32.Find the equation of the hyperbola with

(a) Vertices $(\pm 5, 0)$**, foci** $(\pm 7, 0)$

Ans:

The vertices and the foci both lies on the $x-$ axis so the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

The vertex is given as $(\pm a, 0)$, so on comparing $a = 5$.

The foci is given as $(\pm ae, 0)$, so on comparing $ae = 7$. Here $e = \sqrt{1 + \frac{b^2}{a^2}}$ is eccentricity. On squaring both the sides,

$\Rightarrow a^2e^2 = 49$

$\Rightarrow a^2 + b^2 = 49$

$\Rightarrow 25 + b^2 = 49$

$\Rightarrow b^2 = 24$

Therefore, the equation of the hyperbola is $\left(\frac{x^2}{25} - \frac{y^2}{24} = 1\right)$.

(b) Vertex at $(0, \pm 7)$**,** $e = \frac{4}{3}$

Ans:

Here the vertex lies on the y axis so the hyperbola is of the form $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$.

The vertices is given as $(0, \pm b)$, so on comparing $b=7$.

The eccentricity is given as $e=\sqrt{1+\frac{a^2}{b^2}}$. Since $e=\frac{4}{3}$, so on squaring both the sides,

$$\Rightarrow e^2=\frac{16}{9}$$

$$\Rightarrow 1+\frac{a^2}{b^2}=\frac{16}{9}$$

$$\Rightarrow \frac{a^2}{49}=\frac{7}{9}$$

$$\Rightarrow a^2=\frac{343}{9}$$

Therefore, the equation of the hyperbola is $\left(\frac{y^2}{49}-\frac{9x^2}{343}=1\right)$.

(c) Foci at $\left(0, \pm\sqrt{10}\right)$, passing through $(2,3)$

Ans:

Here the foci lies on the y axis so the hyperbola is of the form $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$.

The foci is given as $(0, \pm be)$, so on comparing $be=\sqrt{10}$.

The eccentricity is given as $e=\sqrt{1+\frac{a^2}{b^2}}$. So on squaring both the sides,

$$\Rightarrow b^2e^2=10$$

$\Rightarrow a^2 + b^2 = 10$ ... (1)

Since, the hyperbola passes through the point $(2, 3)$, so this point will satisfy the equation.

$$\Rightarrow \frac{9}{b^2} - \frac{4}{a^2} = 1$$

$\Rightarrow 9a^2 - 4b^2 = a^2b^2$ ... (2)

Solving equations (1) and (2) gives $a^2 = 5$and $b^2 = 5$.

Therefore, the equation of the hyperbola is $\left(\frac{y^2}{5} - \frac{x^2}{5} = 1\right)$.