28.Find the equation of each of the following parabolas

(a) Directrix $x = 0$, focus at $(6, 0)$

Ans: Let there be a point $P\ (h, k)$ on the parabola. According to the definition of a parabola: -

Distance between the point P and the focus $=$ Perpendicular distance between the point and the directrix.

Using the distance formula the distance between P and the focus is $\sqrt{(h-6)^2 + (k-0)^2}$.

The perpendicular distance from the point P to the line $x = 0$ is $\left|\frac{(h \times 1) + (0 \times k) + 0}{\sqrt{(1)^2 + (0)^2}}\right|$

$$\Rightarrow \sqrt{(h-6)^2 + (k-0)^2} = \left|\frac{(h \times 1) + (0 \times k) + 0}{\sqrt{(1)^2 + (0)^2}}\right|$$

$$\Rightarrow \sqrt{(h-6)^2 + k^2} = |h|$$

Squaring both the sides and simplifying,

$$\Rightarrow (h-6)^2 + k^2 = h^2$$

$$\Rightarrow h^2 + 36 - 12h + k^2 = h^2$$

$$\Rightarrow k^2 = 12\,(h-3)$$

Replacing h with x and k with y gives the equation of the parabola as,

$$\Rightarrow y^2 = 12\,(x-3)$$

(b) Vertex at $(0, 4)$, focus at $(0, 2)$

Ans:

Here the vertex is at $(0, 4)$ and the focus is at $(0, 2)$ that means the equation of the parabola is of the form $(x-h)^2 = 4a(y-k)$ (known as the vertex form), where (h, k) is the vertex and $(h, a+k)$ is the focus. So the equation of the parabola is,

$\Rightarrow (x-0)^2 = 4a(y-4)$

$\Rightarrow x^2 = 4a(y-4)$

Equating the y coordinate of focus,

$\Rightarrow a + k = 2$

$\Rightarrow a + 4 = 2$

$\Rightarrow a = -2$

Therefore, the equation of the parabola is $x^2 = -8(y-4)$.

(c) Focus at $(-1, -2)$, directrix $x - 2y + 3 = 0$

Ans:

Let there be a point $P(h, k)$ on the parabola.

The distance between P and the focus is $\sqrt{(h-(-1))^2 + (k-(-2))^2}$.

The perpendicular distance from the point P to the line $x - 2y + 3 = 0$is $\left|\frac{(h \times 1) + ((-2) \times k) + 3}{\sqrt{(1)^2 + (-2)^2}}\right|$

$$\Rightarrow \sqrt{(h-(-1))^2+(k-(-2))^2} = \left|\frac{(h \times 1)+((-2) \times k)+3}{\sqrt{(1)^2+(-2)^2}}\right|$$

$$\Rightarrow \sqrt{(h+1)^2+(k+2)^2} = \left|\frac{h-2k+3}{\sqrt{5}}\right|$$

Squaring both the sides and simplifying,

$$\Rightarrow (h+1)^2+(k+2)^2 = \frac{(h-2k+3)^2}{5}$$

$$\Rightarrow 5h^2+5+10h+5k^2+20+20k = h^2+4k^2+9-4kh-12k+6h$$

$$\Rightarrow 4h^2+k^2+4h+32k+4kh+16 = 0$$

Replacing h with x and k with y gives the equation of the parabola as,

$$\Rightarrow 4x^2+y^2+4x+32y+4xy+16 = 0$$