A line is such that its segment between the straight lines $5x - y + 4 = 0$ and $3x + 4y - 4 = 0$ is bisected at the point $(1, 5)$. Obtain its equation.

A $107x + y - 92 = 0$

B $17x - 3y + 92 = 0$

C $10x + 3y + 92 = 0$

D $107x - 3y - 92 = 0$

Correct option is D)

Given lines are,

$5x - y + 4 = 0$ ...... (1)

$3x + 4y - 4 = 0$ ...... (2)

Let AB be the segment between the lines (1) and (2) and point P $(1, 5)$ be the mid-point of AB.

Let the points be $A(\alpha_1, \beta_1)$.

It is given that the line segment AB is bisected at the point P $(1, 5)$. Therefore,

$$\Rightarrow (1, 5) = \left(\frac{\alpha_1 + \alpha_2}{2}, \frac{\beta_1 + \beta_2}{2}\right)$$

So,

$$\frac{\alpha_1 + \alpha_2}{2} = 1$$

$$\alpha_1 + \alpha_2 = 2$$

$\alpha_2 = 2 - \alpha_1$ ...... (3)

Also,

$$\frac{\beta_1 + \beta_2}{2} = 5$$

$$\beta_1 + \beta_2 = 10$$

$\beta_2 = 10 - \beta_1$ ...... (4)

Point A and B lie on lines (1) and (2), respectively. Therefore, from lines (1) and (2), we have

$5\alpha_1 - \beta_1 + 4 = 0$ ...... (5)

$3\alpha_2 + 4\beta_2 - 4 = 0$ .... (6)

Now, from equations (3) and (4) and equations (5) and (6), we have,

$3(2-\alpha_1)+4(10-\beta_1)-4=0$

$6-3\alpha_1+40-4\beta_1-4=0$

$-3\alpha_1-4\beta_1+42=0$

$3\alpha_1+4\beta_1-42=0$ ...... (7)

Now, from equations (5) and (7), we have

$\alpha_1=\frac{26}{23}; \beta_1=\frac{222}{23}$

Thus, the line is passing through A $\left(\frac{26}{23}; \frac{222}{23}\right)$ and P $(1, 5)$. Therefore, the equation of the line is,

$$y-5=\frac{\frac{222}{23}-5}{\frac{26}{23}-1}(x-1)$$

$$y-5=\frac{\frac{222-115}{23}}{\frac{26-23}{23}}(x-1)$$

$$y-5=\frac{107}{3}(x-1)$$

$$3y-15=107x-107$$

$$107x-3y-92=0$$

Hence, this is the required equation.