A variable straight line passes through the points of intersection of the lines, x + 2y = 1 and 2x − y = 1 and meets the co-ordinate axes in A and B. The locus of the middle point of AB is

- **A** $x + 3y - 10xy = 0$

- **B** $x - 3y - 10xy = 0$

- **C** $x + 3y + 10xy = 0$

- **D** None

Correct option is A)

Let the equation of any line passing through the point of intersection of the given line be

$$(x + 2y - 1) + a(2x - y - 1) = 0$$

Reducing the equation to its intercept form

$$\frac{x(1 + 2a)}{(1 + a)} + \frac{y(2 - 1)}{(1 + a)} = 1$$

Therefore coordinates of A and B, where this line meets the coordinate axie respectively.

$$A = \left(\frac{1 + a}{1 + 2a}, 0 \right) \text{ on x-axis}$$

$$B = \left(0, \frac{1 + a}{2 - a} \right) \text{ on y-axis}$$

Mid point of AB $= \left(\frac{1 + a}{2 + 4a}, \frac{1 + a}{4 - 2a} \right)$

Now we find the locus of this point by eliminating a between the two expressions

$$x = \frac{1 + a}{2 + 4a}$$

$$y = \frac{1 + a}{4 - 2a}$$

$$y = \frac{x}{10x - 3}$$

$$x = 10xy - 3y$$

$$x + 3y - 10xy = 0$$