

Conic Section: Ellipse - Class XI

Past Year JEE Questions

Questions

Question: 01

Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) be a given ellipse, length of whose latus rectum is 10. If its eccentricity is the maximum value of the function,

$\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to

- A. 145
- B. 126
- C. 135
- D. 116

Solutions

Solution: 01

Explanation

Given ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$)

Length of latus rectum = $\frac{2b^2}{a} = 10$

$$\phi(t) = \frac{5}{12} + t - t^2$$

$$= \frac{8}{12} - \left(t - \frac{1}{2}\right)^2$$

$$\therefore \phi(t)_{\max} = \frac{8}{12} = \frac{2}{3}$$

$$\therefore \text{eccentricity } (e) = \frac{2}{3}$$

$$\text{Also, } e^2 = 1 - \frac{b^2}{a^2}$$

$$\Rightarrow \frac{4}{9} = 1 - \frac{b^2}{a^2}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{5}{9}$$

$$\Rightarrow \frac{b^2}{a} \times \frac{1}{a} = \frac{5}{9}$$

$$\Rightarrow \frac{5}{a} = \frac{5}{9}$$

$$\Rightarrow a = 9$$

$$\therefore b^2 = 5 \times 9 = 45$$

$$\therefore a^2 + b^2 = 81 + 45 = 126$$