

We will continue to discuss the dynamics of a particle and especially what we are studying is when a particle moves in a straight line. In the last class we saw the definition of the length and distance of the displacement path. Running in a straight line

so let's say a particle is in a certain place

so it will be the same position if a particle rests on the x axis x time on the y axis

so a straight line parallel to the t axis it represents a particle at rest because displacement increases with time Does not change

so the particle is now at rest we may have a second case where the particle starts from the origin and as time goes by the particle increases its displacement which means it is moving along a straight line and the value of x continues to increase and if it continues However this xt curve if it is a straight line represents that the particle is moving at equal distances at equal intervals of time and

so call this motion a state of uniform motion. This means that the particle is traveling at a speed that is constant. We may have more complex speeds. For example, we see in the case of a car that is starting, so the speed can be more complex.

Then it moves at the same speed then after a while a pause is applied and it comes to rest

so if we usually want to plot xt curve for this kind of speed then what we get is t is equal to time 0 the car is at rest then it starts to increase its displacement And after some time it comes to the same speed and it continues to move at the same speed and at this point we say that a break is applied if a break is applied then the car will run slower now than before and finally come to rest but its x The component will continue to grow when we take a break here ap we see that the car slows down but it continues to move after a while when it is at rest then the x component or displacement does not change with time and ave The xt curve starts for a car and then moves at a uniform speed and then when it comes to rest it looks like this for a particle traveling which we say with some uniform speed from 0 to p and back to p to 0 how is this

There will be a xt curve for motion. See here. You can better understand that the particle stays at the beginning. It represents this state. Oh, it goes from 0 to p. And now what we understand is that x starts to decrease. The back to ax will come again and

so it represents a particle that is moving forward and then as the value of x starts to decrease we see it start to come down in the curve

so this is now another xt curve which we want is an important element here bec See in this description ome is how fast the position changes with time and we describe it as speed and if we include the directional aspect of motion we get what we call velocity and we will now see the precise measure of speed and velocity. First we define a term called average velocity and average velocity. This is equal to the displacement change which is divided by the change of time. Which is delta t

so the average velocity for delta t on delta x is equal to the average velocity we use a v sign with an over bar and it is equal to the change in displacement we can write it as $x_2 - x_1$ change the time change as $t_2 - t_1$ So now the definition of average velocity if we look at units of average velocity as units of average velocity.

This means that they will be units of length divided by units of time which means we can write units above l in which si units will be the other common units per meter that we have another unit which when we speak we speak of speed and velocity. Kilometers per hour , especially when we talk about vehicles or

aviation we can list them in kilometers per hour which you all need to understand that when you solve a problem and if data is given in different units we should uniform the unit and then the problem Solve You can't add kilometers to meters You can add kilometers to kilometers or meters to meters Now let's look at the graphical meaning of average velocity Let's take a look at some aspects of average velocity average velocity There is a direction and a dimension but when we look at the motion along a straight line the direction is indicated by the sign of displacement and our other to explain the direction There is no need for anything so it is indicated by the sign and it can be positive or negative if it is positive then we are moving with what we call positive x axis if it is negative then we are moving along the negative x axis so it gives us direction now come on Graphically let's see if we have the $x-t$ curve and this $x-t$ curve is given this way now we say this is the gap. Let the time interval t_1 and the particle displace at point p and the particle at point q now displace at x_2 and displace at p_2 be x_1 . So if we look at the average velocity in this interval from t_1 to t_2 . So Δt is equal to t_2 minus t_1 is the average velocity in this interval is equal to t_2 minus t_1 divided by x_2 is equal to x_2 minus x_1 if we see that it is x_2 so x_2 minus x_1 divided t_2 minutes now it will be nothing but If we draw a straight line pq then the slope of this line of pq line will be equal to the distance along the y axis which is Δx and the distance along the x axis which is Δt is the slope. The pq of the line gives us the average velocity $x-t$ The shape of the curve regardless of the average velocity will be given by the slope of the straight line Now the average velocity can be v times positive It can be negative or zero We can also see by the slope of the curve if θ makes the angle $x-t$ curve with the x axis. If this angle is sharp then the slope is positive and here v is positive if the $x-t$ curve looks like this. The slope of the slope forms this line here and here and when we draw it along the x axis we realize that this straight line is the angle θ which makes it with the positive T axis here it is thick it is more than 90° degrees so it represents the negative slope and hence the average velocity here Will be negative and if a particle is at rest it does not change its displacement so this is $x-t$ curve it represents zero average velocity because the particle is not moving at all now what we have said is this velocity v bar If we define average velocity then it involves displacement which means we see the net value x_2 minus x_1 where we talk about the concept of average velocity then what we do is divide the total length of the journey by time interval so when we mean average velocity When we calculate, it says that we calculate the total length, the path is not displacement, and that is what gives us the average speed. There will be meters per second but the dimensions may not be the same because the path length is always greater than or equal to displacement and so the average speed we get will always be greater than or equal to the average velocity and this is because if you go back one way displacement Will decrease where the distance will not decrease and the length of the path will not decrease so average speed now moves like this when we talk about average speed. What we mean by instantaneous velocity by name and name is how we define the velocity of a given moment. If we divide the displacement Δx by the Δt then we get the average velocity now if we measure our time interval. Δt We observe the change we make it from smallest to smallest and when we make it from smallest we finally tell when it comes close to this $\Delta t \rightarrow 0$. So we say it's going to the boundary $\Delta t \rightarrow 0$ then the amount we get is what we call

as v or it's called instantaneous velocity

so instantaneous velocity is the average velocity within the range the time you are considering this average is getting smaller and smaller What we say is that its rate changes with time. Let's try to look at the position

geometrically. If we look at the graph geometrically, what it looks like when we look at it. This is an x vs t graph. What we will see is that we focus on t is equal to t_1 and then we displace a sly short distance from t_1 . We cannot go in either a plus direction or a subtraction. So when the time interval reaches zero, do you know that it will come to ΔT by Δx , it will go to the tangent, or t of the slope of the x vs t curve, if the Δt comes close to zero, then we have All that exists is that the slope of the x vs t curve or the tangent of the x vs t curve gives us instantaneous velocity

so the t of the x vs t curve or the slope t gives a velocity equal to t Equals t is equal to one now you get here if we take instantaneous velocity measure then we call instantaneous motion and in this case when we talk instantaneous velocity and instantaneous velocity then velocity will be equal to velocity which may not be average velocity and average velocity But in case of instantaneous velocity and instantaneous motion we would have it so if we have to look at it graphically, if we have x vs t curve and x vs t curvature slope will give us velocity then instantaneous velocity is defined as instantaneous motion and here we understand that two Dimensions are equal where dimensions may not be equal when we speak of average velocity and average speed because we are only looking at motion at very short intervals. T is the delta around T

so these two dimensions must be the same It is true that instantaneous velocity for one d speed can be negative where instantaneous motion is always positive and its a practical application if we look at ah when we notice the speed of a speedometer in a car then we The reading I get is the instantaneous speed reading that the speedometer gives us as the car moves at every instantaneous speed. The speed of the car at that moment. Had it increased its speed then it was gone. The brakes were then applied between these speeds of uniform speed between us and then it stopped

so now here if I plot the value of instantaneous velocity in the same curve then what we get is that the instantaneous velocity will assume a shape during this period when the speed of the vehicle increases. It will start from rest and keep increasing speed then during this period when we have equal speed velocity is constant

so velocity is going to be constant which means it is constant with some value and then when break is applied during this period velocity starts decreasing and finally when When the rest comes at that time the velocity goes to zero and if it is at the same rate then is this velocity changing? We find that as it starts to decrease it comes to 0 and then the speed goes to 0 when the car is at rest. So the velocity time curve for the car that was started to look like this is what we have to do now we have velocity but as we have seen in this example velocity can also change with time it does not always need to be constant

so we define how velocity velocity The rate of change is changing

so we may not have the speed. Stay constant and actually as we see velocity can change as a function of time or as a function of distance and even bo or both but all we do is define the rate of change of velocity with time and we call it acceleration

so acceleration means how much velocity Acceleration is rapidly changing over time

so where we use the a sign for this and we can define two quantities we can

define the average acceleration at a time interval Δt which is equal to $t_2 - t_1$ and this average acceleration a_{avg} is equal to $\frac{v_2 - v_1}{\Delta t}$ where v_1 and v_2 are instantaneous velocities at times t_1 and t_2 respectively. The symbol we can use again is \bar{a} for averages.

so this is a bar and if we look at units then the unit of acceleration is $\frac{m}{s^2}$ and in SI unit it will be per square meter per second or if we are talking about large measurement for example vehicle measurement it can be per square kilometer now their expression is a unit of acceleration now we can also define instantaneous acceleration. This will be equal to $\frac{dv}{dt}$. Δt is going to zero at the limit of Δt and

so it is equal to the limit of calculus it will be equal to $\frac{dv}{dt}$. so instantaneous acceleration is the rate of change of velocity with time goes to $\Delta t \rightarrow 0$ and this is also derivative of velocity. Now to give a geometric explanation we understand that acceleration is the slope of the tangent of the $v-t$ curve again it is a vector quantity of acceleration and it is positive acceleration can be a vector it can be positive it can be zero now sometimes negative acceleration is also referred to as retardation and if retardation The term is spelled but it is assumed to be negative which means the acceleration decreases with time if we look at these accelerations in terms of our graph

so an $x-t$ curve if we If the curve of $x-t$ is at the top it represents a positive acceleration. If we have the $x-t$ curve at the bottom then it means that this point in this part is a downward curve. A straight line is if the $x-t$ curve is a straight line which will give us the velocity constant which means that the acceleration is zero

so if our $x-t$ curve is like this or $x-t$ curve which means the shape of a straight line $x-t$ curve refers to zero acceleration now if we calculate in terms of derivatives. And looking at these relationships between v but what we have shown is that the way we are describing velocity now will not use the word instantaneous in further discussion because when we talk about velocity and acceleration it will be assumed that instantaneous velocity and instantaneous acceleration only when we average. Speaking of average acceleration of velocity then we will use the term average otherwise it will be kept as instantaneous so what we have now is first as a function of t . We had x then we have $\frac{dx}{dt}$ by dx we now define it as velocity because it is a dimensional motion that we are not writing like this but in general case what we will do is we will use a vector symbol to represent these as vectors \vec{v} of course. We are not going to enter the vectors for the part but when we go to the next two dimensional and three dimensional motion we have to use the vector symbols

so what we have here is that for one dimensional motion we write $\frac{dx}{dt}$ by v and $\frac{dv}{dt}$ it is defined as acceleration. And

so $\frac{dv}{dt}$ since it is $\frac{dx}{dt}$ who have some idea differential calculus it also becomes the second derivative of x over time because $\frac{d}{dt} \left(\frac{dx}{dt} \right)$

so it is also written as $\frac{d^2x}{dt^2}$ if you know the differential calculus. You can understand otherwise you have to understand it. Now let's see some explanation of differential calculus

so $\frac{dx}{dt}$ is equal to v now $\frac{dv}{dt}$ any amount of derivative. Represents the slope

so when we see x is given as a function of t then $\frac{dx}{dt}$ is equal to v by dt

so it means that the slope in the $x-t$ curve gives you velocity now we try to

reverse it and the way we can reverse it means Everything is given here as a function of time

so all we can do is we can write it is equal to $dx = v dt$

so in some sense we are assuming either v is given to us as a constant or a function of time and here if we integrate it we What we get is now $\int dx$ is equal to $\int v dt$

so if I have $v = v(t)$ the curve under this $v(t)$ curve this area $\int_{t_1}^{t_2} v dt$ is represented by this integral from one to t_2 and $\int dx$ gives me $x_2 - x_1$ So the area below the $v(t)$ curve represents the displacement

so the bottom area $\int_{t_1}^{t_2} v dt$ the curve represents the displacement

so what we have seen is that the slope of the $x(t)$ curve gives us v where the area below the $v(t)$ curve gives us the displacement now We can proceed with the same thing with acceleration

so we have $dv = a dt$ is a

so here when we meet we will enter $\int dv = \int a dt$

so now again $\int a dt$ accelerates the slope of curve and area under acceleration and time curve Gives velocity or in this case it will be a change of velocity between two positions or between two periods

so now a slightly more complex situation arises if acceleration is known as a function of x which means that the acceleration we saw is dv/dt as a displacement function to us Is not given as a function of time

so what we can do in this case is $dv = a dx$ now what we can do is we can use the chain rule of difference because acceleration is given as a function of x

so what we do is $dv/dt = a(x)$ we want to write it in terms of x

so it can be written as $dv/dt = dv/dx \cdot dx/dt$ and $dx/dt = v$

so it becomes $v \cdot dv/dx = a(x)$ and what we understand from the product rules of difference is that it is no longer $d(v^2)/dx$ half of the square v^2 once you know v he will do these things very easily in differential formula but otherwise if you do not understand just look at this $v^2 dx/dx$ which is $2v \cdot dv/dx$ and half cancels we v^2 Quality dv/dx

so acceleration is known as a function of x

so we write $dt = dv/v \cdot dx/a$ or $dx = v^2 da/dv$ squared

so what we get is halved by dx of the square v^2 and we take the other side a

so we get d of the square v^2 Half of it is equal to one dx and when we combine the two sides we get the left side position 1 from position 2 to half of the square of v will be equal to the integral of adx and it will be half of the square of $v_2 - v_1$ square is relative to x Integral Integral Now we need to resort to this if acceleration is known as a function of x or we want to express acceleration as a function of x because you understand that there are three variables x, t and one here or and v are basically four variables and our relationship If $a(x)$ is the derivative of x with time velocity and the second derivative of the velocity of velocity with time is acceleration and here is how we are trying to present the acceleration related to $a(x)$ with x or how we use the acceleration as a function of x in x We find velocity as a function of because now we realize that some of you who have seen some mechanics we have a principle of work force where we call change of kinetic energy equal to work performed by force and this relationship which I have brought here actually plays a role. When we talk about change of momentum because if you look at it here, if I multiply both sides by mass, I don't introduce these ideas, I understand, but some of you may know what momentum is and what works for them. Others may appreciate when it comes to the definition of kinetic energy but here if I multiply both sides by m then the left hand side becomes $\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$ square which becomes a change in kinetic energy and if I mass

the right hand side Multiply, but it becomes $ma dx$. For those who have seen it, it comes from Newton's second law. Newton's law tells us that the sum of the external forces is equal to ma and when you take an integral of f relative to x for the motion of a straight line it gives us what we define as ball bar displacement

so we define the work as equal to the change of kinetic energy. Relationships which in some sense are seen as following from here

so here we have seen the general relationship of v and x Now there is a very special case which we call the case of uniform. Acceleration now means uniform acceleration. Acceleration constant. General acceleration itself may change over time. No energy is active but it becomes a very common situation when we sometimes apply it while moving the car. A constant acceleration can occur sometimes when it does not move with constant acceleration but when it is almost constant we assume it is constant and apply the formulas we get for the speed of a vehicle

so in practice it becomes important in case of situation and free fall The acceleration of a body falling freely near the surface of the earth can be considered as constant which we call acceleration due to gravity

so what we will do here is find out the relation between displacement x time.

Taken t and starting velocity v_0 is a final velocity v and acceleration which means that what we are assuming is that there is a particle which has a velocity equal to t zero v_{zero} it is going through uniform acceleration which means velocity of t after a period of time t is v and the displacement that occurs in this interval is x and the acceleration in this whole interval is equal to a and it is constant. This is the point that most people make errors about. They will only work when the acceleration is constant

so what we understand for constant acceleration is acceleration because it is not changing

so it will be equal to time interval v minus v_0 is divided by t

so acceleration v minus v_0 is divided by t Can be done and it gives us v is equal to v_0 plus

so we can think of it as the first formula which is the next velocity. Time will be given at the initial time The initial velocity and uniform acceleration will be multiplied by the time interval,

so now if we look at it as a curve below the vt curve, the velocity at t is equal to v_0 and at the next time t is equal to this velocity

so at this time v_0 Here the velocity of t is equal to v

so now what is the displacement in this space we know that displacement will be vt the area under the curve

so if we calculate this area then we understand that it is $v_0 t$

so the area of this rectangle $v_0 t$ it $v_0 t$ Zero

so this height v minus v_0 this t

so this total area of this triangle is equal to half times v minus v_0 is equal to zero times t

so the total area is equal to $v_0 t$ plus half times v minus v_0 and v minus v_0 from here we can write it like this it is halved in the square and it is equal to $v_0 t$

so this area which is displacement

so displacement is given by $v_0 t$ plus half in the square

so we can write it x is equal to $v_0 t$ plus half square If we follow this formula Extend which means that if we do not express things here in terms of acceleration then what we get is x is equal to $v_0 t$ plus $\frac{a}{2} t^2$ Now if we want to know the velocity in terms of a and x which means we want to subtract time then what we do So we know that time is v minus v_0 divided by a here

so we can write $x + v_0 t = \frac{1}{2} a t^2$ which is equal to $v - v_0 = a t$ and when we do that we get $x = \frac{1}{2} a t^2 + v_0 t$. Dividing the square by two a and it gives us $v = \frac{1}{2} a t + v_0$.

so we can take it as the basic formula

so if I re-list them for the sake of completeness then what we have is the first formula which was equal to $v = v_0 + a t$ and then we had $x = \frac{1}{2} a t^2 + v_0 t$. Now again we should remember that these are not sacred formulas they are only valid if the above formulas. Now the constant is equal to we understand that x is spatial. If x from zero is not equal to zero then $x - x_0$ instead of x because there will be displacement.

so if you choose a reference

so that the initial displacement of the display is not 0 then x will be here. $x - x_0$ has been replaced by zero.

so now when it comes to applying the problem you have to look at how much was given and which was not given. For example in this formula $x - x_0$ or displacement is missing. So $x - x_0$ will not happen anywhere in the problem that you use this formula directly to get the relationship between $v - v_0$ and t in the same way when we look at this formula in this formula the final velocity is not v . There it tells you a relation between displacement $x - x_0$ and t in this formula when we see that t is missing here t is not there.

so if t is not there we use this formula and

so ask for what depends on what is given in the problem. You have to look at it and then there is another catch that we have to be careful about and this is the sign of a because in all these formulas we have plus two axes or halves in the square.

so if we have acceleration the velocity continues to increase towards positive x . But a is positive and what we call acceleration but if the velocity decreases in the direction of positive x then the acceleration is negative and as we have said sometimes it is also referred to as constraint because there are some formulas where you have subtraction halves in square. Be careful when using $x - x_0 = v t + \frac{1}{2} a t^2$ we have plus half in the square but if the acceleration is negative then you place a with a negative sign and for negative acceleration it is $v t - \frac{1}{2} a t^2$. May be half.

These issues need to be kept in mind now because we have discussed a special case where the formulas where this constant acceleration is most often used are free fall. Free fall means a body under the influence of gravity near the earth. This type of body acceleration is constant as long as the body is not too far from the surface of the earth and it is given by a constant which we use as a sign and it will be from the body in the direction of acceleration towards the surface of the earth. Or what we can observe is that the value of g is 9.81

meters per second for the square problem which we will sometimes solve. This value is given as we take it. If you are given 9.8 meters per second square or sometimes for simplification, you can also take it as 10 meters per second square. It should specify which value of G is to be used. Very important, let us say that we are talking about a ball which is thrown up and

so we throw it at some speed and the ball goes up and what happens in the end because if it goes up like this then let it be said from the surface of the earth window we throw. This is the earth

so because the acceleration of this ball will be from the ball to the earth it means it is downwards

so if the ball is thrown upwards with velocity v_0 then it will feel negative acceleration

so this velocity will start descending and finally a point will come where
What happens when this velocity becomes zero? This ball will start now because
its velocity is zero it only feels acceleration due to gravity
so it will start falling down and it will start falling down till it hits the
surface of the earth

so now here you have distance The out sign has to be chosen

so for example if someone chooses it as positive y now what we call as x here
becomes y because this is the direction of motion

so if somebody chooses y as positive upwards

so in this case because your y is like this then acceleration for free fall
will now be minus g and negative signs now will represent a direction which is
downward

so here negative sign represents a downward direction and a positive sign
represents an upward direction

so if you solve a problem you get the value of y or the displacement as
positive that means the final value is the body is at a higher location than
from where it started whereas if y is negative in an answer that means the
location is lower than where it started okay and now for the same problem
another student can choose do the same problem taking y downwards now if you
choose y as downwards then the acceleration will now be plus g because it is
in the direction of motion and here now if you get a positive answer for
displacement that means you are at a lower position than where you started with
so now in the next class we will continue from here and we will take up some
examples of free fall and examples of constant acceleration and we will also
wind up this discussion on the equations of motion by studying
relative velocity and relative acceleration and solve problems of those types
you