Hyperbola - Class XI

Related Questions with Solutions

Questions

Quetion: 01

If a rectangular hyperbola have the equation, $xy = c^2$, find the locus of the middle points of the chords of constant length $2\ \mathrm{d}$ is.

A. $\left(x^2+y^2\right)\left(xy-c^2\right)=d^2xy$

B. $\left(x^2+y^2\right)\left(xy+c^2\right)=dxy$

C. $\left(x^2+y^2\right)\left(xy+c^2\right)=d^2xy$

D. $\left(x^2-y^2\right)\left(xy+c^2\right)=dxy$

Solutions

Solution: 01

$xy = c^2$

$\mathrm{P}\left(\mathrm{x}_1, \frac{\mathrm{c}^2}{\mathrm{x}_1}\right), \mathrm{Q}\left(\mathrm{x}_2, \frac{\mathrm{c}^2}{\mathrm{x}_2}\right)$ let midpoint be $R(h,k)$

$\mathrm{h} = \frac{\mathrm{x}_1+\mathrm{x}_2}{2}, \mathrm{k} = \frac{\mathrm{c}^2}{2}\left(\frac{\mathrm{x}_1+\mathrm{x}_2}{\mathrm{x}_1\times\mathrm{x}_2}\right)$

$\mathrm{k} = \frac{\mathrm{c}^2(\ \mathrm{h})}{\mathrm{x}_1\mathrm{x}_2}$

$\mathrm{x}_1\mathrm{x}_2 = \frac{\mathrm{hc}^2}{\mathrm{k}}$

$2d = \sqrt{\left(x_2-x_1\right)^2 + c^4\left(\frac{1}{x_2}-\frac{1}{x_1}\right)^2}$

$2d = \sqrt{1+\frac{c^4}{x_1x_2^2}}\left(x_2-x_1\right)$

$4d^2 = \left(1+\frac{c^4}{\left(x_1x_2\right)^2}\right)\left(4h^2-\frac{4hc^2}{k}\right)$

$4\ \mathrm{d}^2 = \left(\frac{\mathrm{k}^2+\mathrm{h}^2}{\mathrm{h}^2}\right)4\ \mathrm{h}\left(\frac{\mathrm{hk}-\mathrm{c}^2}{\mathrm{k}}\right)$

$xyd^2 = \left(x^2+y^2\right)\left(xy-c^2\right)$

Correct Options

Answer:01

Correct Options: A