

Exemplar Problem

Conic Section

17. Find the length of the line segment joining the vertex of the parabola $y^2 = 4ax$ and a point on the parabola where the line-segment makes an angle θ to the x -axis.

Ans:

Given

Equation of the parabola $y^2 = 4ax$.

$$y^2 = 4ax \text{ (given)}$$

Let the coordinates of any point P on the parabola be $(at^2, 2at)$

$$\text{In } \triangle POA, \tan \theta = \frac{2at}{at^2} = \frac{2}{t}$$

$$\tan \theta = \frac{2}{t}$$

$$\Rightarrow t = 2 \cot \theta$$

$$\therefore \text{Length of } OP = \sqrt{(0 - at^2)^2 + (0 - 2at)^2}$$

$$= \sqrt{a^2t^4 + 4a^2t^2}$$

$$= at\sqrt{t^2 + 4}$$

$$= 2a \cot \theta \sqrt{4\cot^2 \theta + 4}$$

$$= 4a \cot \theta \sqrt{1 + \cot^2 \theta}$$

$$= 4a \cot \theta \cdot \operatorname{cosec} \theta$$

$$= \frac{4a \cos \theta}{\sin \theta} \cdot \frac{1}{\sin \theta}$$

$$= \frac{4a \cos \theta}{\sin^2 \theta}$$