

Exemplar Problem

Conic Section

14. Find the equation of ellipse whose eccentricity is $\frac{2}{3}$, latusrectum is 5 and the center is $(0, 0)$.

Ans:

Given

$$e = \frac{2}{3}$$

Latusrectum = 5.

Center of the circle i.e., $(h, k) = (0, 0)$

$$e = \frac{2}{3} \text{ and latusrectum} = 5 \text{ (given)}$$

$$\text{i.e., } \frac{2b^2}{a} = 5$$

$$\Rightarrow b^2 = \frac{5a}{2}$$

$$\therefore b^2 = a^2 (1 - e^2)$$

$$\Rightarrow \frac{5a}{2} = a^2 \left(1 - \frac{4}{9}\right)$$

$$\Rightarrow \frac{5}{2} = \frac{5a}{9}$$

$$\Rightarrow a = \frac{9}{2}$$

$$\Rightarrow a^2 = \frac{81}{4}$$

$$b^2 = \frac{5 \times 9}{2 \times 2} = \frac{45}{4}$$

So, the required equation of ellipse is $\frac{4x^2}{81} + \frac{4y^2}{45} = 1$