Past Year JEE Questions

Questions

Quetion: 01

The locus of the vertices of the family of parabolas

$y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$ is

A. $xy = \frac{105}{64}$

B. $xy = \frac{3}{4}$

C. $xy = \frac{35}{16}$

D. $xy = \frac{64}{105}$

Solutions

Solution: 01

Explanation

Given parabola is $y = \frac{a^3x^2}{3} + \frac{a^2x}{2} - 2a$

$\Rightarrow y = \frac{a^3}{3}\left(x^3 + \frac{3}{2a} + x + \frac{9}{16a^2}\right) - \frac{3a}{16} - 2a$

$\Rightarrow y + \frac{35a}{16} = \frac{a^3}{3}\left(x + \frac{3}{4a}\right)^2$

$\therefore$ Vertex of parabola is $\left(\frac{-3}{4a}, \frac{-35a}{16}\right)$

To find locus of this vertex,

$x = \frac{-3}{4a}$ and $y = \frac{-35a}{16}$

$\Rightarrow a = \frac{-3}{4x}$ and $a = -\frac{16y}{35}$

$\Rightarrow \frac{-3}{4x} = \frac{-16y}{35} \Rightarrow 64xy = 105$

$\Rightarrow xy = \frac{105}{64}$ which is the required locus.