

Exemplar Problem

Conic Section

40. The locus of the point of intersection of lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different value of k is a hyperbola whose eccentricity is 2.

Ans:

Given The locus of the point of intersection of lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$ for different value of k is a hyperbola whose eccentricity is 2.

$$\text{From the line } \sqrt{3}x - y - 4\sqrt{3}k = 0, k = \frac{\sqrt{3}x - y}{4\sqrt{3}}.$$

$$\text{And from the line } \sqrt{3}kx + ky - 4\sqrt{3} = 0, k = \frac{4\sqrt{3}}{\sqrt{3}x + y}.$$

Equate the values of k .

$$\Rightarrow \frac{\sqrt{3}x - y}{4\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{3}x + y}$$

$$\Rightarrow 3x^2 - y^2 = 48$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{48} = 1 \text{ which is an equation of a hyperbola.}$$

$$\Rightarrow a^2 = 16 \text{ and } b^2 = 48$$

$$\Rightarrow e^2 = 1 + \frac{48}{16}$$

$$\Rightarrow e^2 = 1 + 3$$

$$\Rightarrow e^2 = 4$$

$$\Rightarrow e = 2$$

Hence, the given statement is true.