

Conic Section: Ellipse - Class XI

Past Year JEE Questions

Questions

Question: 01

Let e_1 and e_2 be the eccentricities of the ellipse,

$$\frac{x^2}{25} + \frac{y^2}{b^2} = 1 \quad (b < 5) \text{ and the hyperbola,}$$

$$\frac{x^2}{16} - \frac{y^2}{b^2} = 1 \text{ respectively satisfying } e_1 e_2 = 1. \text{ If } \alpha$$

and β are the distances between the foci of the

ellipse and the foci of the hyperbola

respectively, then the ordered pair (α, β) is equal to :

A. (8, 10)

B. (8, 12)

C. $(\frac{24}{5}, 10)$

D. $(\frac{20}{5}, 12)$

Solutions

Solution: 01

Explanation

For ellipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1 \quad (b < 5)$

Let e_1 is eccentricity of ellipse

$$\therefore b^2 = 25(1 - e_1^2) \dots\dots(1)$$

Again for hyperbola

$$\frac{x^2}{16} - \frac{y^2}{b^2} = 1$$

Let e_2 is eccentricity of hyperbola.

$$\therefore b^2 = 16(e_2^2 - 1) \dots\dots(2)$$

by (1) & (2)

$$25(1 - e_1^2) = 16(e_2^2 - 1)$$

Now $e_1 \cdot e_2 = 1$ (given)

$$\therefore 25(1 - e_1^2) = 16\left(\frac{1 - e_1^4}{e_1^2}\right)$$

$$\text{or } e_1 = \frac{4}{5} \therefore e_2 = \frac{5}{4}$$

Now distance between foci is $2ae$

$$\therefore \text{distance for ellipse} = 2 \times 5 \times \frac{4}{5} = 8 = \alpha$$

$$\text{distance for hyperbola} = 2 \times 4 \times \frac{5}{4} = 10 = \beta$$

$$\therefore (\alpha, \beta) \equiv (8, 10)$$