13. If the eccentricity of an ellipse is $\dfrac{5}{8}$ and the distance between its foci is 10, then find the latus rectum of the ellipse.

Ans:

Given

Eccentricity $= \dfrac{5}{8}$

Distance between its foci $= 10$

Eccentricity $= \dfrac{5}{8}$ i.e., $e = \dfrac{5}{8}$

Let equation of the ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

Since, the foci of the ellipse is $(\pm ae, 0)$,

$\therefore$ Distance between foci $= \sqrt{(ae + ae)^2}$

$\Rightarrow 2\sqrt{a^2 e^2} = 10$ (distance between its foci $= 10$)

$\Rightarrow \sqrt{a^2 e^2} = 5$

$\Rightarrow a^2 e^2 = 25$

$\Rightarrow a^2 = \dfrac{25 \times 64}{25}$

$\therefore a = 8$

$\Rightarrow b^2 = a^2 \left(1 - e^2\right)$

$\Rightarrow b^2 = 64 \left(1 - \dfrac{25}{64}\right)$

$\Rightarrow b^2 = 64 \left(\dfrac{64 - 25}{64}\right)$

$b^2 = 39$

Length of latus rectum of ellipse$= \dfrac{2b^2}{a} = 2\left(\dfrac{39}{8}\right) = \dfrac{39}{4}$