed and a clear frame then we do not write it and then the relation will become v_p with respect v_p equal to v_b of b Plus v_{ba} This means that the velocity of the point p observed from the ground is equal to the velocity of p which is observed from the frame b plus the velocity of b observed from the ground

so this and what we understand from it gives us a relation to the velocity of the p point observed from frame b v_p is equal to the velocity of p minus velocity

so this is why we can write the velocity of p as the velocity of p observed by frame b is equal to p minus velocity of b where nothing is said to mean that they are now being measured relative to the ground if we assume Both frames a and b move with constant velocity which means that the acceleration of a and the acceleration of b are both 0 .

We find that the acceleration of b differs in the expression of velocity and because the acceleration of b will be equal to $\frac{dv_{ba}}{dt}$ minus a_a and since we get 0 from both the acceleration of p relative to a is equal to the acceleration of

b relative to b . That means if two frames are moving with constant velocity and you can see acceleration of a point p which is moving with respect to this frame. If the frames are moving with constant velocity then you will measure the same acceleration by these two frames. And the reason why acceleration and the measurement of this speed become important relative to the frame is that eventually we will use these relations of acceleration we will relate it to force and we will know Newton's second law for those of you who say that force is equal to mass acceleration and there we have to be very careful because when we say that the force mass is equal to the mass acceleration then the acceleration a_c reference acceleration cannot be measured in terms of an arbitrary frame. Another word of caution when we come to acceleration is that when we use this relation $v_{pa} = v_{pb} + v_{ba}$ this relation is valid for low speed and to say low is what we mean. What we want is when v_{pb} and v_{ba} are much less than each of them c where c is the speed of light if these velocities come close to the scale then the speed of light this relationship does not work and we have to use the theory of relativity to make it work because when we when it comes to the speed of light, the theory of relativity tells us that the speed of something cannot be greater than the speed of light. So we have to be careful. Next we will look at some examples of relative motion, so the first example we have is a person walking up to a stagnant escalator so there is an escalator and if a person walks from bottom to top he takes s time t_1 now it is also given that if the escalator is running on a moving escalator and the person is standing there, it reaches from bottom to top in time t_2 . Now we need to find out the time if the person is moving at the speed we specified and the escalator is also moving.

We want to figure this out

so we have the person here 1 time and the escalator itself sorry to go from top to bottom it takes two times to go from bottom to top and now we put if the person moves with his speed on the moving escalator then time T_3 comes out and it is given to us that The length of the escalator

so let's do this, when the escalator is off let's first find the speed of the person

so that we know that the speed of the person is equal to the total length of the escalator. Now the time he travels is divided by t_1 . Now when the person moves to the escalator, the velocity of this V_p becomes the person related to the escalator and

so with a person running on the escalator, the person running on the escalator is equal to 1 l and the escalator. The velocity itself is given as l on t_2 . Because the escalator itself takes a time t_2 . So now when the person moves with the escalator speed of the person relative to the ground, it will be equal to the speed of the person with the escalator and equal to the speed of the escalator relative to the ground. This will be equal to l on t_1 and it will be given as l on 2 and the velocity of the person on the ground will now be given as t_3 on l where t_3 is the final time taken by the moving person is the escalator

so what we get from here l is equal to 1 over t_3 is equal to 1 over t_1 plus 1 over t_2 and we can get t_3 equal to $t_1 t_2$ over $t_1 + t_2$ and we can understand the final answer here is independent of l

so we Let's find the time taken by the person in the running escalator or now let's look at another problem, the two cities a and b are connected by a bus service where a bus is leaving in any direction within minutes. He saw a bus go. Passing it every 18 minutes towards its speed and every six minutes in reverse now it is given that the bus speed v_b constant we need to find v_b and t so the bus speed a_h given that constant we need to find the bus speed we have

in the bus We need to find out the duration given to us the speed of the cyclist

so let's see the speed of the cyclist it is known to us we also know the time T_1 when the buses pass after this time is in the direction of the cyclist and time t_2 is the time when the buses pass the cyclist in opposite direction So t_1 and t_2 are the distances between consecutive bus passes

so let v_b be the speed of the bus

so let us use this problem a method where we will analyze everything d reference frame

so let's see when the cyclist is in a position and a bus passes him Then the bus number one passes him and the cyclist is equal to zero. Now the next bus which is the number two bus one is behind the bus number one. The distance of v_b time capital t at this moment because after all bus time passing is equal to v_c to t_1 . And the distance traveled by bus number two at the same time will be equal to v_b times t plus v_c times t_1 and this total distance is also equal to the distance traveled by bus number two

so let's write this distance traveled by bus two when it is from t When the cyclist crosses it will be equal to zero then the time observed will be equal to v_b times t plus v_c times one and how much time this bus has traveled at this time this bus has traveled for one time is equal to v_b times t one

so we are here We get a relation from. This is a relation and we now analyze the other side. Now if we look at this relation we have two unknown v_b and t v_c is given to t_1

so we have an equation about this but two unknown

so we need one more equation. And for that we look at the buses traveling in the opposite direction

so now let's say here a bus equal to t_0 has just passed the number 3 cyclist and there is a bus number 4 it will be a v_b times distance cyclist in the number three bus behind t This is how it is now What will happen is that after two times the bus number four will pass the cyclist. Now the distance this distance traveled by the cyclist is equal to v_c times t_2 and the distance traveled by bus here is v_b times t_2 and the total distance is v_b times t

so we P_2 is the second relation that gives us v_b times t is equal to v_c times t_2 plus v_b times t_2

so now again if we calculate we have the same unknown v_b and t_2 unknown v_c familiar to us t_2 familiar to us v_b same unknown

so when we find it When we work we have two equations and two unknowns and we can solve it and we get our answer

so there is a way to solve it and you can now omit

so for to solve it for example in this case what we have if you solve this equation The number one gives us $v_b t$ is equal to v_b minus v_c times t_1 and the equation number two gives us $v_b t$ is equal to v_b plus v_c times t_2 . will give the value of v_b because we know v_c t_1 and t_2 and its Then we can now find the same question. When you work with numbers we can figure out the numbers. Please note that your unit should be consistent if you express the time in hours. Time is given to m inutes

so it is better to convert everything if you speak in our terms we should convert minutes to hours When the bus and the cyclist travel in the same direction

so this bus is traveling it is v_b and the cyclist is traveling in the same direction which is given by v_c

so we will keep the way the cyclist saw the speed of the bus as v_b relative to c it will be v_b minus v_c Equal and in this frame we look at it when the cyclist is riding towards it. The cyclist bus travels the distance between

two consecutive buses when it overtakes him
 so the distance to the cyclist's frame is $v_b t$ and if you have any difficulty in trying to understand it you will assume that the cyclist is at rest if the cyclist is at rest v_c is equal to zero and Then when he sees a bus passing him I will say bus number one then when bus number two will pass this bus number two $v_b t$ is far away
 so he will $v_b t$ before the distance this bus has crossed it and the same thing will continue. When the cyclist is running and
 so now the distance traveled in this reference frame is $v_b t$ and time taken is not one
 so we have bus speed as seen by multiplying the cyclist frame by time it will be equal to the distance traveled by the bus shown in the cyclist frame and it will be $v_b t$ The product will be equal to t
 so we have $v_b - v_c$ times t is equal to $v_b t$ and this relationship is the same as we observed earlier in equation number one now let us look at the opposite direction of motion The cyclist is moving like this. The bus is going in the opposite direction. So now the bus speed in the ground frame is equal to $-v_b$ because we are taking it as a positive x direction. So it will be equal to $-v_b$ and the speed of the cyclist as seen from the ground is $+v_c$ from the side
 so with the minus sign it becomes $-v_b - v_c$ and it is equal to $-v_b - v_c$
 so it is the bus speed cyclist now The cyclist observes in the reference frame that the bus minimizes $v_b t$ before it passes him and when it passes the first bus number three it is equal to zero when we start and it is bus number four so the cyclist observes the distance traveled by bus number four before it reaches him will be $-v_b t$ the minus sign comes because the bus is traveling in the opposite direction
 so now that we again use the fact that velocity of the bus with respect to the cyclist this time taken is t_2 and this will be equal to $-v_b t_2$ and velocity of bus as seen from the cyclist we have already worked this out this is $-v_b + v_c$ times t_2 is equal to $-v_b t_2$ and this is the same as relation number two which we have obtained earlier
 so you with when we observe things in a relative frame we can also solve the same problem with this method
 so we have seen this concept of relative motion and with this will end up our lectures on motion in one dimension in the next class we will talk of motion in a plane where we look at position displacement velocity when a body is traveling in a plane but to understand that will also need a crash course on vectors
 so we will start with vectors and then we will talk of motion in a plane you