

Today we will start with dynamics. Kinematics is a branch of mechanics where we study the motion of a point or a particle and describe the motion without describing the cause of motion

so when we study dynamics we do not go into details of what is causing the cause. Speed but we only analyze motion but before we start with dynamics let us look at some basic mathematical concepts you will understand that we need the help of mathematics while studying physics and you will study in detail about the mathematical concepts we will talk about. In mathematics, however, we will introduce some concepts when we need to, so we are talking about the motion of a point, and we will limit our discussion to planar motion, which means that a particle is moving in a plane .

And what we do is we take two mutually perpendiculars and say one of them is  $x$  and the other is  $y$

so we call their two courtesies an axis and they are  $x$  and  $y$  and these axes are mutually perpendicular which means they are at  $90$  degree angles to each other.

The intersection is called the origin  $o$  is represented by any point  $p$  which is described here as  $x$  and  $y$  as the coordinates of these points which we call and what we mean by  $x$  is that if we look at the distance  $x$ , this point is from the origin along the  $x$  axis. And distance is the amount of distance given by  $x$ .

The point is given by  $y$  along the  $y$  axis

so  $x$  comma  $y$  is called the coordinate of the point  $p$

so let's write it the position of a point  $p$  coordinates  $x$  comma  $y$  is now explicitly given as  $p$  point moves it means that it is moving some way then  $x$  And the value of  $y$  will change and this is the analysis of what we will do when we talk about dynamics. Now there is another term that we define. It is point  $p$ .

The segment from the origin to the position of  $p$  which we call position vector is now a directed line segment because what we understand is that  $op$  has two quantities one is the length of  $p$  which is also called its dimension vector and the other thing is the length of  $op$  with the same length. If the direction I travel along a circle I get different points  $p$  whose length is the same but to give the exact direction of the point  $p$  we then mark from  $o$  to  $p$  and this direction gives the direction of our vector

so you can give a vector a magnitude As  $o$  can think of two properties one is magnitude which is the length of a position vector and the other is direction and both together they define a vector and we can uniquely define a position vector with these quantities. And two you can call them a branch of calculus. First we will look at the elements of difference .

Rental calculus and what we are describing here is very limited math. It should not be taken as a class of mathematics. So the first thing we have to understand is what we call the concept of derivative in differential calculus based on all the derivatives. Now suppose we have a line where it changes with  $x$

so we call it equal to the function of  $x$

so we define a curve  $y$  is Equal to the function of  $x$  means we have different values of  $y$  for different values of  $x$  and when we have them When we add, we get this curve which we call a function of  $x$ . Now we see two points  $p$  and  $q$  in this curve

so we have the same curve. This curve has two points  $p$  and  $q$ . Now we say that the coordinates of point  $p$  are  $x$  and  $y$  point  $q$ . The coordinates of the point  $q$  are the coordinates of the point  $p$   $x$   $x$  is located in the  $\Delta x$  direction and it is at a distance  $\Delta y$  away  $y$  from point  $p$

so the coordinates of the point  $q$

so the coordinates of point  $p$  are  $x$  comma  $y$  and the coordinates of point  $q$  will be  $x$  plus  $\Delta x$  The coordinates of  $x$  and  $y$  are  $y$  plus  $\Delta y$

so these are coordinates now if we look at a straight line with  $pq$  then if we

see that we make a right triangle like this then we will continue till p until the range of q along the qx axis and then the vertical point goes up to q and if this angle is theta then we can clearly see that the theoretical tangent will be equal to delta y on delta x

so now what we say is that if the point q approaches the point p then we give the q approach p which means we Here Not taking it closer and closer to point p but it does not go exactly to point p

so we say that it comes close to qp which means we say that delta x comes close to zero and we have delta y which distance along y will also be approx 0 but delta The delta x by y will not be the separation of these two ap in a general case it can go to zero in a certain case but usually delta x by delta x it is a small quantity which will not go to zero and what you will see here is straight line pq it is the tangent to the curve Approaches the point p

so the pq line now moves to the tangent of the curve if we mean the slope of the line and the slope of the line is again something you see in ah coordinate geometry and

so in math course if we point this slope by the line m then we have the slope of the line The slope of the m line goes from delta x to delta x 0 at the boundary of delta x. So now what we do is define a function of y relative to x where y is equal to f of x

so we use the dy symbol by dx and write it as a derivative of f of x relative to x it is x plus The function of delta x subtraction is equal to the value of x

so the value of y with x is delta x which is the delta of x x minus f which divides y by delta x and we take it and we call it as the boundary process because delta x is towards 0. So we write it as the limit going to delta x 0 and it is called the derivative of the function position x and physically it is equal to the slope or it is equal to the slope of the tangent at that point so this is how we define the derivative. If you have two math courses now we have two simple formulas for derivatives that we may need

so I will give you some of these formulas

so let's say we have two functions x has x and v has two functions just like we have x F has two of these then what we have is equal to the sum of u plus v derivatives

so any amount that is defined as sum you can take derivatives of individual functions we can add them to the product if we have a different rule u and v is the product of m and the derivative of en is given by adding dx by u times dv as v multiplied by du dx

so it does not follow the same rule as we gave for addition but not derivative for product. The form is in this form and then we have a derivative rule that the quotient of u is divisible by d for v divided by dx or the derivative of u divided by v is equal to one over v squared by dx minus u multiplied by dv and again Ah these inventions you will now do in detail in your math course. If you have a power if we have the power of x then the power of n then the derivative of x is equal to the power of n with x multiplied by n is given by the power of n minus 1 and if we have a function then u if u of ux So what we have then is the power of n relative to x is a derivative of u times n times the power of n minus 1 times du dx and actually this formula that I did is a general form of a formula that we have called chain rules And what we have in the chain rule is if we have a function that is equal to a function of u and u is equal to u of x

so now if we want to find f here let us know as a function of u we say f is equal to u square and u is equal to 2 x

so a function of u xf is known as a function of u and if we now want to find

the derivative of  $f$  relative to  $x$  now notice here we know that  $f$  is known as a function of  $u$  and  $u$  is equal to  $x$  Known as a function. What we do here is it is given first we subtract  $f$  from  $u$  and then we multiply it by the derivative of  $u$  relative to  $x$  and this is termed as chain rule it often comes out very easily and you should remember The way we use it now is that we have the formula for tangent and cotangent seconds to make the derivative of the  $x$  equal to the cosine of  $x$  equal to  $x$  and the derivative of the  $\cos$  of  $x$  equal to  $x$  equal to minus  $x$ . And using some rules we have product rules so to find the derivative of tangent we can take a sign divided by cosine and make it

so these things can be

so these are some of the differential calculus that may be needed at the moment. But what we will see when we need something else that we need a little bit is some element of what we call integral calculus and here if we have this problem Either if we have a function of  $x$  and what we want to find is the area between the  $f$  and  $x$  axes of  $x$  when  $x$  is equal to  $a$  or  $x$  is equal to  $x$  in  $x$  and  $x$  is equal to  $b$  means  $x$  axis at this point  $x$  at another point  $a$  Equals  $x$  is equal to  $b$  is a function  $fx$  we want to find the area of the shaded part which lies between the  $x$  axis and  $fx$  and on the left we have the boundary  $x$  is equal to  $a$  on the right side it is equal to  $x = b$  if  $x = f$  is a straight line Either way the area will be the area of a rectangle that we can easily find out but how we will do this if it is a common curve of  $x$  and to find this area between  $f$  and  $ab$  and  $x$  axes  $a$  to  $b$  of  $x$  What we do is we divide this area so we have this curve of  $x = f$  it is  $x$  axis is equal to  $x$  is equal to  $x$  is equal to  $b$  we divide this distance by a very small distance from  $a$  to  $b$  Let's say a position between position  $x_i$  Given and each of these intervals has a length of  $\Delta x_i$

so now that means  $x_i$  and The next interval is looking at  $\Delta x_i$

so I write this area of this small part as  $\Delta a_i$

so  $\Delta a_i$  will be equal to this area this will be equal to the height here  $f$  is multiplied by  $f(x_i)$  of what is here

so it is the width of the strip and This is the height

so the product of these two gives me the area of the strip

so the total area is going to be equal to the sum of all of these. The area will be the sum of the capital sigma  $\Delta a_i$

so it will be equal to the sum of  $x_i \Delta x_i$  from one to  $n$  if we want all the sums of the area to match correctly the area means we should make these rectangles smaller

so what we will do is What I mean is that if  $n$  tends to infinity means  $n$

becomes too large then the sum sign that we have is what we call integral and we use  $\int$ . The symbol which is like an extended  $s$  is a symbol of integral

so when it is sigma when  $n$  is inclined towards infinity we call it integral and what we say is the area of  $x$  is integral of  $f$  and  $\Delta x$  we write it  $dx$  and it goes away when  $x$  is equal to  $a$  is equal to  $x$  is equal to  $b$  and at the moment

I mean to write  $x$  is equal to  $a$  and  $x$  is equal to  $b$  but usually we write it as an integral from  $a$  to  $b$   $\int_a^b f(x) dx$  and we call it as integral

so below the curve in an integral of a curve The field is the lower boundary and the upper boundary is the first boundary here it is called the lower boundary of the unification and it is called the upper boundary of the unification

so and such an unbroken boundary that we have when we have area under our boundary is called a definite whole and this definite whole is below the curve

Area of  $x$  then  $\int_a^b f(x) dx$  is equal to the integral of  $fx$  with respect to  $x$

so integration in this sense is the opposite of a division because if  $dg$  is

equal to  $\int_a^b f(x) dx$  then  $f$  will be integral of  $g$  and when we write something we can write if we are from  $a$  to  $b$   $\int_a^b f(x) dx$  up to say integral but it will be equal to  $g$  of  $x$  and in terms of when we put a certain integration here we put the limit then it will be equal to  $g(b) - g(a)$  and this is how we limit if we do justice to combine it and leave it as a function then the integral function is evaluated to an arbitrary constant and if we look again for some formula for the integral function you will find it in any of your books and they will take the full form of  $x$  with the power of  $n$  relative to  $x$   $x^n$  is equal to  $x^{n+1}$  divided by the power of  $n+1$  is a constant in any one of these indeterminate integers where there is no limit. Will be and it does not work for  $n$  subtraction is not equal to one then we have another formula  $\int \frac{1}{x} dx$  means in this case when  $x^n$  is equal to minus 1 it is given by a function called The logarithmic function of  $x$  is natural log. You don't know much about this function at the moment but you will see it later in some of your math courses and when we need it here we will do it this way and again simple because these are indefinite integrals we have an arbitrary constant. Let us add two more things when we see that these integrals of the sign of  $x dx$  are equal to the minus cosine of  $x$  plus  $c$  and the integral of the cosine of  $x dx$  is equal to the sign of  $x$  plus  $c$  and these two functions you get directly from the inverse rule. Can I say that derivative is inverse of integration now these are some of the mathematical preliminary that we saw now come back to mechanics so take a short break that we took but we need all this so we do it now

so we come back to mechanics and physics Let's start with the definition of a particle in our initial state. Let's talk about mechanics which we call particle mechanics. We would call it a point but a finite mass so an object we call it a particle now it could be eh if we look at it from a physical point to find something very small ah size It may not be possible and what we will see is that when we study mechanics we will treat and often we will consider things like balls that can be one. Cricket balls or footballs can also be considered as a particle so when can we consider these things as a particle if the distance moved by these entities by the individual body is much greater than its size and we assume that everything there has the same point of motion So we can consider them as particles

so what we have here is that if we need to estimate the overall path of movement by the body without thinking about the details of different parts of our body, then we can treat it as a particle. Must be much larger than size When I mark two points on a rubber band I see the distance between the two points as I extend it. The point will change

so our rubber band cannot be considered a rigid body where if I see a cricket ball in motion and I now mark two points on the cricket ball the points can move if the ball moves but if I see the distance between any two points of the ball which will always be constant

so we now call it a rigid body. Whenever we talk about mechanics here we are talking about the motion of a point and we sometimes need an idea called a reference frame to observe how a point moves or how it moves. .

A reference frame is called a reference frame. It is very important to understand this concept in a position where the distance between two points is always constant

so we can say for example if I am standing on the ground then ground is a reference frame and if I look at the ground two points between them. Distance will not change

so this is how I define a reference frame and what I do in a reference frame is

I have a device with which I measure length and I have a clock with which I measure time

so my two things with the frame A reference frame would require that we have a device for measuring length and a device and a device for measuring time which would normally be a clock

so now when I fix myself in a reference frame and I want to observe

so now I am in this reference frame I can observe the motion of a point p

so let's say it's ground I have a person here I'm sitting here and I have a point p here As time goes on

so time is equal to  $t_0$  I measure the distance to the point p over time and I keep measuring how p changes over time I have an instrument to measure the length and I have an instrument to measure time So using this I can see that they observe the motion of the p point and it is observed that the motion is relative to the reference frame one. Now the motion of the same point can be observed in the case of the second reference frame

so the reference frame can have multiple reference frames Suppose there is a frame on the ground and suppose a car is moving on the ground,

so we observe the speed of the vehicle relative to the ground,

so I would say I am observing with reference to the reference frame or I can

see the speed of the vehicle relative to the ground now let my reference frame be 2 car which means another There is an observer who is sitting in the car seat

so now the second observer who is sitting in the car seat is not moving at all.

Where a person is on the ground, if

so let us think of this then what we have is suppose a car is running we have a car a car is running on the road a reference frame is on the ground and a car

attached to the reference frame 2 And we have a dot on the back seat of our car now if I observe it from the reference frame now one thing I have to do is

if I want to observe the motion of a dot then all I have to do is refer to the reference frame with i reference frame Will connect a coordinate axis where I

want to measure and suppose I observe I connect the coordinate axis with the reference frame to the ground and I observe the motion of this point p which is in the rear seat a fixed point p which I find is the car moves As the car moves along the straight path, I will observe this point p moving with the car where the second reference frame is attached to the reference frame on top of the car.

And from this reference frame let's now place the axis in the front seat of the car. When I notice the same point p I have to go to the negative side

because I am sitting in front Moving along and what is going to happen in this frame is that point p will show steady where point p is moving away from

reference frame one and we have looked at point p and if I see what is fixed on the ground at point q then point q from reference frame 1 seems to be moving at

all. Not where if I look at this point q from the car then the q point can be seen moving as the car moves. Point q which is fixed with reference frame

running one reference frame two and point p which is fixed with reference frame one running reference frame two. The position is always there. Now let's

pause and think if there is a reference frame that is absolutely static which means now we are talking about the reference frame, can we find a reference ?

ence frame that is not moving at all

so in the example I took earlier we think that if I fixed the coordinate axis to the ground then there is nothing moving in that reference frame which would be good for some real observation but if you actually see the earth itself

Revolving around its own axis

so if we explicitly see that a reference frame fixed to the car is moving with respect to the ground now I take a reference frame which is fixed to the earth

or fixed to the ground in a perfect sense this reference frame is rotating with the earth. What I am saying is that maybe I will fix my reference frame not on the earth but in the center of the sun. If I do that the frame will move the earth but this reference frame will not move but the sun itself revolves around another body.

so is it possible to find a reference frame that is absolutely fixed? I don't know the answer but the question is why do we do that? The idea of the static reference frame and the reason is that next we will talk about Newton's law of motion where we will relate the force to the acceleration and that relation is valid if the acceleration of a particle is measured with a frame which is static. What we also call inertial frames of reference is Newton's law is not valid for some accelerating frames if acceleration is measured and so on but then if we say that if we do not know the existence of perfectly fixed frames then Newton's law can never be valid. But then the problem arises that we can consider motion as negligible. For example, in most of the cases that we calculate for the motion of the body on the earth, if we neglect the rotation of the earth which works as a fairly good approximation then we consider the ground as inertial frame or a fixed frame. If we have to calculate for the motion of the earth then this frame cannot be considered as inert.

so one has to keep in mind this kind of consideration.

so now after observing a reference frame what I am going to do is I start discussing the dynamics of a general discussion of a point. Before I do that I want to give a basic definition of two quantities which is velocity and acceleration. We talked about the speed of a point. Now the velocity of a point will give us an idea of what speed means. Moving point means its distance is changing with time. The idea of how fast this distance is changing is given to us by velocity and I will talk about the exact meaning of what we call velocity and how fast velocity is. The concept that we understand by acceleration is changing over time.

so velocity and acceleration give us the rate of change of distance and the rate of change of velocity respectively.

so now let us see again we are taking flat motion in this case we suppose it is a path running at point  $p$ .

so The point or particle is in  $p$  and this is the origin.

so what we do is we draw up the position vector in time  $t$  we mark it as vector  $r$  as time function in  $t + \Delta t$ .

so it is in time  $t + \Delta t$  particle in  $p'$ . So the position vector is given by  $op'$  this vector is the position vector at  $t + \Delta t$ .

so it is  $r$  at  $t + \Delta t$  it is now  $r$  at  $t$  vector  $pp'$  it is called displacement vector and if I see  $r$  vector  $t$  at  $t + \Delta t$ . By subtracting  $t$  the vector is divided by the  $\Delta t$  in the range that we keep the  $\Delta t$  in the range very small. The  $\Delta t$  moves to  $t \rightarrow 0$ . This is what we call velocity vector at  $t$  which is the basic definition of velocity vector.

so the velocity of point  $p$  at  $t$  is velocity vector  $T$  is equal to the derivative of the position vector which I am discussing. I am giving you a very general definition here. We have not even talked about how to add or subtract vectors which will come later. So we look at the displacement of the vector between these two. Now you can understand one thing and this is again some thought that I want to leave with you. Anywhere again if you see the vector  $pp'$  which will be the same position of the new vector.

so for example if here let me use a red pen.

so suppose there is another coordinate system  $x'$   $y'$  mounted in the same reference frame the original is now different if different directions. I see point  $p$  position vector point  $p'$  position vector they are different.

from  $r$  and  $r \Delta t$  and  $r t$  plus  $\Delta t$  but if I see vector  $pp'$  it The new coordinates are the same and the velocity is given. The  $\Delta t$  is bounded by the  $pp'$  vector. The  $\Delta t$  is going to zero so the velocity vector will be the same. It is independent of the coordinate axis if they are both fixed in the same reference frame. Although the orientation is different, the velocity of point  $v$  will be the same as that of a vector because it depends only on  $pp'$ . The definitions are simple and they will work for all one dimensional two dimensional speeds so  $p$  point acceleration is given as  $p$  point velocity  $t$  plus  $\Delta t$  minus the same point  $p$  velocity so you can call it  $p'$  because we call it the same point so  $p$  And  $pp'$  different position of the same element point so  $\Delta t$  moves to zero at the boundary divided by you and  $\Delta t$ . In terms of our initial definitions we would call it instantaneous acceleration so with these concepts we want to move towards the concept of simplest kinetics which is motion in a straight line so now what we see is a straight line running along a particle so we have some lines like this. There is a particle which we call  $p$ . It moves in different places. Now it does not need to move forward. After a while it can reverse its path. So for example the particle can move like this if it is moving it will show curve but all I can do is I can always align my  $x$  along the axis curve as long as it is in a straight line, then all we have is the particle when it When we move along a straight line we can generalize that it travels along the  $x$  axis or minus  $x$  axis so we will tilt and this is called rectilinear motion and it is the opposite of curved or planar motion if If the particle is then we say that the position in rectilinear motion along a curved path is now the position of space where the particle is at time  $t$ . So when the particle moves, so do we. For example, if this is the path, then let there be a particle equal to  $t = 0$  at the time. There is a distance of 200 meters. This is the point  $p_0$ . The time  $t$  is equal to one second the particle  $p$  is equal to  $t$  for two seconds the particle is in  $q$  and the  $t$  is equal to three seconds the second particle is in  $p$  which means the particle moves from  $o$  to  $t$  from  $pp$  to  $q$  and then returns to  $p$  so if we call then we Let us write what we call as the length of the path. Let us write the path level. The length of the path is the total distance at which the particle moves or the length of the path is equal to one now so the length of the path is the distance. If we try then the length of the path at  $t$  is equal to 1 and the length of the path at  $t$  is equal to 200 meters . One hundred meters away so the length of the path is equal to 400 meters so the length of the path Is the distance of the scalar moved by the particle and it has no direction. This quantity is always positive because when a particle moves it covers a distance and the distance covered is always positive. Net change so suppose if the particle is at  $x_1$  position at  $t_1$  and at  $x_2$  at  $t_2$  then displacement is defined as  $\Delta x / \Delta t$  symbol  $\Delta$  which I have used here. This means that in the change  $x$  the final subtraction is the initial so it will be equal to  $x_2$  minus  $x_1$  divided by  $t_2$  minus  $t_1$ . So this gives us displacement and if  $x_2$  is greater than  $x_1$  in st it means the particle is moving along the positive  $x$  axis and if  $x_2$  is less than  $x_1$  then the particle is moving along the negative  $x$  axis so if the particle moves then displacement is positive Or can be negative so displacement can be positive or negative is positive if particle moves

along the  $x$  axis and it is negative if particle is moving against the  $x$  axis or it is moving as we would say along the minus  $x$  axis it is moving in the minus  $x$  direction then we say this is displacement

so now displacement is a vector quantity but in one dimensional motion we do not need to give its direction in the sense that it is always along  $x$  so therefore in 1d motion direction of displacement is given by the sign if displacement is positive then it is along plus  $x$  axis and if it is negative then it is along minus  $x$  axis

so this is how one write the displacement as a vector and it is clear from here that displacement is not the same as path length  $t$  Here are two different quantities ah

so in in the next class we will continue from here we will talk about how how we look at the graph of  $x$  versus  $t$  what what is the meaning of those graphs and ah we will then talk about how fast particle is moving that means we will introduce the concept of velocity and how fast the velocity is changing that is acceleration for one dimensional motion in the next class thank you you